A Large Creditor in Contagious Liquidity Crises

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This paper presents a contagious liquidity crises model for nonfinancial firms in which a large creditor influences the extent to which the contagion spreads across firms. We consider a sequential framework where two rollover games occur one after another. A liquidity crisis in one firm triggers a liquidity crisis in another firm through changes in the risk attitudes of creditors from the wealth effect. We show that the presence of a large creditor with a sufficient asset size reduces the contagion effect. Moreover, a concentration of a large creditor's loan portfolio towards the former firm increases the contagion effect. (JEL G01, G33, D82, D83)

Key Words: Contagion; Large Creditor; Liquidity Crisis; Global Game; Wealth Effect; Coordination Failure

1. INTRODUCTION

Large creditors appear to play a role in almost every financial crisis. Among the most important large players in credit markets, commercial banks stand out as having superior positions in terms of fund size. In fact, many countries have a bank-centered financial system in which banks play a critical role in providing funds to corporations. As the so-called Asian *Flu* generated severe financial crises across most Asian countries via a contagion effect in the late 1990s, many large South Korean firms went bankrupt as the liquidity crises spread from one firm to another. Kia Motors (Korea's eighth largest conglomerate) and Jinro (Korea's nineteenth largest conglomerate and also the largest liquor group) are two such large firms that went bankrupt during the crises. These firms had one common large creditor, the Korea First Bank, through a unique bank—enterprise relationship based on government credit control, namely, "the principal transactions bank system."[2] This brings us to the question of how the presence of large creditors influences the stability of a financial system.

In this regard, we investigate how the presence of a large creditor influences the magnitude of the contagion of a liquidity crisis between two unrelated nonfinancial firms. Following Corsetti, Dasgupta, Morris, and Shin [2004], we focus on the role of a large creditor in a coordination problem interacting with a continuum of small creditors. Our study extends their model to a two-firm model of contagion in which each firm sequentially faces a coordination problem among creditors. The creditors—consisting of a large creditor and a continuum of small creditors—have their own private information about each firm and decide whether or not to roll over loans in each firm's coordination problem with other creditors based on their own

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[2] For more information on the principal transactions bank system, see Nam [1996].

information. This study analyzes the effects of a large creditor's characteristics such as fund size and investment portfolio on the contagion of a liquidity crisis between two firms.

This paper focuses on "self-fulfilling crises," that is, the crises resulting from a coordination failure among creditors from their pessimistic beliefs about outcomes. This self-fulfilling nature plays an important role in explaining the liquidity crises of corporations; however, considering a crisis to be self-fulfilling often leads to multiple equilibrium outcomes and makes it difficult to characterize the cases where crises emerge and quantify the contagion effect. By employing the global game method introduced by Carlsson and van Damme [1993] and developed by Morris and Shin [1998], we can obtain unique equilibrium outcomes for the coordination problem of each firm and thus capture the contagion effect by which a liquidity crisis in one firm influences the likelihood of a liquidity crisis in another firm. Specifically, the global game setting between firms and creditors in our model is similar to that in Morris and Shin [2004]. The rollover game among creditors considered by Morris and Shin [2004] leads to a unique equilibrium outcome as well as a uniquely determined probability of failure, given that the creditors' private information about the fundamentals is sufficiently precise. Bruche [2011] develops a continuous-time version of Morris and Shin's [2004] model. However, these studies address the rollover game among creditors of the same type, which corresponds to the case where there are no large creditors.

In contrast to most studies in the literature employing the global game setting, we consider the coordination problem of creditors with asymmetries in fund size. Corsetti, Dasgupta, Morris, and Shin [2004] develop a currency crises model with one large trader and a continuum of small traders and examine how the presence of a large trader affects the likelihood of a currency crisis emerging. They show that the presence of the large creditor has a positive impact on the occurrence of a currency crisis. Bannier [2005] highlights the role of market sentiment in the impact a large trader has on a currency crisis. Takeda and Takeda [2008] investigate the role of large creditors in determining the price of corporate bonds in line with Morris and Shin [2004]. However, these studies consider only one coordination problem between players and do not address the question of what affects the contagion of a crisis, which is the main interest of this paper. In this regard, we extend Corsetti, Dasgupta, Morris, and Shin [2004] to a two-firm case with a common pool of creditors consisting of one large creditor and a continuum of small creditors. In doing so, we provide a better understanding of the effects of a large creditor on the stability of the financial chain of an economy.

Following Goldstein and Pauzner [2004], we focus on the wealth effect of creditors' risk attitudes as a mechanism triggering the contagion of a liquidity crisis.[3] Goldstein and Pauzner [2004] consider the contagion between two countries sharing the same group of investors. In their model, a decrease in investor wealth due to a crisis in one country makes them more cautious against the strategic risks associated with the behavior of other investors, increasing the incentives to withdraw their investments in the other country. Since we consider the contagion between two firms sharing a group of creditors, Goldstein and Pauzner's [2004] contagion mechanism fits well with our setting. Indeed, most of the participants in credit markets are banks and institutional investors. When the value of one asset in their investment

[3] Note that a series of studies on financial contagion focus on the investors' learning behavior; for example, Angeletos, Hellwig, and Pavan [2007], Manz [2010], Oh [2013, 2015], and Taketa [2004].

portfolio decreases from an idiosyncratic shock, they are forced to sell their other assets in the portfolio to meet their margin calls or cash requirements. This pattern of creditor behavior is consistent with the portfolio channel of contagion.

One particular type of large creditors diversifying their assets has been extensively addressed in the literature of financial contagion: financial institutions. For example, Allen and Gale [2000] and Dasgupta [2004] investigate how interbank dealings (e.g., cross-holdings of deposits) can lead to a banking crisis. Wagner [2010] examines how the diversification of risk in financial institutions can make a systemic crisis more likely. Allen, Babus, and Carletti [2012] show that asset commonality and short-term debts of banks generate excessive systemic risk. In contrast to these studies focusing on the contagion of crises facing large creditors (e.g., banks), we highlight the influence of large creditors on the contagion of debt crises between non-financial firms, which has not been explicitly addressed in the literature.

For the contagion setting, we consider a sequential framework in which the creditors' rollover game for firm 1 takes place before that for firm 2. For each rollover game, the creditors decide whether or not to roll over loans until the maturity date from their noisy signals about the firm's fundamentals.[4] After the rollover game for firm 1, the creditors observe whether a liquidity crisis occurs for firm 1 as well as the firm's actual state of fundamentals. A liquidity crisis in firm 1 would make the creditors more cautious against rollover risk in the rollover game that follow for firm 2 through the wealth effect. This leads to the "contagion" of a liquidity crisis from firm 1 to firm 2. Note that the liquidity crisis would not have occurred in firm 2 if there were no liquidity crisis in firm 1. Furthermore, this paper refers to the increased probability of firm 2 having a liquidity crisis as a result of contagion as the "contagion effect" for firm 2's liquidity crisis. We distinguish two types of contagion effects based on the role of the large creditor in the occurrence of a liquidity crisis. That is,

- Conditioned on the large creditor not rolling over his loans for firm 2, the contagion effect is defined as the (positive) difference in firm 2's threshold fundamental strength[5] between (1) a situation where firm 1 suffered a liquidity crisis and (2) a situation where firm 1 did not suffer a liquidity crisis;
- Conditioned on the large creditor rolling over his loans for firm 2, the contagion effect is defined as the (positive) difference in firm 2's threshold fundamental strength between (1) a situation where firm 1 suffered a liquidity crisis and (2) a situation where firm 1 did not suffer a liquidity crisis.

After demonstrating the effects of contagion of a liquidity crisis from firm 1 to firm 2, we analyze the effect of asset allocation across creditors and firms on the contagion effects. The impact of the large creditor's size on the contagion effects is negative: As the size of the large creditor becomes sufficiently large, the contagion effects are less than that when there is no large creditor, because the rollover coordination problem that small creditors are faced with becomes irrelevant.

[4] As noted by Allen, Babus, and Carletti [2012], the presence of an interim decision stage in this type of short-term financing entails rollover risk, possibly leading to a liquidity crisis in a firm. Nevertheless, creditors may prefer short-term financing instead of long-term financing for several reasons: Short-term financing reduces the asymmetric information problem in credit markets (e.g., Diamond, 1991; Flannery, 1986) and disciplines the behavior of managers for alignment with creditors' interests (e.g., Calomiris & Kahn, 1991; Diamond & Rajan, 2001).

[5] The threshold fundamental strength is the (endogenous) minimum firm fundamental value below which the firm suffers a liquidity crisis.

This argument gives a rather different insight compared to the findings of previous studies on financial contagion, which typically address the effects of a large trader on a currency crisis. Such studies find that the larger the large trader, the less stable is the financial system of the economy (e.g., Corsetti, Dasgupta, Morris & Shin, 2004; Corsetti, Pesenti & Roubini, 2002). Moreover, we examine the role of the large creditor's investment portfolio across the two firms in the contagion effects. Given that the large creditor is sufficiently large, the contagion effects increase as the large creditor shifts his loan portfolio toward firm 1, which is in line with Oh's [2013] results on learning-based contagion.

We further consider the effects of other variables related to creditor characteristics and payoff structure on the contagion effects: Contagion effects reach a peak at some intermediate level of the collateral value and approach zero at either extremely high or extremely low levels of the value. To investigate the influence of creditors' utility functions (i.e., risk attitudes) on the contagion effects, we parameterize them using extended power functions and find that the contagion effects decrease to zero as the large creditor and small creditors' risk aversion parameters respectively become either extremely high or extremely low. Noteworthy is that the accuracy of private information of the large creditor or small creditors does not influence the contagion effects by itself. Large and small creditors' risk aversion determines the role of the creditors' accurate private information in the contagion.

The rest of this paper is organized as follows: Section 2 presents the model. Section 3 solves the equilibrium for firms 1 and 2. Section 4 first defines the contagion effects of a liquidity crisis between two firms and then discusses the role of a large creditor in a contagious liquidity crisis with comparative statics analyses. Section 5 concludes the paper, and the Appendix presents the proofs and derivations of the paper.

2. THE MODEL

There are two firms in our model, firm 1 and firm 2—each firm having an investment project requiring funds to invest. The two firms share a common pool of creditors consisting of one large creditor and a continuum of small creditors such that the stake of any individual small creditor in the whole is negligible. The sequence of events (see FIG. 1) proceeds as follows: First, the creditors lend money to both firms. Second, firm 1's state of fundamentals (θ_1) is realized. Third, the creditors receive private signals for the fundamentals of firm 1: y_1 for the large creditor and x_{1j} for a typical small creditor j. Fourth, the creditors decide whether or not to roll over loans for firm 1. Fifth, as a result of the rollover game for firm 1, all the creditors get to know whether the firm's project is successful or not as well as the exact realization of the firm's fundamentals. Sixth, the state of firm 2's fundamentals (θ_2) is realized. Seventh, the creditors receive private signals for the state of firm 2's fundamentals: y_2 for the large creditor and x_{2j} for a small creditor j. Eighth, the creditors decide whether or not to roll over loans for firm 2. Ninth, firm 2's exact realization of fundamentals and project results are known to all creditors.

We assume that the investment projects of both firms are financed only through loans provided by creditors. The size of the large creditor's loans to both firms is λ whereas that of the small creditors' loans sums up to $1-\lambda$, where $0 < \lambda < 1$. Each creditor has a loan portfolio diversified across both firms. The large creditor invests a proportion α of his total funds in firm 1 and the rest in firm 2, where $0 < \alpha < 1$.

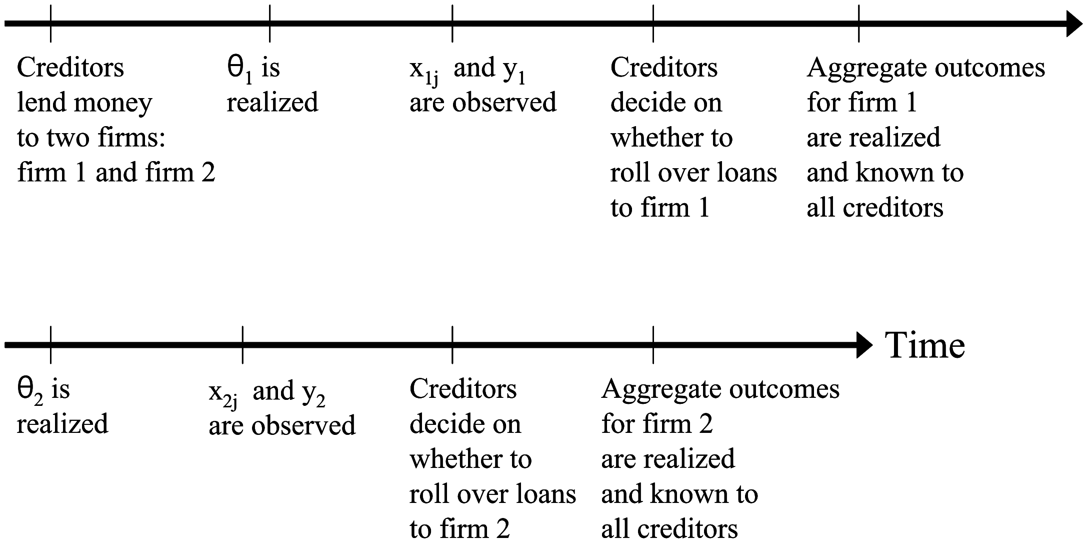

FIG. 1 Timeline

On the other hand, each small creditor invests a proportion β of his funds in firm 1 and the rest in firm 2, where $0 < \beta < 1$. That is, firm 1 receives $\alpha\lambda + \beta(1-\lambda)$ in funding, while firm 2 receives $(1-\alpha)\lambda + (1-\beta)(1-\lambda)$.

The state of firm i's fundamentals is denoted by θ_i, where $i = 1, 2$. Here, θ_i can be interpreted as the measure of firm i's ability to meet the short-term claims of creditors, where a high θ_i value indicates better fundamentals. After each firm i raises its fund requirements from the creditors and invests them in its projects, θ_i is randomly drawn from the real line, each realization considered equally likely.[6] In addition, we assume that θ_1 and θ_2 are independent of each other, indicating no linkage of fundamentals (e.g., no capital or trade linkages) between firms 1 and 2.

For each $i = 1, 2$, after θ_i is realized, a rollover game takes place among the creditors for firm i. For each rollover game, the large creditor and small creditors decide on their action based on their utility function. The utility functions for wealth are given by $u_L(w)$ and $u_S(w)$ for the large creditor and small creditors respectively, where w is the creditor's total wealth. For simplicity's sake, we assume $u_L(0) = u_S(0) = 0$. We also assume that $u_L(w)$ and $u_S(w)$ are twice continuously differentiable, are increasing, and satisfy a decreasing absolute risk aversion; that is, $-u_L''(w)/u_L'(w)$ and $-u_S''(w)/u_S'(w)$ are decreasing.

From Morris and Shin's [2004] model, each rollover game consists of two periods, period 1 (interim stage) and period 2 (maturity). During these periods, the large and small creditors finance the investment project of each firm. When the project is completed in period 2, it yields a return of ν_i for one unit of investment depending on creditors' actions in period 1. Both the firms are financed by a standard debt contract under which the gross return from one unit of investment is I. Each creditor receives the full amount of repayment in period 2 if the realized value of ν_i is large enough to cover debt repayment.

In period 1, the creditors decide whether or not to roll over their loans for the firms until period 2. Since the loans are collateralized, if the creditors decide not to

[6] As argued by Corsetti, Dasgupta, Morris, and Shin [2004], this improper prior distribution with infinite mass simplifies our analysis when evaluating the creditors' updated beliefs conditional on their signals without considering the information contained in the prior distribution.

roll over the loans in period 1, they seize the collateral and obtain the liquidation value, which corresponds to $K^* \in (0, I)$ for one unit of investment. However, if the creditors seize and liquidate the collateral (in period 2) after they roll over the loans, the liquidation value of the seized collateral is $K_* \in (0, K^*)$. As in Morris and Shin [2004], for the sake of simplicity, we normalize the payoffs so that $I = 1$, $K^* = K$, and $K_* = 0$.

The gross return of the investment project for firm i in period 2 (ν_i) depends on two factors, firm i's fundamentals (θ_i) and the proportion of the firm's funds not rolled over in period 1 (l_i). Firm i succeeds in its project as long as θ_i is large enough to meet the creditors' claims, and, otherwise, it is pushed to a default. Specifically, if $\theta_i \geq l_i$, the firm's investment project succeeds and the realized value of $\nu_i(\theta_i, l_i)$ becomes equal to $V(> 1)$. However, if $l_i > \theta_i$, the project fails and $\nu_i(\theta_i, l_i) = 0$.

Note that multiple equilibrium outcomes may arise when the creditors perfectly know the value of θ_i before deciding on whether or not to roll over their loans (period 1). Given the value of θ_i, the creditors' optimal strategy reflects Obstfeld's [1996] self-fulfilling features as follows: If $\theta_i > 1$, the creditors optimally roll over their loans regardless of other creditors' actions because the project would succeed even if every other creditor recalls his loans. Conversely, if $\theta_i \leq 0$, the creditors will not roll over their loans because the project will fail even if all the other creditors roll over their loans. When $\theta_i \in (0, 1]$, a coordination problem arises among creditors. If all other creditors roll over their loans, the payoff for rolling over loans is one at maturity (period 2), which is greater than the premature liquidation value K. However, if all creditors recall their loans, the payoff for rolling over loans is zero, which is less than K, and thus, early liquidation is optimal. Hence, the assumption of creditors' common knowledge about θ_i leads to multiple equilibrium outcomes.

Instead, we can employ a global game method where θ_i is not common knowledge and obtain a unique equilibrium outcome. Creditors have private information on θ_i in period 1, but it is not perfect. Specifically, creditors receive noisy private signals: $y_i = \theta_i + \eta_i$ for a large creditor and $x_{ij} = \theta_i + \epsilon_{ij}$ for a small creditor j, where $\tau\eta_i \sim F(\cdot)$ and $\sigma\epsilon_{ij} \sim G(\cdot)$ for constants $\tau > 0$ and $\sigma > 0$. Here, τ and σ are parameters representing the precision of the large and small creditors' signals respectively. We assume that the distribution functions F and G are symmetric, with a mean of zero and variance of one. In addition, we assume that both the noise terms (i.e., η_i and ϵ_{ij}) are independent of each other.

For each firm i's rollover game, a *strategy* for creditors is a decision rule mapping each realization of private signals (i.e., y_i and x_{ij}) to an action, that is, rolling over or not rolling over loans. An *equilibrium* consists of (1) *a firm's switching fundamentals* below which the project fails—and a liquidity crisis occurs—(i.e., θ_i^* if the large creditor rolls over loans and θ_i^{**} if the large creditor does not roll over loans) and (2) *the creditors' switching private signals* below which they do not roll over loans (i.e., y_i^* for the large creditor and x_i^* for small creditors).

3. SOLVING THE MODEL

In this section, we first solve the equilibrium for firm 1, following Oh [2013, 2015]. After the rollover game for firm 1, every creditor will know whether there was a crisis for firm 1 as well as the exact realization of θ_1. We then solve the equilibrium for firm 2, which is influenced by whether a crisis occurred for firm 1. Specifically, the creditors' changed wealth due to what happened to firm 1 affects their action in the rollover game for firm 2. This forward induction approach shows how a crisis in firm 1 triggers a liquidity crisis in firm 2 (i.e., the contagion of a liquidity crisis from firm 1 to firm 2) through a change in the large and small creditors' wealth.

3.1. Equilibrium for Firm 1

An equilibrium for firm 1 means (1) *the firm's switching fundamentals* below which its project fails and a liquidity crisis occurs (i.e., θ_1^* if the large creditor rolls over loans and θ_1^{**} if the large creditor does not roll over loans) and (2) *the creditors' switching private signals* below which they do not roll over loans (i.e., y_1^* for the large creditor and x_1^* for the small creditors).

After receiving the private signals in period 1, the creditors have to decide whether to roll over loans or not. At $y_1 = y_1^*$, the large creditor is indifferent between these two options and the expected payoff for rolling over loans equals the payoff for recalling them as follows:

$$\underbrace{u_L(\alpha K + (1-\alpha))}_{\text{Total utility if recalling at firm 1}} = \underbrace{\left\{ \begin{array}{c} u_L(\alpha + (1-\alpha)) \Pr(\text{Project succeeds}|y_1^*) \\ +u_L(0 + (1-\alpha)) \Pr(\text{Project fails}|y_1^*) \end{array} \right\}}_{\text{Total expected utility if rolling-over at firm 1}}. \quad (1)$$

Similarly, at $x_{1j} = x_1^*$, each small creditor j is indifferent between the two options, and thus, the following equation holds:

$$\begin{aligned} u_S(\beta K + (1-\beta)) = {} & u_S(\beta + (1-\beta)) \Pr(\text{Project succeeds}|x_1^*) \\ & + u_S(0 + (1-\beta)) \Pr(\text{Project fails}|x_1^*). \end{aligned} \quad (2)$$

Further, the critical value of firm 1's fundamentals (θ_1^* and θ_1^{**}) is determined at the level of fundamentals at which the proportion of firm 1's funds not rolled over (l_1) equals the critical value itself. Thus, the creditors' indifference conditions and switching fundamental conditions are combined to determine the unique equilibrium values, that is, switching fundamentals of firm 1 (θ_1^* and θ_1^{**}) and switching private signals (y_1^* and x_1^*). Firm 1's equilibrium is summarized in the following proposition:

PROPOSITION 1. *A unique equilibrium for firm 1 consists of (1) the firm's switching fundamentals (θ_1^* if the large creditor rolls over loans and θ_1^{**} if the large creditor does not roll over loans) below which the project fails (i.e., a liquidity crisis arises in firm 1) and (2) the creditors' switching private signals (y_1^* for the large creditor and x_1^* for the small creditors) below which they do not roll over loans.*

3.2. Equilibrium for Firm 2

After the rollover game is undertaken for firm 1, all the creditors observe what occurs for firm 1, including the exact value of θ_1. This provides the market with information on the distribution of the creditors' wealth and hence risk aversion. Specifically, a liquidity crisis in firm 1 would lead to a decrease in wealth of the creditors who roll over loans for firm 1, which, in turn, leads to an increased risk aversion of those creditors for firm 2.

If $\theta_1 \notin [\theta_1^*, \theta_1^{**}]$, a liquidity crisis in firm 1 is independent of the large creditor's decision whether or not to roll over loans. In particular, if $\theta_1 < \theta_1^*$, there would be a liquidity crisis in firm 1 regardless of whether the large creditor rolls over loans or not. On the other hand, if $\theta_1 > \theta_1^{**}$, no liquidity crisis would arise in firm 1 regardless of whether the large creditor decides to roll over loans.

Let us now consider the other case where $\theta_1 \in [\theta_1^*, \theta_1^{**}]$. Conditional on such θ_1, a liquidity crisis occurs for firm 1 if and only if the large creditor decides not to roll over loans for firm 1. Hence, the existence of a liquidity crisis in firm 1 influences the structure of firm 2's rollover game through a decrease in the creditors' wealth level.

Conditional on $\theta_1 \in [\theta_1^*, \theta_1^{**}]$, we now discuss the two scenarios based on the occurrence of a liquidity crisis in firm 1. In each scenario, that is, conditional on the realization of firm 1's fundamentals (θ_1) and the occurrence of a liquidity crisis in firm 1, we derive a unique equilibrium for firm 2 (i.e., θ_2^*, θ_2^{**}, y_2^*, and x_2^*).

3.2.1. *Scenario 1: No crisis facing firm 1 when $\theta_1 \in [\theta_1^*, \theta_1^{**}]$*

This scenario corresponds to the case where the large creditor rolls over loans for firm 1. In this case, no liquidity crisis occurs in firm 1 and the large creditor and the small creditors who roll over loans for firm 1 earn α and β, respectively. On the other hand, the small creditors who do not roll over loans for firm 1 obtain βK through the liquidation of the collaterals of firm 1.

The equilibrium consists of five variables as follows (i.e., two in (1) and three in (2)): (1) Conditional on the large creditor's decision to roll over loans for the firm, two different switching fundamentals exist below which a liquidity crisis would occur for firm 2. That is, we define $\underline{\theta}_2^*$ and $\underline{\theta}_2^{**}$ as the switching fundamentals when the large creditor rolls over loans and decides not to roll over loans for firm 2, respectively. (2) Creditors have three switching private signals below which they decide not to roll over loans for firm 2. That is, we define $\underline{y}_2^*$, $\underline{x}_{2,1}^*$, and $\underline{x}_{2,K}^*$ as the switching private signals for the large creditor, the small creditors who roll over loans for firm 1, and the small creditors who do not roll over loans for firm 1, respectively.

As with firm 1's rollover game, the equilibrium involves the indifference conditions for creditors. In particular, at $\underline{y}_2^*$, the large creditor's expected payoff for rolling over loans equals that for recalling them as follows:

$$\underbrace{u_L(\alpha + (1-\alpha)K)}_{\text{Total utility if recalling at firm 2}} = \underbrace{\left\{ \begin{array}{l} u_L(\alpha + (1-\alpha))\,\Pr(\text{Project succeeds}|\underline{y}_2^*) \\ +u_L(\alpha + 0)\,\Pr(\text{Project fails}|\underline{y}_2^*) \end{array} \right\}}_{\text{Total expected utility if rolling-over at firm 2}}. \tag{3}$$

On the other hand, the small creditors' switching private signals also follow similar indifference conditions, as shown below. Note that the small creditors who roll over loans and decide not to roll over loans for firm 1 have different indifference conditions, which, in turn, lead to different switching private signals (i.e., $\underline{x}_{2,1}^*$ and $\underline{x}_{2,K}^*$, respectively):

$$\begin{aligned} u_S(\beta + (1-\beta)K) &= u_S(\beta + (1-\beta))\,\Pr(\text{Project succeeds}|\underline{x}_{2,1}^*) \\ &\quad + u_S(\beta + 0)\,\Pr(\text{Project fails}|\underline{x}_{2,1}^*), \end{aligned} \tag{4}$$

$$\begin{aligned} u_S(\beta K + (1-\beta)K) &= u_S(\beta K + (1-\beta))\,\Pr(\text{Project succeeds}|\underline{x}_{2,K}^*) \\ &\quad + u_S(\beta K + 0)\,\Pr(\text{Project fails}|\underline{x}_{2,K}^*). \end{aligned} \tag{5}$$

Further, firm 2's switching fundamentals (i.e., $\underline{\theta}_2^*$ and $\underline{\theta}_2^{**}$) are determined at the level of fundamentals equalizing the proportion of firm 2's funds whose creditors recall loans (l_2) at the critical value itself. Using the indifference conditions for creditors and the condition of the critical threshold value of firm 2's fundamentals, we obtain firm 2's equilibrium $(\underline{\theta}_2^*, \underline{\theta}_2^{**}, \underline{y}_2^*, \underline{x}_{2,1}^*, \underline{x}_{2,K}^*)$ for Scenario 1.

3.2.2. *Scenario 2: Firm 1's liquidity crisis when $\theta_1 \in [\theta_1^*, \theta_1^{**}]$*

This scenario comes true when the large creditor does not roll over loans for firm 1. In this case, the large creditor earns αK from the liquidation of firm 1's collaterals. On the other hand, the small creditors who roll over loans for firm 1 obtain zero payoffs from firm 1, whereas those who decide not to roll over loans obtain βK through the liquidation of firm 1's collaterals.

As in Scenario 1, the equilibrium consists of five variables as follows: (1) Firm 2's switching fundamentals below which a liquidity crisis would occur for firm 2 (i.e., $\overline{\theta}_2^*$ when the large creditor rolls over loans for firm 2 and $\overline{\theta}_2^{**}$ when the large creditor does not roll over loans for firm 2) and (2) the creditors' switching private signals below which they decide not to roll over loans for firm 2 (i.e., $\overline{y}_2^*$ for the large creditor, $\overline{x}_{2,0}^*$ for the small creditors who roll over loans for firm 1, and $\overline{x}_{2,K}^*$ for the small creditors who do not roll over loans for firm 1). As in Scenario 1, the indifference conditions for the creditors and the conditions equalizing the proportion of firm 2's funds that the creditors recall (l_2) with the switching fundamentals lead to a unique equilibrium $(\overline{\theta}_2^*, \overline{\theta}_2^{**}, \overline{y}_2^*, \overline{x}_{2,0}^*, \overline{x}_{2,K}^*)$.

3.2.3. *Scenario 1 versus Scenario 2*

We now compare Scenario 1 (i.e., when $\theta_1 \in [\theta_1^*, \theta_1^{**}]$ and there is no liquidity crisis in firm 1) with Scenario 2 (i.e., when $\theta_1 \in [\theta_1^*, \theta_1^{**}]$ and there is a liquidity crisis in firm 1). Scenario 1 provides firm 2's benchmark switching fundamentals (i.e., $\underline{\theta}_2^*$ and $\underline{\theta}_2^{**}$), whereas Scenario 2 provides firm 2's new switching fundamentals (i.e., $\overline{\theta}_2^*$ and $\overline{\theta}_2^{**}$). By comparing the values of these switching fundamentals, we obtain the following lemma:

LEMMA 1. *$\underline{\theta}_2^* < \overline{\theta}_2^*$ and $\underline{\theta}_2^{**} < \overline{\theta}_2^{**}$.*

The intuition behind this lemma is as follows: When a liquidity crisis occurs in firm 1 (i.e., Scenario 2), the small creditors who roll over loans for firm 1 obtain nothing from their investment in firm 1. Further, the large creditor who recalls his loans and thus causes the crisis also becomes worse off compared to the case in Scenario 1, because the large creditor would have earned a higher return if he had rolled over loans for firm 1 and did not cause the crisis in firm 1. Thus, the assumption of decrease in absolute risk aversion implies that the large and small creditors become more risk averse and more likely to choose the safe option of recalling their loans from firm 2. Therefore, given the large creditor's decision regarding his loans to firm 2, a liquidity crisis in firm 1 would lead to a higher conditional probability for a liquidity crisis in firm 2 as well.

4. CONTAGION WITH A LARGE CREDITOR

4.1. What is Contagion?

The contagion effect from firm 1 to firm 2 indicates an increase in the switching fundamentals that appear when a liquidity crisis arises in firm 2 compared to the benchmark switching fundamentals (i.e., those of Scenario 1). Specifically, depending on which switching fundamentals are in focus, two different contagion effects can be defined as follow (see FIG. 2):

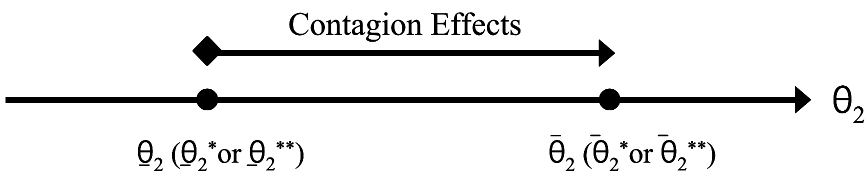

FIG. 2 Contagion Effects ($\mathbf{CE}_L$ and $\mathbf{CE}_S$)

First, we focus on the liquidity crisis arising from the large creditor's decision to recall his loans. In this case, the liquidity crisis arises for firm 1 with the state of fundamentals $\theta_1 \in [\theta_1^*, \theta_1^{**}]$. A liquidity crisis may arise for firm 2 as well from the large creditor's right on his significant share of claims. In particular, if no liquidity crisis arises in firm 1, a possibility for this could arise when $\theta_2 \leq \underline{\theta}_2^{**}$. On the other hand, if a liquidity crisis does arise for firm 1, the large creditor's decision to recall his loans gives rise to a liquidity crisis in firm 2 when $\theta_2 \leq \overline{\theta}_2^{**}$. Hence, the contagion effect between the two firms can be defined as follows:

Definition 1. Focusing on a liquidity crisis arising from the large creditor's decision to recall his loans, the contagion effect of a liquidity crisis from firm 1 to firm 2 is defined as follows:

$$\mathbf{CE}_L := \overline{\theta}_2^{**} - \underline{\theta}_2^{**}.$$

Second, we may also consider a liquidity crisis that may arise independent of the large creditor's decision on his share of claims. For the crisis in firm 1, this happens when $\theta_1 < \theta_1^*$. A liquidity crisis may also arise for firm 2 independent of the large creditor's decision to recall his loans. In particular, if no liquidity crisis arises in firm 1, we have $\theta_2 \leq \underline{\theta}_2^*$. On the other hand, if a liquidity crisis does arise in firm 1, a liquidity crisis will always occur in firm 2 regardless of the large creditor's decision when $\theta_2 \leq \overline{\theta}_2^*$. Hence, these arguments lead to the following definition of the contagion effect between two firms:[7]

Definition 2. Focusing on a liquidity crisis that could arise independent of the large creditor's right on his share of claims, the contagion effect of a liquidity crisis from firm 1 to firm 2 is defined as follows:

$$\mathbf{CE}_S := \overline{\theta}_2^* - \underline{\theta}_2^*.$$

4.2. Comparative Statics Analyses

In this subsection, we provide the comparative statics of the two contagion effects defined above (i.e., $\mathbf{CE}_L := \overline{\theta}_2^{**} - \underline{\theta}_2^{**}$ and $\mathbf{CE}_S := \overline{\theta}_2^* - \underline{\theta}_2^*$). The numerical calculations of the contagion effects are also reported with reasonably given initial values and utility functions as follows: $\lambda = 0.5$, $\alpha = \beta = 0.5$, $K = 0.5$, $\tau = \sigma = 1$, and $u_L(w) = u_S(w) = \sqrt{1 + 2w} - 1$. For the numerical results, F and G are standard normal, and we set $\theta_1 = \theta_1^{**}$.

[7] Note that no inequality generally holds between two contagion effects (i.e., $\mathbf{CE}_L$ can be either larger or smaller than $\mathbf{CE}_S$).

4.2.1. Changes in size of large creditor (λ)

First, we analyze the influence of a large creditor's size (λ) on contagion effects. The following proposition states that the contagion effects decrease when a large creditor with a sufficiently large size is present (see FIG. 3):

PROPOSITION 2. *The presence of a large creditor with a sufficiently large size (λ) reduces the contagion effects. In particular, it holds that*

$$\mathbf{CE}_L > \lim_{\lambda \to 1} \mathbf{CE}_L = 0 \text{ and } \mathbf{CE}_S > \lim_{\lambda \to 1} \mathbf{CE}_S = 0.$$

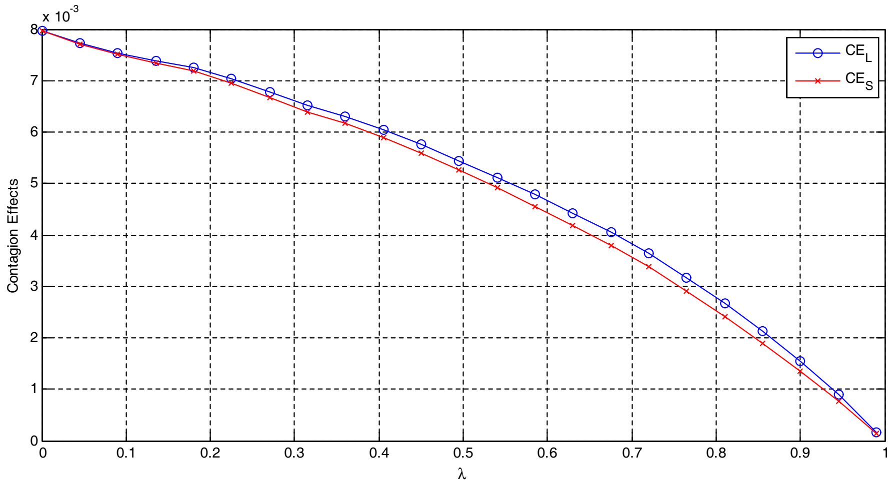

FIG. 3 Effects of λ on $\mathbf{CE}_L$ and $\mathbf{CE}_S$

The intuition behind Proposition 2 is as follows: As the large creditor takes a dominant position in terms of relative size, the liquidity crisis arising in firm 2 is largely determined by the large creditor's decision on his loans. That is, a liquidity crisis would rarely occur for firm 2 if the large creditor rolls over loans for firm 2, whereas it is highly likely to occur when he does not do so. Thus, the occurrence of a liquidity crisis in firm 1 is less sensitive to changes in the large creditor's wealth level compared to the case with only small creditors. This argument stands in marked contrast to most previous studies on the role of large traders in the stability of the financial system (e.g., Corsetti, Dasgupta, Morris & Shin, 2004; Corsetti, Pesenti & Roubini, 2002). Moreover, as indicated by Bolton and Scharfstein [1996], Proposition 2 implies that financing from multiple (small) creditors might not be an optimal debt structure from a firm (borrower)'s perspective, because it encourages a contagious liquidity crisis in the financial market.

4.2.2. Changes in portfolio weight (α and β)

We now examine the impact of the creditors' investment portfolio on contagion effects. The investment portfolio toward firm 1 of the large creditor is represented as α and that of the small creditors is represented as β. Here, we assume a negative impact of the presence of the large creditor on contagion effects, as indicated by Proposition 2.

Proposition 3. *(1) The contagion effects will increase if the large creditor shifts his loan portfolio toward firm* 1*. In particular, it holds that*

$$\lim_{\alpha \to 1} \mathbf{CE}_L > \mathbf{CE}_L \text{ and } \lim_{\alpha \to 1} \mathbf{CE}_S > \mathbf{CE}_S.$$

(2) The contagion effects will decrease to zero as small creditors shift their loan portfolio toward firm 1*. In particular, it holds that*

$$\mathbf{CE}_L > \lim_{\beta \to 1} \mathbf{CE}_L = 0 \text{ and } \mathbf{CE}_S > \lim_{\beta \to 1} \mathbf{CE}_S = 0.$$

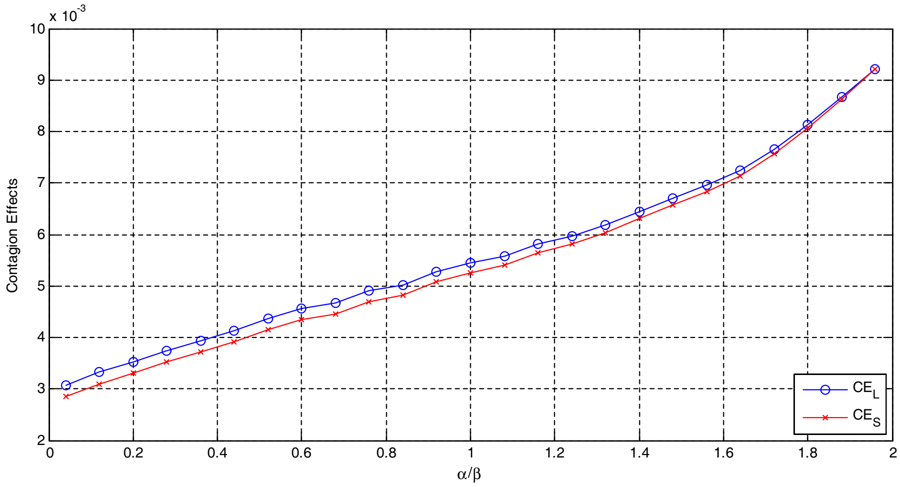

FIG. 4 Effects of (α/β) on $\mathbf{CE}_L$ and $\mathbf{CE}_S$

As the large creditor's funds in firm 2 disappear, the influence of his decision on his loans in firm 2 disappears as well. Meanwhile, this leads to an increase in the contagion effects from firm 1 to firm 2. That is, a concentration of the large creditor's loan portfolio toward firm 1 leads to an increase in the contagion effects. On the other hand, a concentration of the small creditors' portfolios toward firm 1 leads to changes in the contagion effects in the opposite direction. As the small creditors shift their loan portfolios toward firm 1, the large creditor's influence on firm 1 becomes relatively small, but that on firm 2 becomes relatively large, leading to a decrease in the contagion effects. FIG. 4 shows that both the contagion effects (i.e., $\mathbf{CE}_L$ and $\mathbf{CE}_S$) monotonically increase in the large creditor's relative portfolio weight in firm 1 (i.e., α/β).

Moreover, as discussed in Goldstein and Pauzner [2004], Proposition 3 highlights the potential trade-off of large creditors' diversified portfolios. Indeed, in most emerging economies, banks are the typical "large" creditors to nonfinancial firms, and their portfolios are rarely consisting of only two firms but are rather diversified across borrowers, industries, and regions, with the obvious objective to reduce the volatility of their portfolio credit performances. Proposition 3 points out the possibility that such diversification comes at a cost to the economy, because the contagion effects identified by our proposed model are the increased vulnerability of firms whose creditors have suffered credit losses in other parts of their portfolios.

4.2.3. Changes in collateral value (K)

As noted in Morris and Shin [2003, 2004], an increase in the value of collateral (K) has a strategic effect: It reduces the creditors' incentives to roll over loans by decreasing the cost of not rolling over loans and thereby increasing the range of fundamentals at which a default can occur. In our proposed model, an increase in the value of collateral always leads to an increase in all the switching fundamentals of both firms (i.e., θ_1^*, θ_1^{**}, $\underline{\theta}_2^*$, $\underline{\theta}_2^{**}$, $\overline{\theta}_2^*$, and $\overline{\theta}_2^{**}$). That is, increasing the value of collateral has a negative impact on the value of debt, as opposed to the direct effect of enhancing the value of debt in the event of a default.

In the contagion context, the impact of changes in the collateral value on contagion effects is not monotonic due to the interaction between the strategic effect described above and the wealth effect from the occurrence of a liquidity crisis in firm 1. Of course, both effects influence the switching fundamentals of firm 2 in the same direction. However, when the collateral value is either too low or too high, the strategic effect is so influential that the contagion effects are no longer sensitive to the changes in risk attitudes arising from wealth effect, as indicated by the following proposition (see FIG. 5):

Proposition 4. *As the collateral value approaches either zero or one, the contagion effects approach zero. In particular, it holds that*

$$\lim_{K\to 0} \mathbf{CE}_L = \lim_{K\to 1} \mathbf{CE}_L = 0 \text{ and } \lim_{K\to 0} \mathbf{CE}_S = \lim_{K\to 1} \mathbf{CE}_S = 0.$$

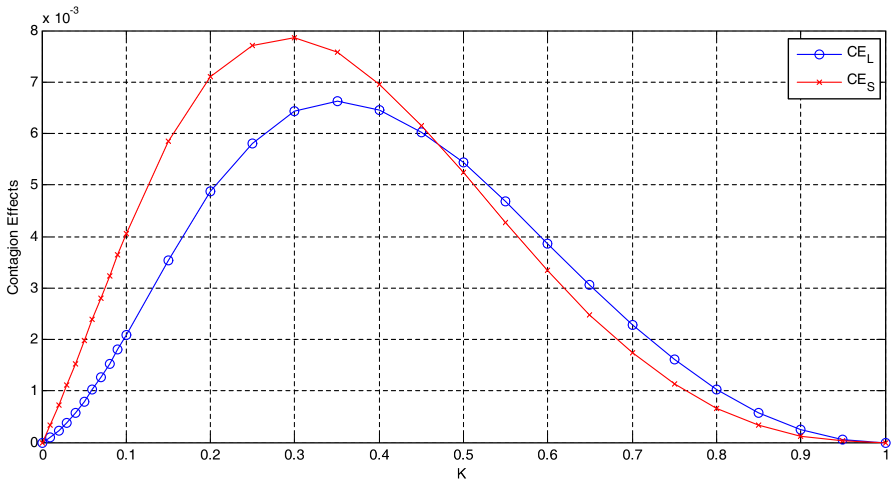

FIG. 5 Effects of K on $\mathbf{CE}_L$ and $\mathbf{CE}_S$

4.2.4. Changes in risk aversion parameters (c_L and c_S)

The utility functions of the large creditor and small creditors (i.e., u_L and u_S, respectively) reflect the sensitivities of creditors' risk attitudes to changes in wealth, which, in turn, affect the contagion effects. To parameterize these characteristics, we assume that, for the rollover game of each firm, the utilities of the large creditor and small creditors from wealth are given by the extended power utility functions $u_L(w) = \frac{1}{c_L-1}(A + c_L w)^{1-\frac{1}{c_L}} - \frac{1}{c_L-1}A^{1-\frac{1}{c_L}}$ and $u_S(w) = \frac{1}{c_S-1}(A + c_S w)^{1-\frac{1}{c_S}} -$

$\frac{1}{c_S-1}A^{1-\frac{1}{c_S}}$, where $A > 0$ and $w \geq 0$. Note that the creditors' utility functions in the numerical results in the previous analyses correspond to the case where $A = 1$ and $c_L = c_S = 2$. For any given wealth, parameters c_L and c_S represent respectively the extent to which the large creditor and small creditors are less risk averse. For each type of creditor, if its risk aversion parameter is either too low or too high, then its absolute risk aversion is no longer sensitive to wealth, and hence, there is no wealth effect for that type of creditor.[8] When it is the case for both the large creditor and small creditors, the contagion effects disappear, as indicated by the following proposition (see FIG. 6 and 7):

PROPOSITION 5. *As the utilities of the large creditor and small creditors have either too low or too high risk aversion parameters, respectively, the contagion effects decrease to zero. In particular, either as $c_L \to 0$ or $c_L \to \infty$, and either $c_S \to 0$ or $c_S \to \infty$, it holds that $\mathbf{CE}_L \to 0$ and $\mathbf{CE}_S \to 0$.*

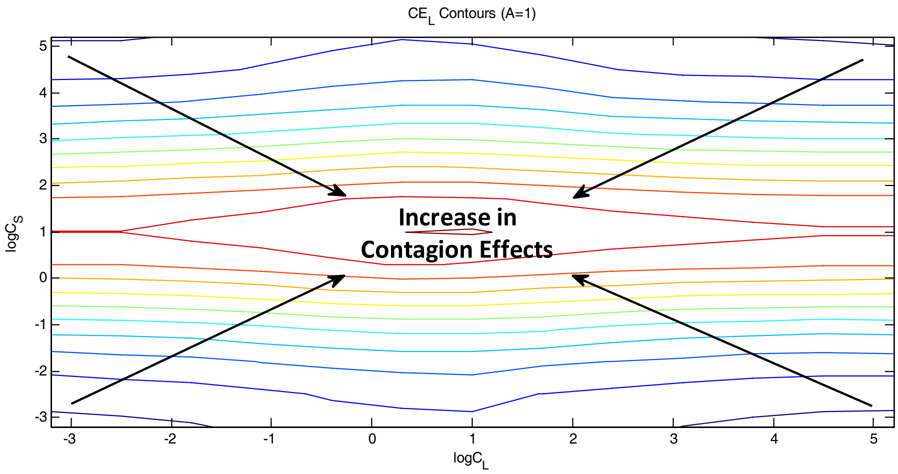

FIG. 6 Contagion Effect Contours in (c_L, c_S)-Space ($\mathbf{CE}_L$ at $A = 1$)

The influence of the creditors' signal precisions (i.e., τ and σ) on contagion effects interacts with other factors, and thus, it is hard to clearly identify. For example, we find that as the large creditor's private signal becomes very accurate (i.e., $\tau \to \infty$), his wealth effect disappears. However, the contagion effects do not disappear completely in this case because the small creditors' wealth effect persists. Only when the small creditors' risk aversion parameters are extremely low or high does their wealth effect also disappear so that the contagion effects disappear.[9]

5. CONCLUDING REMARKS

In this paper, we focus on the liquidity crises involving a large creditor and explore the financial contagion: A liquidity crisis occurring in one firm triggers a liquidity crisis in another firm through changes in the behavioral pattern of creditors. The contagion mechanism between two nonfinancial firms originates in the

[8] Note that the Arrow-Pratt measures of absolute risk aversion (see Arrow, 1965; Pratt, 1964) of $u_L(w)$ and $u_S(w)$ are $\frac{1}{A+c_L w}$ and $\frac{1}{A+c_S w}$, respectively. For any given wealth $w > 0$, the measure of absolute risk aversion of $u_L(w)$ approaches $\frac{1}{A}$ as c_L goes to zero. On the other hand, it approaches zero as c_L goes to infinity. The same argument holds for $u_S(w)$.

[9] The formal proof for this example is presented with the proof of Proposition 5 in the Appendix.

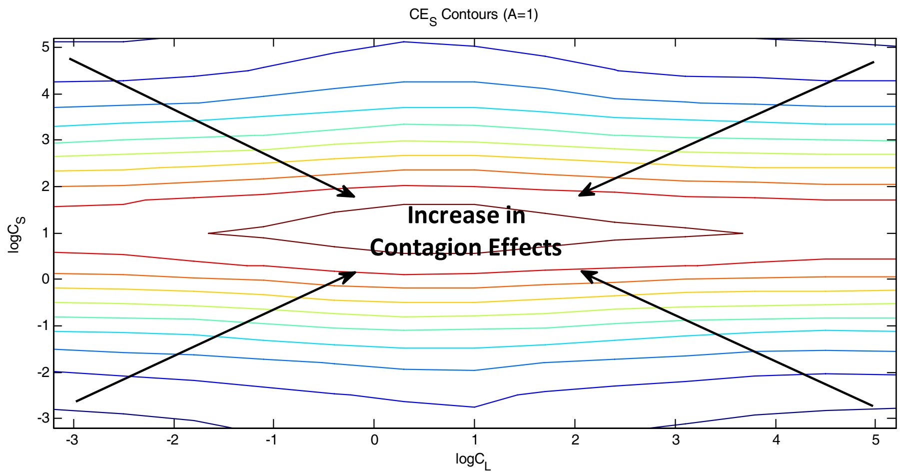

FIG. 7 Contagion Effect Contours in (c_L, c_S)-Space ($\mathbf{CE}_S$ at $A = 1$)

wealth effect, which is especially important for a large creditor because of his high leverage ratio and capital management requirements. By learning from what happens in one firm, creditors revise their beliefs about the risk attitudes of other creditors and decide their own action for another firm; this influences the probability of a liquidity crisis arising in the firm. Relative to small creditors, a large creditor has certain distinct characteristics such as size and investment portfolio. These characteristics influence the extent to which an event in a firm triggers another firm's event, namely, the contagion effect, because the large creditor influences a significant portion of the economy. The analyses of contagion effects involving a large creditor indicate a noteworthy feature of financial contagion: The presence of a large creditor with a sufficient size makes a firm more vulnerable to a liquidity crisis, but it reduces contagion effects in the whole market. In addition, the contagion effects become more severe if the large creditor shifts his loan portfolio toward the former firm.

APPENDIX: PROOFS AND DERIVATIONS

Proof of Proposition 1

First, we consider a large creditor's decision on whether or not to roll over loans for firm 1. Given his private signal y_1, the posterior cumulative distribution function of the state of fundamentals is $G(\tau(\theta_1 - y_1))$. If the large creditor decides to roll over loans, the switching fundamental value of firm 1 becomes θ_1^*, and hence, the conditional probability that the project succeeds becomes $\Pr(\theta_1 \geq \theta_1^*) = 1 - G(\tau(\theta_1^* - y_1)) = G(\tau(y_1 - \theta_1^*))$, because G is assumed to be symmetric around zero. Accordingly, Equation (1) is equivalent to

$$G(\tau(y_1^* - \theta_1^*)) = \frac{u_L(\alpha K + (1-\alpha)) - u_L(1-\alpha)}{u_L(\alpha + (1-\alpha)) - u_L(1-\alpha)}. \tag{A1}$$

Second, we consider the decisions of small creditors. Given private signal x_{1j}, the posterior cumulative distribution function of θ_1 is $F(\sigma(\theta_1 - x_{1j}))$. Given θ_1, the likelihood of firm 1's project succeeding is as follows: If $\theta_1 < \theta_1^*$, the project never succeeds. If $\theta_1^* \leq \theta_1 \leq \theta_1^{**}$, the project succeeds if and only if the large creditor rolls over loans, implying that the large creditor receives y_1, which is greater than y_1^*; conditional on θ_1, this probability equals $\Pr(y_1 = \theta_1 + \eta_1 \geq y_1^*) = \Pr(\eta_1 \geq y_1^* - \theta_1) = 1 - G(\tau(y_1^* - \theta_1))$. If $\theta_1 > \theta_1^{**}$, the project always succeeds. To summarize, the conditional probability that the project succeeds is as follows:

$$\Pr(\text{Project succeeds}|\theta_1) = \begin{cases} 0 & \text{if } \theta_1 < \theta_1^*; \\ 1 - G(\tau(y_1^* - \theta_1)) & \text{if } \theta_1 \in [\theta_1^*, \theta_1^{**}]; \\ 1 & \text{if } \theta_1 > \theta_1^{**}. \end{cases}$$

Thus, given x_{1j}, the conditional probability that the project succeeds can be calculated by the Bayes rule as follows:

$$\begin{aligned} \Pr(\text{Project succeeds}|x_{1j}) &= \int_{-\infty}^{-\infty} \sigma f(\sigma(\theta_1 - x_{1j})) \Pr(\text{Project succeeds}|\theta_1) d\theta_1 \\ &= \int_{\theta_1^*}^{\theta_1^{**}} \sigma f(\sigma(\theta_1 - x_{1j}))[1 - G(\tau(y_1^* - \theta_1))] d\theta_1 + \int_{\theta_1^{**}}^{\infty} \sigma f(\sigma(\theta_1 - x_{1j})) d\theta_1. \end{aligned}$$

At $x_{1j} = x_1^*$, Equation (2) is equivalent to

$$\frac{u_S(\beta K + (1-\beta)) - u_S(1-\beta)}{u_S(\beta + (1-\beta)) - u_S(1-\beta)} = \int_{\theta_1^*}^{\theta_1^{**}} \sigma f(\sigma(\theta_1 - x_1^*))[1 - G(\tau(y_1^* - \theta_1))] d\theta_1 + \int_{\theta_1^{**}}^{\infty} \sigma f(\sigma(\theta_1 - x_1^*)) d\theta_1. \tag{A2}$$

Finally, we consider the switching fundamentals of firm 1. Given θ_1, the proportion of small creditors who decide not to roll over loans is $\Pr(x_{1j} = \theta_1 + \epsilon_{1j} < x_1^* | \theta_1) = \Pr(\epsilon_{1j} < x_1^* - \theta_1) = F(\sigma(x_1^* - \theta_1))$. If the large creditor rolls over loans, the proportion of funds that the creditors do not roll over is

$$\begin{aligned} l_1(\theta_1|\text{Large creditor rolls over}) &= (1 - \lambda_1) \Pr(x_{1j} < x_1^*) \\ &= (1 - \lambda_1) F(\sigma(x_1^* - \theta_1)), \end{aligned}$$

where λ_1 is the proportion of firm 1's funds provided by the large creditor. Specifically, we define λ_1 as $\frac{\lambda\alpha}{\lambda\alpha + (1-\lambda)\beta}$. Thus, the critical threshold value of firm 1's fundamentals (i.e., switching fundamentals) when the large creditor rolls over loans can be determined by

$$\theta_1^* = (1 - \lambda_1) F(\sigma(x_1^* - \theta_1^*)). \tag{A3}$$

On the other hand, if the large creditor does not roll over loans, the proportion of funds that the creditors do not roll over is

$$\begin{aligned} l_1(\theta_1|\text{Large creditor does not roll over}) &= \lambda_1 + (1 - \lambda_1) Pr(x_{1j} < x_1^*) \\ &= \lambda_1 + (1 - \lambda_1) F(\sigma(x_1^* - \theta_1)). \end{aligned}$$

Accordingly, the switching fundamental value in this case can be determined by

$$\theta_1^{**} = \lambda_1 + (1 - \lambda_1) F(\sigma(x_1^* - \theta_1^{**})). \tag{A4}$$

The system of Equations (A1), (A2), (A3), and (A4) is exactly identical to that considered by Corsetti, Dasgupta, Morris, and Shin [2004]. Thus, closely following their arguments, we can show the uniqueness of the solution as well as that $\theta_1^* < \theta_1^{**}$.[10] Here, we prove the uniqueness of our solutions: First, we define

[10] See Figure 2 in Corsetti, Dasgupta, Morris, and Shin [2004].

$z := \sigma(\theta_1 - x_1^*)$, $\delta_1^* := \sigma(\theta_1^* - x_1^*)$, and $\delta_1^{**} := \sigma(\theta_1^{**} - x_1^*)$. Now, the right-hand side of Equation (A2) becomes

$$\int_{\delta_1^*}^{\delta_1^{**}} f(z)\left[1 - G\left(-\frac{\tau}{\sigma}z - \tau x_1^* + \tau y_1^*\right)\right] dz + \int_{\delta_1^{**}}^{\infty} f(z)\, dz$$

$$= \int_{\delta_1^*}^{\delta_1^{**}} f(z)\left[1 - G\left(-\frac{\tau}{\sigma}z + \underbrace{\tau(\theta_1^* - x_1^*)}_{=:\frac{\delta_1^*}{\sigma}} + \underbrace{\tau(y_1^* - \theta_1^*)}_{=:G^{-1}\left(\frac{u_L(\alpha K + (1-\alpha)) - u_L(1-\alpha)}{u_L(\alpha+(1-\alpha)) - u_L(1-\alpha)}\right)}\right)\right] dz + \int_{\delta_1^{**}}^{\infty} f(z)\, dz,$$

which monotonically decreases with δ_1^* and δ_1^{**}, respectively.

Next, it suffices to show that both δ_1^* and δ_1^{**} monotonically decrease with x_1^*, which guarantees that the right-hand side of Equation (A2) monotonically decreases with x_1^*, leading to at most one x_1^* satisfying Equation (A2). From $\delta_1^* := \sigma(\theta_1^* - x_1^*)$ and Equation (A3), we obtain $\frac{d\delta_1^*}{dx_1^*} = \frac{-\sigma}{1+(1-\lambda_1)\sigma f(-\delta_1^*)} < 0$. Likewise, we can derive $\frac{d\delta_1^{**}}{dx_1^*} = \frac{-\sigma}{1+(1-\lambda_1)\sigma f(-\delta_1^{**})} < 0$ from $\delta_1^{**} := \sigma(\theta_1^{**} - x_1^*)$ and Equation (A4).

Derivation of Firm 2's Equilibrium

Here, we derive the equilibrium $(\underline{\theta}_2^*, \underline{\theta}_2^{**}, \underline{y}_2^*, \underline{x}_{2,1}^*, \underline{x}_{2,K}^*)$ for Scenario 1. We then apply the same argument to derive the equilibrium $(\overline{\theta}_2^*, \overline{\theta}_2^{**}, \overline{y}_2^*, \overline{x}_{2,0}^*, \overline{x}_{2,K}^*)$ for Scenario 2.

First, we consider the large creditor's decision on whether or not to roll over loans. Given the large creditor's private signal y_2, the posterior cumulative distribution function of the state of fundamentals is $G(\tau(\theta_2 - y_2))$. If the large creditor decides to roll over loans, the switching fundamental value of firm 2 becomes $\underline{\theta}_2^*$, and hence, the conditional probability that the project succeeds is $\Pr(\theta_2 \geq \underline{\theta}_2^*) = 1 - G(\tau(\underline{\theta}_2^* - y_2)) = G(\tau(y_2 - \underline{\theta}_2^*))$, because G is assumed to be symmetric around zero. Accordingly, Equation (3) is equivalent to

$$G(\tau(\underline{y}_2^* - \underline{\theta}_2^*)) = \frac{u_L(\alpha + (1-\alpha)K) - u_L(\alpha)}{u_L(1) - u_L(\alpha)}. \tag{A5}$$

Second, we consider the decisions of small creditors. Given the small creditors' private signal x_{2j}, the posterior cumulative distribution function of θ_2 is $F(\sigma(\theta_2 - x_{2j}))$. Given θ_2, the conditional probability that the project succeeds is as follows:

$$\Pr(\text{Project succeeds}|\theta_2) = \begin{cases} 0 & \text{if } \theta_2 < \underline{\theta}_2^*; \\ 1 - G(\tau(\underline{y}_2^* - \theta_2)) & \text{if } \theta_2 \in [\underline{\theta}_2^*, \underline{\theta}_2^{**}]; \\ 1 & \text{if } \theta_2 > \underline{\theta}_2^{**}. \end{cases}$$

Thus, given x_{2j}, the conditional probability that the project succeeds can be calculated by the Bayes rule as follows:

$$\begin{aligned}\Pr(\text{Project succeeds}|x_{2j}) &= \int_{-\infty}^{-\infty} \sigma f(\sigma(\theta_2 - x_{2j}))\Pr(\text{Project succeeds}|\theta_2)d\theta_2 \\ &= \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - x_{2j}))[1 - G(\tau(\underline{y}_2^* - \theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - x_{2j}))d\theta_2.\end{aligned}$$

Now, we derive the indifference conditions for small creditors. For small creditors who roll over loans for firm 1, it follows from Equation (4) that

$$\frac{u_S(\beta+(1-\beta)K)-u_S(\beta)}{u_S(1)-u_S(\beta)} = \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2-\underline{x}_{2,1}^*))[1-G(\tau(\underline{y}_2^*-\theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2-\underline{x}_{2,1}^*))d\theta_2. \tag{A6}$$

On the other hand, for small creditors who do not roll over loans for firm 1, it follows from Equation (5) that

$$\frac{u_S(K)-u_S(\beta K)}{u_S(\beta K+(1-\beta))-u_S(\beta K)} = \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2-\underline{x}_{2,K}^*))[1-G(\tau(\underline{y}_2^*-\theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2-\underline{x}_{2,K}^*))d\theta_2. \tag{A7}$$

Finally, we consider the switching fundamentals of firm 2. We denote by $q(\theta_1)$ the proportion of small creditors who roll over loans for firm 1. That is, $q(\theta_1) = 1 - F(\sigma(x_1^*-\theta_1))$. Given θ_2, the proportion of small creditors who decide not to roll over loans is

$$\begin{aligned} q\Pr(x_2<\underline{x}_{2,1}^*)+(1-q)\Pr(x_2<\underline{x}_{2,K}^*) &= q\Pr(\epsilon_{2j}<\underline{x}_{2,1}^*-\theta_2)+(1-q)\Pr(\epsilon_{2j}<\underline{x}_{2,K}^*-\theta_2) \\ &= qF(\sigma(\underline{x}_{2,1}^*-\theta_2))+(1-q)F(\sigma(\underline{x}_{2,K}^*-\theta_2)). \end{aligned}$$

If the large creditor rolls over loans, the proportion of funds that the creditors do not roll over is

$$\begin{aligned} l_2(\theta_2|\text{Large creditor rolls over}) &= (1-\lambda_2)\left[q\Pr(x_{2j}<\underline{x}_{2,1}^*)+(1-q)\Pr(x_{2j}<\underline{x}_{2,K}^*)\right] \\ &= (1-\lambda_2)[qF(\sigma(\underline{x}_{2,1}^*-\theta_2))+(1-q)F(\sigma(\underline{x}_{2,K}^*-\theta_2))], \end{aligned}$$

where λ_2 is the proportion of firm 2's funds provided by the large creditor. Specifically, we define λ_2 as $\frac{\lambda(1-\alpha)}{\lambda(1-\alpha)+(1-\lambda)(1-\beta)}$. Thus, the critical threshold value of firm 2's fundamentals (i.e., switching fundamentals) when the large creditor rolls over loans can be determined by

$$\underline{\theta}_2^* = (1-\lambda_2)[qF(\sigma(\underline{x}_{2,1}^*-\underline{\theta}_2^*))+(1-q)F(\sigma(\underline{x}_{2,K}^*-\underline{\theta}_2^*))]. \tag{A8}$$

On the other hand, if the large creditor does not roll over loans, the proportion of funds that the creditors do not roll over is

$$l_2(\theta_1|\text{Large creditor does not roll over}) = \lambda_2+(1-\lambda_2)[qF(\sigma(\underline{x}_{2,1}^*-\theta_2))+(1-q)F(\sigma(\underline{x}_{2,K}^*-\theta_2))].$$

Accordingly, the switching fundamental value in this case can be determined by

$$\underline{\theta}_2^{**} = \lambda_2+(1-\lambda_2)[qF(\sigma(\underline{x}_{2,1}^*-\underline{\theta}_2^{**}))+(1-q)F(\sigma(\underline{x}_{2,K}^*-\underline{\theta}_2^{**}))]. \tag{A9}$$

The system of Equations (A5), (A6), (A7), (A8), and (A9) leads to a unique solution $(\underline{\theta}_2^*, \underline{\theta}_2^{**}, \underline{y}_2^*, \underline{x}_{2,1}^*, \underline{x}_{2,K}^*)$.

Likewise, we can derive a unique equilibrium $(\overline{\theta}_2^*, \overline{\theta}_2^{**}, \overline{y}_2^*, \overline{x}_{2,0}^*, \overline{x}_{2,K}^*)$ satisfying the following conditions:

$$G(\tau(\overline{y}_2^* - \overline{\theta}_2^*)) = \frac{u_L(K) - u_L(\alpha K)}{u_L(\alpha K + (1-\alpha)) - u_L(\alpha K)}, \tag{A5$'$}$$

$$\frac{u_S((1-\beta)K)}{u_S(1-\beta)} = \int_{\overline{\theta}_2^*}^{\overline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - \overline{x}_{2,0}^*))[1 - G(\tau(\overline{y}_2^* - \theta_2))]d\theta_2 \tag{A6$'$}$$
$$+ \int_{\overline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \overline{x}_{2,0}^*))d\theta_2,$$

$$\frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)} = \int_{\overline{\theta}_2^*}^{\overline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - \overline{x}_{2,K}^*))[1 - G(\tau(\overline{y}_2^* - \theta_2))]d\theta_2 \tag{A7$'$}$$
$$+ \int_{\overline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \overline{x}_{2,K}^*))d\theta_2,$$

$$\overline{\theta}_2^* = (1-\lambda_2)[qF(\sigma(\overline{x}_{2,0}^* - \overline{\theta}_2^*)) + (1-q)F(\sigma(\overline{x}_{2,K}^* - \overline{\theta}_2^*))], \tag{A8$'$}$$
$$\overline{\theta}_2^{**} = \lambda_2 + (1-\lambda_2)[qF(\sigma(\overline{x}_{2,0}^* - \overline{\theta}_2^{**})) + (1-q)F(\sigma(\overline{x}_{2,K}^* - \overline{\theta}_2^{**}))]. \tag{A9$'$}$$

Proof of Lemma 1

We start by transforming Equations (A5), (A6), (A7), (A8), and (A9) with variables $(\underline{\theta}_2^*, \underline{\theta}_2^{**}, \underline{y}_2^*, \underline{x}_{2,1}^*, \underline{x}_{2,K}^*)$ into a more general form of equations with variables $(\theta^*, \theta^{**}, y^*, x_{S1}^*, x_{S2}^*)$ (i.e., Equations (A10), (A11), (A12), (A13), and (A14)) by generalizing some terms in the equations into parameters. The generalized form of equations involves firm 2's equilibrium conditions for both Scenarios 1 and 2 for some particular settings of these parameters. Thus, the contagion effects (i.e., $\mathbf{CE}_L$ and $\mathbf{CE}_S$) are interpreted as changes in θ^{**} and θ^* in response to changes in these parameters.

Now, we define the generalized form of equations. For notational convenience, we define λ_L, λ_{S1}, and λ_{S2} as follows: $\lambda_L := \frac{\lambda(1-\alpha)}{\lambda(1-\alpha)+(1-\lambda)(1-\beta)}$, $\lambda_{S1} := (1 - F(\sigma(x_1^* - \theta_1)))(1-\lambda_L)$, and $\lambda_{S2} := F(\sigma(x_1^* - \theta_1))(1-\lambda_L)$. First, we define two equations that are exactly identical to Equations (A8) and (A9) as follows:

$$\theta^* := \lambda_{S1} F(\sigma(x_{S1}^* - \theta^*)) + \lambda_{S2} F(\sigma(x_{S2}^* - \theta^*)), \tag{A10}$$
$$\theta^{**} := \lambda_L + \lambda_{S1} F(\sigma(x_{S1}^* - \theta^{**})) + \lambda_{S2} F(\sigma(x_{S2}^* - \theta^{**})). \tag{A11}$$

Next, we generalize the left-hand side of Equation (A5) to parameter κ_L as follows:

$$\kappa_L := G(\tau(y^* - \theta^*)). \tag{A12}$$

Similarly, we generalize the left-hand side terms of Equations (A6) and (A7) to parameters κ_{S1} and κ_{S2}, respectively, as follows:

$$\kappa_{S1} := \int_{\theta^*}^{\theta^{**}} \sigma f(\sigma(\theta - x_{S1}^*))[1 - G(\tau(y^* - \theta))]d\theta + \int_{\theta^{**}}^{\infty} \sigma f(\sigma(\theta - x_{S1}^*))d\theta, \tag{A13}$$
$$\kappa_{S2} := \int_{\theta^*}^{\theta^{**}} \sigma f(\sigma(\theta - x_{S2}^*))[1 - G(\tau(y^* - \theta))]d\theta + \int_{\theta^{**}}^{\infty} \sigma f(\sigma(\theta - x_{S2}^*))d\theta. \tag{A14}$$

From the system of Equations (A10), (A11), (A12), (A13), and (A14), we obtain a solution $(\theta^*, \theta^{**}, y_2^*, x_{S1}^*, x_{S2}^*)$, which depends on the relevant parameters κ_L, κ_{S1}, and κ_{S2}. For some settings of these relevant parameters, the solution corresponds to firm 2's equilibrium for Scenarios 1 and 2, respectively. In particular, the equilibrium of firm 2's rollover game for Scenario 1 (i.e., $\underline{\theta}_2^*, \underline{\theta}_2^{**}, \underline{y}_2^*, \underline{x}_{2,1}^*$, and $\underline{x}_{2,K}^*$) corresponds to the case where the relevant parameters κ_L, κ_{S1}, and κ_{S2} are given as follows:

$$\begin{aligned}
\underline{\kappa}_L &= \frac{u_L(\alpha + (1-\alpha)K) - u_L(\alpha)}{u_L(1) - u_L(\alpha)}, \\
\underline{\kappa}_{S1} &= \frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)}, \\
\underline{\kappa}_{S2} &= \frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)}.
\end{aligned}$$

On the other hand, the equilibrium of firm 2's rollover game for Scenario 2 (i.e., $\overline{\theta}_2^*, \overline{\theta}_2^{**}, \overline{y}_2^*, \overline{x}_{2,0}^*$, and $\overline{x}_{2,K}^*$) corresponds to the case where κ_L, κ_{S1}, and κ_{S2} are given as follows:

$$\begin{aligned}
\overline{\kappa}_L &= \frac{u_L(K) - u_L(\alpha K)}{u_L(\alpha K + (1-\alpha)) - u_L(\alpha K)}, \\
\overline{\kappa}_{S1} &= \frac{u_S((1-\beta)K)}{u_S(1-\beta)}, \\
\overline{\kappa}_{S2} &= \frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)}.
\end{aligned}$$

Note that $\underline{\theta}_2^*$ and $\underline{\theta}_2^{**}$ correspond to θ^* and θ^{**} for the cases $\kappa_L = \underline{\kappa}_L$, $\kappa_{S1} = \underline{\kappa}_{S1}$, and $\kappa_{S2} = \underline{\kappa}_{S2}$. Similarly, $\overline{\theta}_2^*$ and $\overline{\theta}_2^{**}$ correspond to θ^* and θ^{**} for the cases $\kappa_L = \overline{\kappa}_L$, $\kappa_{S1} = \overline{\kappa}_{S1}$, and $\kappa_{S2} = \overline{\kappa}_{S2}$. Thus, we can show that θ^* and θ^{**} are greater for the cases $\kappa_L = \overline{\kappa}_L$, $\kappa_{S1} = \overline{\kappa}_{S1}$, and $\kappa_{S2} = \overline{\kappa}_{S2}$ than for the cases $\kappa_L = \underline{\kappa}_L$, $\kappa_{S1} = \underline{\kappa}_{S1}$, and $\kappa_{S2} = \underline{\kappa}_{S2}$. For this, we first prove that $\underline{\kappa}_L < \overline{\kappa}_L$, $\underline{\kappa}_{S1} < \overline{\kappa}_{S1}$, and $\underline{\kappa}_{S2} = \overline{\kappa}_{S2}$. Then, we prove that θ^* and θ^{**} are increasing in κ_L and κ_{S1}.

Let us now show that $\underline{\kappa}_L < \overline{\kappa}_L$, $\underline{\kappa}_{S1} < \overline{\kappa}_{S1}$, and $\underline{\kappa}_{S2} = \overline{\kappa}_{S2}$. The third one, that is, $\underline{\kappa}_{S2} = \overline{\kappa}_{S2}$, trivially holds by definition. To show that $\underline{\kappa}_L < \overline{\kappa}_L$, we define $v_1(w) := u_L(\alpha + w)$ and $v_2(w) := u_L(\alpha K + w)$ for $w \geq 0$. A decreasing absolute risk aversion of u_L implies that $v_1(w)$ has a lower absolute risk aversion than $v_2(w)$ for every w. This, in turn, implies that, from Theorem 1(e) of Pratt [1964],

$$\frac{v_1(1-\alpha) - v_1((1-\alpha)K)}{v_1((1-\alpha)K) - v_1(0)} > \frac{v_2(1-\alpha) - v_2((1-\alpha)K)}{v_2((1-\alpha)K) - v_2(0)}.$$

By adding 1 and then taking the inverse of the left- and right-hand sides, respectively, we find that

$$\frac{v_1((1-\alpha)K) - v_1(0)}{v_1(1-\alpha) - v_1(0)} < \frac{v_2((1-\alpha)K) - v_2(0)}{v_2(1-\alpha) - v_2(0)}.$$

From this, it follows that $\underline{\kappa}_L < \overline{\kappa}_L$, because the left-hand side corresponds to $\underline{\kappa}_L$ and the right-hand side corresponds to $\overline{\kappa}_L$. Similarly, we obtain $\underline{\kappa}_{S1} < \overline{\kappa}_{S1}$ by defining $v_3(w) := u_S(\beta + w)$ and $v_4(w) := u_S(w)$ for $w \geq 0$ and then applying Theorem 1(e) of Pratt [1964].

Now, we proceed to show that θ^* and θ^{**} are increasing in κ_L and κ_{S1}. We first define the normalized variables as follows: $\delta^*_{S1} := \sigma(\theta^* - x^*_{S1})$, $\delta^*_{S2} := \sigma(\theta^* - x^*_{S2})$, $D := \sigma(\theta^{**} - \theta^*)$, and $r := \tau/\sigma$. By substituting $z = \sigma(\theta - \theta^*)$ for θ in the integral terms of (A13) and (A14), respectively, we have

$$\kappa_{S1} = \int_0^D f(z + \delta^*_{S1})[1 - G(-rz + G^{-1}(\kappa_L))]dz + \int_D^\infty f(z + \delta^*_{S1})dz, \qquad \text{(A15)}$$

$$\kappa_{S2} = \int_0^D f(z + \delta^*_{S2})[1 - G(-rz + G^{-1}(\kappa_L))]dz + \int_D^\infty f(z + \delta^*_{S2})dz, \qquad \text{(A16)}$$

because $\tau(y^* - \theta) = \tau(y^* - \theta^*) + \tau(\theta^* - \theta) = G^{-1}(\kappa_L) - rz$ from Equation (A12).

On the other hand, from the definition of the normalized variables δ^*_{S1}, δ^*_{S2}, and D, Equations (A10) and (A11) are respectively equivalent to

$$\theta^* = \lambda_{S1}F(-\delta^*_{S1}) + \lambda_{S2}F(-\delta^*_{S2}),$$
$$\theta^{**} = \lambda_L + \lambda_{S1}F(-\delta^*_{S1} - D) + \lambda_{S2}F(-\delta^*_{S2} - D).$$

By applying these equations to the definition of D, we obtain the following equality:

$$\begin{aligned} D &= \sigma(\theta^{**} - \theta^*) \\ &= \sigma(\lambda_L + \lambda_{S1}F(-\delta^*_{S1} - D) + \lambda_{S2}F(-\delta^*_{S2} - D) - \lambda_{S1}F(-\delta^*_{S1}) - \lambda_{S2}F(-\delta^*_{S2})). \end{aligned}$$

By dividing both sides by σ and arranging the terms, we have

$$\lambda_L - \frac{D}{\sigma} = \lambda_{S1}(F(-\delta^*_{S1}) - F(-\delta^*_{S1} - D)) + \lambda_{S2}(F(-\delta^*_{S2}) - F(-\delta^*_{S2} - D)). \quad \text{(A17)}$$

For the sake of notational convenience, we define the function $\varphi(\delta, \kappa_L, r, D)$ as follows:

$$\varphi(\delta, D, \kappa_L, r) := \int_0^D f(z + \delta)[1 - G(-rz + G^{-1}(\kappa_L))]dz + \int_D^\infty f(z + \delta)dz. \quad \text{(A18)}$$

Then, since Equations (A15) and (A16) are equivalent to $\kappa_{S1} = \varphi(\delta^*_{S1}, D, \kappa_L, r)$ and $\kappa_{S2} = \varphi(\delta^*_{S2}, D, \kappa_L, r)$, respectively, the following lemma holds:

Lemma 2. *$\varphi_1(\delta, D, \kappa_L, r) < \varphi_2(\delta, D, \kappa_L, r) < 0$, $\varphi_3(\delta, D, \kappa_L, r) < 0$, and $\varphi_4(\delta, D, \kappa_L, r) < 0$.*

Proof. Note that $\int_D^\infty f'(z + \delta)dz = -f(D + \delta)$ holds because

$$\begin{aligned} f(D + \delta) + \int_D^\infty f'(z + \delta) &= \int_{-\infty}^\infty f'(z + \delta)dz = \int_{-\infty}^\infty \frac{d}{d\delta} f(z + \delta)dz \\ &= \frac{d}{d\delta}\int_{-\infty}^\infty f(z + \delta)dz = \frac{d}{d\delta}(1) = 0. \end{aligned}$$

By differentiating φ with respect to δ and then using the integration by part together with the above equality, we have

$$\begin{aligned}
\varphi_1(\delta, D, \kappa_L, r) &= \int_0^D f'(z+\delta))[1-G(-rz+G^{-1}(\kappa_L))]dz + \int_D^\infty f'(z+\delta)dz \\
&= \left[f(z+\delta)[1-G(-rz+G^{-1}(\kappa_L))]\right]_0^D \\
&- \int_0^D rf(z+\delta)g(-rz+G^{-1}(\kappa_L))dz - f(D+\delta) \\
&= f(D+\delta)[1-G(-rD+G^{-1}(\kappa_L))] - f(\delta)[1-G(G^{-1}(\kappa_L))] \\
&- \int_0^D rf(z+\delta)g(-rz+G^{-1}(\kappa_L))dz - f(D+\delta) \\
&= -f(\delta)(1-\kappa_L) - G(-rD+G^{-1}(\kappa_L))f(D+\delta) \\
&- \int_0^D rf(z+\delta)g(-rz+G^{-1}(\kappa_L))dz.
\end{aligned}$$

On the other hand, by differentiating φ with respect to D, we have

$$\varphi_2(\delta, D, \kappa_L, r) = -G(-rD+G^{-1}(\kappa_L))f(D+\delta).$$

Now, we can straightforwardly show that $\varphi_1(\delta, D, \kappa_L, r) < 0$ and $\varphi_2(\delta, D, \kappa_L, r) < 0$. Further, we have $\varphi_1(\delta, D, \kappa_L, r) < \varphi_2(\delta, D, \kappa_L, r)$, because

$$\varphi_1(\delta, D, \kappa_L, r) - \varphi_2(\delta, D, \kappa_L, r) = -f(\delta)(1-\kappa_L) - \int_0^D rf(z+\delta)g(-rz+G^{-1}(\kappa_L))dz < 0.$$

Similarly, by differentiating φ with respect to κ_L and r, we have

$$\begin{aligned}
\varphi_3(\delta, D, \kappa_L, r) &= -\frac{1}{g(G^{-1}(\kappa_L))}\int_0^D f(z+\delta)g(-rz+G^{-1}(\kappa_L))dz < 0, \\
\varphi_4(\delta, D, \kappa_L, r) &= -r\int_0^D f(z+\delta)g(-rz+G^{-1}(\kappa_L))dz < 0.
\end{aligned}$$

∎

Now, we can show that θ^* and θ^{**} are increasing in κ_L. By differentiating Equations (A15), (A16), and (A17) with respect to κ_L, we have

$$0 = \varphi_3(\delta_{S1}^*, D, \kappa_L, r) + \varphi_1(\delta_{S1}^*, D, \kappa_L, r)\frac{d\delta_{S1}^*}{d\kappa_L} + \varphi_2(\delta_{S1}^*, D, \kappa_L, r)\frac{dD}{d\kappa_L}, \quad \text{(A19)}$$

$$0 = \varphi_3(\delta_{S2}^*, D, \kappa_L, r) + \varphi_1(\delta_{S2}^*, D, \kappa_L, r)\frac{d\delta_{S2}^*}{d\kappa_L} + \varphi_2(\delta_{S2}^*, D, \kappa_L, r)\frac{dD}{d\kappa_L}, \quad \text{(A20)}$$

$$\begin{aligned}
\frac{1}{\sigma}\frac{dD}{d\kappa_L} &= \lambda_{S1}f(-\delta_{S1}^*)\frac{d\delta_{S1}^*}{d\kappa_L} - \lambda_{S1}f(-\delta_{S1}^*-D)\left(\frac{d\delta_{S1}^*}{d\kappa_L} + \frac{dD}{d\kappa_L}\right) \qquad \text{(A21)} \\
&+ \lambda_{S2}f(-\delta_{S2}^*)\frac{d\delta_{S2}^*}{d\kappa_L} - \lambda_{S2}f(-\delta_{S2}^*-D)\left(\frac{d\delta_{S2}^*}{d\kappa_L} + \frac{dD}{d\kappa_L}\right).
\end{aligned}$$

From now on, we abbreviate $\varphi(\delta, D, \kappa_L, r)$ as $\varphi(\delta)$ for notational convenience. By solving the system of linear equations (A19), (A20), and (A21) with respect to $\frac{d\delta_{S1}^*}{d\kappa_L}$,

$\frac{d\delta^*_{S2}}{d\kappa_L}$, and $\frac{dD}{d\kappa_L}$, we have

$$\begin{aligned}\frac{d\delta^*_{S1}}{d\kappa_L} &= -\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})} + \frac{1}{\Delta_1}\frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\lambda_{S1}\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}(f(-\delta^*_{S1}) - f(-\delta^*_{S1} - D)) \qquad \text{(A22)}\\ &\quad + \frac{1}{\Delta_1}\frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\lambda_{S2}\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}(f(-\delta^*_{S2}) - f(-\delta^*_{S2} - D)),\\ \frac{d\delta^*_{S2}}{d\kappa_L} &= -\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})} + \frac{1}{\Delta_1}\frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}\lambda_{S1}\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}(f(-\delta^*_{S1}) - f(-\delta^*_{S1} - D)) \qquad \text{(A23)}\\ &\quad + \frac{1}{\Delta_1}\frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}\lambda_{S2}\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}(f(-\delta^*_{S2}) - f(-\delta^*_{S2} - D)),\\ \frac{dD}{d\kappa_L} &= -\frac{1}{\Delta_1}\lambda_{S1}\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\left(f(-\delta^*_{S1}) - f(-\delta^*_{S1} - D)\right) \qquad \text{(A24)}\\ &\quad - \frac{1}{\Delta_1}\lambda_{S2}\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}\left(f(-\delta^*_{S2}) - f(-\delta^*_{S2} - D)\right),\end{aligned}$$

where Δ_1 is defined as

$$\begin{aligned}\Delta_1 &= \frac{1}{\sigma} + \lambda_{S1}\frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}f(-\delta^*_{S1}) + \lambda_{S1}\left(1 - \frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\right)f(-\delta^*_{S1} - D)\\ &\quad + \lambda_{S2}\frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}f(-\delta^*_{S2}) + \lambda_{S2}\left(1 - \frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}\right)f(-\delta^*_{S2} - D).\end{aligned}$$

By differentiating $\theta^* = \lambda_{S1}F(-\delta^*_{S1}) + \lambda_{S2}F(-\delta^*_{S2})$ with respect to κ_L and then substituting $\frac{d\delta^*_{S1}}{d\kappa_L}$ and $\frac{d\delta^*_{S2}}{d\kappa_L}$ with (A22) and (A23), we have

$$\begin{aligned}\frac{d\theta^*}{d\kappa_L} &= -\lambda_{S1}f(-\delta^*_{S1})\frac{d\delta^*_{S1}}{d\kappa_L} - \lambda_{S2}f(-\delta^*_{S2})\frac{d\delta^*_{S2}}{d\kappa_L} \qquad \text{(A25)}\\ &= \lambda_{S1}\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}f(-\delta^*_{S1}) + \lambda_{S2}\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}f(-\delta^*_{S2})\\ &\quad + \left(\lambda_{S1}\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}f(-\delta^*_{S1}) + \lambda_{S2}\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}f(-\delta^*_{S2})\right)\frac{1}{\Delta_1}\\ &\quad \times \left(-\lambda_{S1}\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}[f(-\delta^*_{S1}) - f(-\delta^*_{S1} - D)] - \lambda_{S2}\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}[f(-\delta^*_{S2}) - f(-\delta^*_{S2} - D)]\right)\\ &= \lambda_{S1}\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}f(-\delta^*_{S1})\left(1 - \frac{\Delta_2}{\Delta_1}\right) + \lambda_{S2}\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}f(-\delta^*_{S2})\left(1 - \frac{\Delta_2}{\Delta_1}\right)\\ &\quad + \lambda_{S1}\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}f(-\delta^*_{S1} - D)\frac{\Delta_2}{\Delta_1} + \lambda_{S2}\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}f(-\delta^*_{S2} - D)\frac{\Delta_2}{\Delta_1},\end{aligned}$$

where $\Delta_2 := \lambda_{S1}f(-\delta^*_{S1})\frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})} + \lambda_{S2}f(-\delta^*_{S2})\frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}$. Lemma 2 implies that $0 < \frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})} < 1$ and $0 < \frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})} < 1$, leading to $\Delta_1 > 0$ and $\Delta_2 > 0$. Further, this holds that

$$\Delta_1 - \Delta_2 = \frac{1}{\sigma} + \lambda_{S1}f(-\delta^*_{S1} - D)\left(1 - \frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\right) + \lambda_{S2}f(-\delta^*_{S2} - D)\left(1 - \frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}\right) > 0,$$

implying that $0 < \frac{\Delta_2}{\Delta_1} < 1$. On the other hand, from Lemma 2, it holds that $\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})} > 0$ and $\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})} > 0$. Therefore, these facts (i.e., $0 < \frac{\Delta_2}{\Delta_1} < 1$, $\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})} > 0$, and $\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})} > 0$) lead to $\frac{d\theta^*}{d\kappa_L} > 0$ from Equation (A25). Similarly, by differentiating $\theta^{**} = \lambda_L + \lambda_{S1}F(-\delta^*_{S1}) + \lambda_{S2}F(-\delta^*_{S2})$ with respect to κ_L, we have

$$\frac{d\theta^{**}}{d\kappa_L} = -\lambda_{S1}f(-\delta^*_{S1} - D)\left(\frac{d\delta^*_{S1}}{d\kappa_L} + \frac{dD}{d\kappa_L}\right) - \lambda_{S2}f(-\delta^*_{S2} - D)\left(\frac{d\delta^*_{S2}}{d\kappa_L} + \frac{dD}{d\kappa_L}\right) \tag{A26}$$
$$= \lambda_{S1}f(-\delta^*_{S1} - D)\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})} + \lambda_{S2}f(-\delta^*_{S2} - D)\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}$$
$$+ \left(\lambda_{S1}f(-\delta^*_{S1} - D)\left(1 - \frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\right) + \lambda_{S2}f(-\delta^*_{S2} - D)\left(1 - \frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}\right)\right)\frac{1}{\Delta_1}$$
$$\times\left(\lambda_{S1}\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}[f(-\delta^*_{S1}) - f(-\delta^*_{S1} - D)] + \lambda_{S2}\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}[f(-\delta^*_{S2}) - f(-\delta^*_{S2} - D)]\right)$$
$$= \lambda_{S1}f(-\delta^*_{S1})\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\frac{\Delta_3}{\Delta_1} + \lambda_{S2}f(-\delta^*_{S2})\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}\frac{\Delta_3}{\Delta_1}$$
$$+ \lambda_{S1}f(-\delta^*_{S1} - D)\frac{\varphi_3(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\left(1 - \frac{\Delta_3}{\Delta_1}\right) + \lambda_{S2}f(-\delta^*_{S2} - D)\frac{\varphi_3(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}\left(1 - \frac{\Delta_3}{\Delta_1}\right),$$

where $\Delta_3 := \lambda_{S1}f(-\delta^*_{S1} - D)\left(1 - \frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\right) + \lambda_{S2}f(-\delta^*_{S2} - D)\left(1 - \frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}\right)$. We have $\Delta_3 > 0$, because Lemma 2 implies $0 < \frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})} < 1$ and $0 < \frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})} < 1$. Also, $\Delta_1 - \Delta_3 = \frac{1}{\sigma} + \lambda_{S1}\frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}f(-\delta^*_{S1}) + \lambda_{S2}\frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}f(-\delta^*_{S2}) > 0$ holds leading to $0 < \frac{\Delta_3}{\Delta_1} < 1$. Therefore, from Equation (A26), we have $\frac{d\theta^{**}}{d\kappa_L} > 0$.

Finally, we can show that both θ^* and θ^{**} are increasing in κ_{S1}. By differentiating Equations (A15), (A16), and (A17) with respect to κ_{S1}, we have

$$1 = \frac{d\delta^*_{S1}}{d\kappa_{S1}}\varphi_1(\delta^*_{S1}, D, \kappa_L, r) + \frac{dD}{d\kappa_{S1}}\varphi_2(\delta^*_{S1}, D, \kappa_L, r), \tag{A27}$$
$$0 = \frac{d\delta^*_{S2}}{d\kappa_{S1}}\varphi_1(\delta^*_{S2}, D, \kappa_L, r) + \frac{dD}{d\kappa_{S1}}\varphi_2(\delta^*_{S2}, D, \kappa_L, r), \tag{A28}$$
$$\frac{1}{\sigma}\frac{dD}{d\kappa_{S1}} = \lambda_{S1}f(-\delta^*_{S1})\frac{d\delta^*_{S1}}{d\kappa_{S1}} + \lambda_{S1}f(-\delta^*_{S1} - D)\left(-\frac{d\delta^*_{S1}}{d\kappa_{S1}} - \frac{dD}{d\kappa_{S1}}\right) \tag{A29}$$
$$+ \lambda_{S2}f(-\delta^*_{S2})\frac{d\delta^*_{S2}}{d\kappa_{S1}} + \lambda_{S2}f(-\delta^*_{S2} - D)\left(-\frac{d\delta^*_{S2}}{d\kappa_{S1}} - \frac{dD}{d\kappa_{S1}}\right).$$

By solving the system of linear equations (A27), (A28), and (A29) with respect to $\frac{d\delta^*_{S1}}{d\kappa_L}$, $\frac{d\delta^*_{S2}}{d\kappa_L}$, and $\frac{dD}{d\kappa_L}$, we have

$$\frac{d\delta^*_{S1}}{d\kappa_{S1}} = \frac{1}{\varphi_1(\delta^*_{S1})} - \lambda_{S1}\frac{\varphi_2(\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}[f(-\delta^*_{S1}) - f(-\delta^*_{S1} - D)]\frac{1}{\varphi_1(\delta^*_{S1})\Delta_1}, \tag{A30}$$
$$\frac{d\delta^*_{S2}}{d\kappa_{S1}} = -\lambda_{S1}\frac{\varphi_2(\delta^*_{S2})}{\varphi_1(\delta^*_{S2})}[f(-\delta^*_{S1}) - f(-\delta^*_{S1} - D)]\frac{1}{\varphi_1(\delta^*_{S1})\Delta_1}, \tag{A31}$$
$$\frac{dD}{d\kappa_{S1}} = \lambda_{S1}[f(-\delta^*_{S1}) - f(-\delta^*_{S1} - D)]\frac{1}{\varphi_1(\delta^*_{S1})\Delta_1}. \tag{A32}$$

By differentiating $\theta^* = \lambda_{S1}F(-\delta^*_{S1}) + \lambda_{S2}F(-\delta^*_{S2})$ with respect to κ_{S1} and then substituting $\frac{d\delta^*_{S1}}{d\kappa_L}$, $\frac{d\delta^*_{S2}}{d\kappa_L}$, and $\frac{dD}{d\kappa_L}$ with (A30), (A31), and (A32), we have

$$\begin{aligned}\frac{d\theta^*}{d\kappa_{S1}} &= -\lambda_{S1}f(-\delta^*_{S1})\frac{d\delta^*_{S1}}{d\kappa_{S1}} - \lambda_{S2}f(-\delta^*_{S2})\frac{d\delta^*_{S2}}{d\kappa_{S1}} \\ &= -\lambda_{S1}\frac{f(-\delta^*_{S1})}{\varphi_1(\delta^*_{S1})} + \lambda_{S1}\frac{f(-\delta^*_{S1}) - f(-\delta^*_{S1} - D)}{\varphi_1(\delta^*_{S1})}\frac{\Delta_2}{\Delta_1} \\ &= -\lambda_{S1}\frac{f(-\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\left(1 - \frac{\Delta_2}{\Delta_1}\right) - \lambda_{S1}\frac{f(-\delta^*_{S1} - D)}{\varphi_1(\delta^*_{S1})}\frac{\Delta_2}{\Delta_1} > 0,\end{aligned} \tag{A33}$$

because $0 < \Delta_2 < \Delta_1$ and $\varphi_1(\delta^*_{S1}) < 0$ from Lemma 2. Further, by differentiating $\theta^{**} = \lambda_L + \lambda_{S1}F(-\delta^*_{S1} - D) + \lambda_{S2}F(-\delta^*_{S2} - D)$ with respect to κ_{S1} and then substituting $\frac{d\delta^*_{S1}}{d\kappa_L}$, $\frac{d\delta^*_{S2}}{d\kappa_L}$, and $\frac{dD}{d\kappa_L}$ with (A30), (A31), and (A32), we have

$$\frac{d\theta^{**}}{d\kappa_{S1}} = -\lambda_{S1}f(-\delta^*_{S1} - D)\left(\frac{d\delta^*_{S1}}{d\kappa_{S1}} + \frac{dD}{d\kappa_{S1}}\right) - \lambda_{S2}f(-\delta^*_{S2} - D)\left(\frac{d\delta^*_{S2}}{d\kappa_{S1}} + \frac{dD}{d\kappa_{S1}}\right) \tag{A34}$$

$$\begin{aligned}&= -\lambda_{S1}\frac{f(-\delta^*_{S1} - D)}{\varphi_1(\delta^*_{S1})} - \frac{\Delta_3}{\Delta_1}\lambda_{S1}\frac{f(-\delta^*_{S1}) - f(-\delta^*_{S1} - D)}{\varphi_1(\delta^*_{S1})} \\ &= -\lambda_{S1}\frac{f(-\delta^*_{S1})}{\varphi_1(\delta^*_{S1})}\frac{\Delta_3}{\Delta_1} - \lambda_{S1}\frac{f(-\delta^*_{S1} - D)}{\varphi_1(\delta^*_{S1})}\left(1 - \frac{\Delta_3}{\Delta_1}\right) > 0,\end{aligned}$$

because $0 < \Delta_3 < \Delta_1$ and $\varphi_1(\delta^*_{S1}) < 0$ from Lemma 2.

Proof of Proposition 2

If $\lambda \to 1$, by the assumption that $0 \le \alpha < 1$, we have $\lambda_2 := \frac{\lambda(1-\alpha)}{\lambda(1-\alpha)+(1-\lambda)(1-\beta)} \to 1$. Then, Equations (A8) and (A9) imply that $\underline{\theta}^*_2 = 0$ and $\underline{\theta}^{**}_2 = 1$ for firm 2's rollover game in Scenario 1. Similarly, for firm 2's rollover game in Scenario 2, we have $\overline{\theta}^*_2 = 0$ and $\overline{\theta}^{**}_2 = 1$. Therefore, it follows that $\mathbf{CE}_L = \mathbf{CE}_S = 0$.

Proof of Proposition 3

For the first part of the proof, let us denote by $\mathbf{CE}_{L0}$ and $\mathbf{CE}_{S0}$ the contagion effects when $\lambda = 0$. First, we will show that $\mathbf{CE}_{L0} > 0$ and $\mathbf{CE}_{S0} > 0$. Then, these facts and Proposition 2 imply that $\mathbf{CE}_L < \mathbf{CE}_{L0}$ and $\mathbf{CE}_S < \mathbf{CE}_{S0}$ for sufficiently large $\lambda > 0$. Next, we proceed to show that $\lim_{\alpha\to 1}\mathbf{CE}_L > \mathbf{CE}_{L0}$ and $\lim_{\alpha\to 1}\mathbf{CE}_S > \mathbf{CE}_{S0}$ because these imply that $\lim_{\alpha\to 1}\mathbf{CE}_L > \mathbf{CE}_L$ and $\lim_{\alpha\to 1}\mathbf{CE}_S > \mathbf{CE}_S$.

If $\lambda = 0$, the equilibrium conditions of Proposition 1 (i.e., (A3) and (A4)) would imply that $\theta^*_1 = F(\sigma(x^*_1 - \theta^*_1))$ and $\theta^{**}_1 = F(\sigma(x^*_1 - \theta^{**}_1))$. That is, $\theta^*_1 = \theta^{**}_1$. Then, from Equation (A2), we have

$$\begin{aligned}u_S(\beta K + (1-\beta)) - u_S(1-\beta) &= \{u_S(\beta + (1-\beta)) - u_S(1-\beta)\}\int_{\theta^{**}_1}^{\infty}\sigma f(\sigma(\theta_1 - x^*_1))d\theta_1 \\ &= \{u_S(1) - u_S(1-\beta)\}(1 - F(\sigma(\theta^{**}_1 - x^*_1)) \\ &= \{u_S(1) - u_S(1-\beta)\}F(\sigma(x^*_1 - \theta^{**}_1)),\end{aligned}$$

leading to

$$F(\sigma(x_1^* - \theta_1^*)) = F(\sigma(x_1^* - \theta_1^{**})) = \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)}. \tag{A35}$$

For firm 2's rollover game following the case of no liquidity crisis in firm 1, we apply $\lambda = 0$ and Equation (A35) to (A8) and (A9) to obtain respectively

$$\begin{aligned}
\underline{\theta}_2^* =& \left(1 - \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)}\right) F(\sigma(\underline{x}_{2,1}^* - \underline{\theta}_2^*)) \\
&+ \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)} F(\sigma(\underline{x}_{2,K}^* - \underline{\theta}_2^*)), \\
\underline{\theta}_2^{**} =& \left(1 - \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)}\right) F(\sigma(\underline{x}_{2,1}^* - \underline{\theta}_2^{**})) \\
&+ \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)} F(\sigma(\underline{x}_{2,K}^* - \underline{\theta}_2^{**})).
\end{aligned}$$

That is, $\underline{\theta}_2^{**} = \underline{\theta}_2^*$. From (A6), (A7), and $\underline{\theta}_2^* = \underline{\theta}_2^{**}$, we have

$$\begin{aligned}
\frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)} &= \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,1}^*))d\theta_2 = 1 - F(\sigma(\underline{\theta}_2^{**} - \underline{x}_{2,1}^*)), \\
\frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)} &= \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,K}^*))d\theta_2 = 1 - F(\sigma(\underline{\theta}_2^{**} - \underline{x}_{2,K}^*)).
\end{aligned}$$

Accordingly, by applying $\underline{\theta}_2^{**} = \underline{\theta}_2^*$ and the above conditions to Equation (A8), we obtain the following results:

$$\begin{aligned}
\underline{\theta}_2^* =& \left(1 - \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)}\right) F(\sigma(\underline{x}_{2,1}^* - \underline{\theta}_2^*)) \\
&+ \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)} F(\sigma(\underline{x}_{2,K}^* - \underline{\theta}_2^*)) \\
=& \left(1 - \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)}\right) (1 - F(\sigma(\underline{\theta}_2^* - \underline{x}_{2,1}^*))) \\
&+ \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)} (1 - F(\sigma(\underline{\theta}_2^* - \underline{x}_{2,K}^*))) \\
=& \left(1 - \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)}\right) \frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)} \\
&+ \left(\frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)}\right) \frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)}
\end{aligned}$$

as well as $\underline{\theta}_2^{**} = \underline{\theta}_2^*$. For the rollover game of firm 2 following a liquidity crisis in firm 1, the same argument holds. In particular, we have

$$\begin{aligned}
\overline{\theta}_2^* =& \left(1 - \frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)}\right) \frac{u_S((1-\beta)K)}{u_S(1-\beta)} \\
&+ \left(\frac{u_S(\beta K + (1-\beta)) - u_S\,(1-\beta)}{u_S(1) - u_S\,(1-\beta)}\right) \frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)}
\end{aligned}$$

as well as $\overline{\theta}_2^{**} = \overline{\theta}_2^{*}$. Hence, the contagion effects are as follows:

$$\begin{aligned}\mathbf{CE}_L &= \mathbf{CE}_S \\ &= \left(1 - \frac{u_S(\beta K + (1-\beta)) - u_S(1-\beta)}{u_S(1) - u_S(1-\beta)}\right)\left(\frac{u_S((1-\beta)K)}{u_S(1-\beta)} - \frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)}\right) > 0.\end{aligned}$$

Our next step is to show that $\lim_{\alpha\to 1}\mathbf{CE}_L > \mathbf{CE}_{L0}$ and $\lim_{\alpha\to 1}\mathbf{CE}_S > \mathbf{CE}_{S0}$. As $\alpha \to 1$, for firm 2's equilibrium $(\underline{\theta}_2^*, \underline{\theta}_2^{**}, \underline{y}_2^*, \underline{x}_{2,1}^*, \underline{x}_{2,K}^*)$ for Scenario 1, Equations (A8) and (A9) imply that

$$\underline{\theta}_2^* = \underline{\theta}_2^{**} = qF(\sigma(\underline{x}_{2,1}^* - \underline{\theta}_2^*)) + (1-q)F(\sigma(\underline{x}_{2,K}^* - \underline{\theta}_2^*)), \tag{A36}$$

where $q(\theta_1) = 1 - F(\sigma(x_1^* - \theta_1))$ is the proportion of small creditors who roll over loans for firm 1 when the state of fundamentals of firm 1 is $\theta_1 \in [\theta_1^*, \theta_1^{**}]$. On the other hand, by applying $\alpha = 1$ and $\underline{\theta}_2^* = \underline{\theta}_2^{**}$ to Equations (A6) and (A7), we obtain

$$\frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)} = \int_{\underline{\theta}_2^*}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,1}^*))d\theta_2 = F(\sigma(\underline{x}_{2,1}^* - \underline{\theta}_2^*)), \tag{A37}$$

$$\frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)} = \int_{\underline{\theta}_2^*}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,K}^*))d\theta_2 = F(\sigma(\underline{x}_{2,K}^* - \underline{\theta}_2^*)). \tag{A38}$$

By substituting $F(\sigma(\underline{x}_{2,1}^* - \underline{\theta}_2^*))$ and $F(\sigma(\underline{x}_{2,K}^* - \underline{\theta}_2^*))$ of Equation (A36) with Equations (A37) and (A38), we have

$$\underline{\theta}_2^* = \underline{\theta}_2^{**} = q\frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)} + (1-q)\frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)}.$$

Similarly, the switching fundamentals of firm 2 for Scenario 2 are represented as follows:

$$\overline{\theta}_2^* = \overline{\theta}_2^{**} = q\frac{u_S((1-\beta)K)}{u_S(1-\beta)} + (1-q)\frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)}.$$

Accordingly, the contagion effects are as follows:

$$\mathbf{CE}_L = \mathbf{CE}_S = q\left(\frac{u_S((1-\beta)K)}{u_S(1-\beta)} - \frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)}\right).$$

That is, $\mathbf{CE}_L$ and $\mathbf{CE}_S$ are increasing with q.

What we have to show is that, conditional on $\alpha = 1$, q increases as the large creditor's relative size increases from zero to λ, implying that $\lim_{\alpha\to 1}\mathbf{CE}_L > \mathbf{CE}_{L0}$ and $\lim_{\alpha\to 1}\mathbf{CE}_S > \mathbf{CE}_{S0}$. If we define the normalized variables as $\delta_1^* := \sigma(\theta_1^* - x_1^*)$, $D_1 := \sigma(\theta_1^{**} - \theta_1^*)$, and $r := \tau/\sigma$, then we have $1 - F(-\delta_1^*) \le q(\theta_1) \le 1 - F(-\delta_1^* - D_1)$ because $\theta_1 \in [\theta_1^*, \theta_1^{**}]$. We first show that, as the large creditor's size increases from zero to some positive value λ, D_1 also increases from zero to some positive value. Then, we show that δ_1^* also increases. For firm 1's rollover game, Equations (A3) and (A4) are equivalent to

$$\theta_1^* = (1-\lambda_1)F(-\delta_1^*), \tag{A39}$$

$$\theta_1^{**} = \lambda_1 + (1-\lambda_1)F(-\delta_1^* - D_1), \tag{A40}$$

where $\lambda_1 = \frac{\lambda\alpha}{\lambda\alpha+(1-\lambda)\beta}$ is the proportion of firm 1's loans held by the large creditor.

If $\lambda = 0$, it holds that $\theta_1^* = \theta_1^{**}$, because by applying $\lambda = 0$ to Equations (A3) and (A4), we have $\theta_1^* = F(\sigma(x_1^* - \theta_1^*))$ and $\theta_1^{**} = F(\sigma(x_1^* - \theta_1^{**}))$. Also, by applying $\lambda = 0$ to Equations (A39) and (A40), we have $\theta_1^* = F(-\delta_1^*)$ and $\theta_1^{**} = F(-\delta_1^* - D_1)$. Combining these with the equality $\theta_1^* = \theta_1^{**}$, we obtain $D_1 = 0$. This implies that D_1 increases as the size of the large creditor increases from zero to $\lambda(> 0)$.

Since $q(\theta_1) \in [1 - F(-\delta_1^*), 1 - F(-\delta_1^* - D_1)]$, we need to show that δ_1^* increases as the large creditor's size increases from zero to λ. From the definitions of the normalized variables δ_1^*, D_1, and r, and by substituting $z = \sigma(\theta_1 - \theta_1^*)$ into the integral terms in Equation (A2), we have

$$
\begin{aligned}
&\frac{u_S(\beta K + (1-\beta)) - u_S(1-\beta)}{u_S(1) - u_S(1-\beta)} \\
&= \int_0^{D_1} f(z + \delta_1^*)\left[1 - G\left(-rz + G^{-1}\left(\frac{u_L(\alpha K + (1-\alpha)) - u_L(1-\alpha)}{u_L(1) - u_L(1-\alpha)}\right)\right)\right] dz \\
&+ \int_{D_1}^{\infty} f(z + \delta_1^*) dz.
\end{aligned} \tag{A41}
$$

This is equivalent to

$$
\frac{u_S(\beta K + (1-\beta)) - u_S(1-\beta)}{u_S(1) - u_S(1-\beta)} = \int_0^{\infty} f(z + \delta_1^*) T(z, D_1) dz,
$$

where

$$
T(z, D_1) = \begin{cases} 0 & \text{if } z < 0; \\ 1 - G\left(-rz + G^{-1}\left(\frac{u_L(\alpha K + (1-\alpha))}{u_L(1)}\right)\right) & \text{if } z \in [0, D_1]; \\ 1 & \text{if } z > D_1. \end{cases}
$$

Here, $T(z, D_1)$ is increasing in z. This implies that the right-hand side of the equation is increasing in δ_1^*. Further, the right-hand side of the equation is decreasing in D_1, which means that δ_1^* should increase with an increase in D_1 because the left-hand side of the equation is constant. Therefore, as the large creditor's size increases from zero to λ, D_1 increases from zero to some positive value and δ_1^* also increases. That is, $q(\theta_1)$ increases as well, because $q(\theta_1)\mid_{\lambda=0} = (1 - F(-\delta_1^*))\mid_{\lambda=0} < 1 - F(-\delta_1^*) \leq q(\theta_1)$.

Now, we prove the second part of the proposition. As $\beta \to 1$, for firm 2's equilibrium for Scenario 1, Equations (A8) and (A9) imply that $\underline{\theta}_2^* = 0$ and $\underline{\theta}_2^{**} = 1$, because it holds that

$$
\frac{(1-\lambda)(1-\beta)}{\lambda(1-\alpha) + (1-\lambda)(1-\beta)} \to 0 \text{ and } \frac{\lambda(1-\alpha)}{\lambda(1-\alpha) + (1-\lambda)(1-\beta)} \to 1.
$$

Similarly, as $\beta \to 1$, firm 2's equilibrium for Scenario 2 also satisfies $\overline{\theta}_2^* = 0$ and $\overline{\theta}_2^{**} = 1$. Therefore, we have $\lim_{\beta \to 1} \mathbf{CE}_L = \lim_{\beta \to 1} \mathbf{CE}_S = 0$.

Proof of Proposition 4

First, we consider the case $K \to 0$. For firm 2's equilibrium for Scenario 1, Equations (A5), (A6), and (A7) are equivalent to

$$0 = G(\tau(\underline{y}_2^* - \underline{\theta}_2^*)), \tag{A42}$$

$$0 = \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - \underline{x}_{2,1}^*))[1 - G(\tau(\underline{y}_2^* - \theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,1}^*))d\theta_2, \tag{A43}$$

$$0 = \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - \underline{x}_{2,K}^*))[1 - G(\tau(\underline{y}_2^* - \theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,K}^*))d\theta_2. \tag{A44}$$

From Equation (A42), we have $\underline{y}_2^* \to -\infty$. By applying this to Equations (A43) and (A44), we have $0 = 1 - F(\sigma(\underline{\theta}_2^* - \underline{x}_{2,1}^*))$ and $0 = 1 - F(\sigma(\underline{\theta}_2^* - \underline{x}_{2,K}^*))$, which, in turn, lead to $\underline{x}_{2,1}^* \to -\infty$ and $\underline{x}_{2,K}^* \to -\infty$. By applying $\underline{x}_{2,1}^* \to -\infty$ and $\underline{x}_{2,K}^* \to -\infty$ to Equations (A8) and (A9), we have $\underline{\theta}_2^* \to 0$ and $\underline{\theta}_2^{**} \to \frac{\lambda(1-\alpha)}{\lambda(1-\alpha)+(1-\lambda)(1-\beta)}$. Similarly, for firm 2's equilibrium for Scenario 2, we obtain $\overline{\theta}_2^* \to 0$ and $\overline{\theta}_2^{**} \to \frac{\lambda(1-\alpha)}{\lambda(1-\alpha)+(1-\lambda)(1-\beta)}$. Therefore, it follows that $\lim_{K\to 0} \mathbf{CE}_L = \lim_{K\to 0} \mathbf{CE}_S = 0$.

Next, we consider the case $K \to 1$. For firm 2's equilibrium for Scenario 1, Equations (A5), (A6), and (A7) imply that

$$1 = G(\tau(\underline{y}_2^* - \underline{\theta}_2^*)), \tag{A45}$$

$$1 = \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - \underline{x}_{2,1}^*))[1 - G(\tau(\underline{y}_2^* - \theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,1}^*))d\theta_2, \tag{A46}$$

$$1 = \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - \underline{x}_{2,K}^*))[1 - G(\tau(\underline{y}_2^* - \theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,K}^*))d\theta_2. \tag{A47}$$

From Equation (A45), we have $\underline{y}_2^* \to \infty$. By applying this to Equations (A46) and (A47), we have $1 = 1 - F(\sigma(\underline{\theta}_2^{**} - \underline{x}_{2,1}^*))$ and $1 = 1 - F(\sigma(\underline{\theta}_2^{**} - \underline{x}_{2,K}^*))$, which, in turn, lead to $\underline{x}_{2,1}^* \to \infty$ and $\underline{x}_{2,K}^* \to \infty$. By applying $\underline{x}_{2,1}^* \to \infty$ and $\underline{x}_{2,K}^* \to \infty$ to Equations (A8) and (A9), we have $\underline{\theta}_2^* \to 1$ and $\underline{\theta}_2^{**} \to 1$. Similarly, for firm 2's equilibrium for Scenario 2, we obtain $\overline{\theta}_2^* \to 1$ and $\overline{\theta}_2^{**} \to 1$. Therefore, we have $\lim_{K\to 1} \mathbf{CE}_L = \lim_{K\to 1} \mathbf{CE}_S = 0$.

Proof of Proposition 5

First, we consider the case where $c_L \to 0$ and $c_S \to 0$. As $c_L \to 0$, we have

$$\frac{u_L(\alpha+(1-\alpha)K)-u_L(\alpha)}{u_L(1)-u_L(\alpha)} = \frac{(A+c_L\alpha+c_L(1-\alpha)K)^{1-\frac{1}{c_L}}-(A+c_L\alpha)^{1-\frac{1}{c_L}}}{(A+c_L\alpha+c_L(1-\alpha))^{1-\frac{1}{c_L}}-(A+c_L\alpha)^{1-\frac{1}{c_L}}} \tag{A48}$$

$$= \frac{\left(1+\frac{c_L(1-\alpha)K}{A+c_L\alpha}\right)^{1-\frac{1}{c_L}}-1}{\left(1+\frac{c_L(1-\alpha)}{A+c_L\alpha}\right)^{1-\frac{1}{c_L}}-1}$$

$$\to \frac{e^{-\frac{1}{A}(1-\alpha)K}-1}{e^{-\frac{1}{A}(1-\alpha)}-1}.$$

Similarly, we also have

$$\frac{u_L(K)-u_L(\alpha K)}{u_L(\alpha K+(1-\alpha))-u_L(\alpha K)} \to \frac{e^{-\frac{1}{A}(1-\alpha)K}-1}{e^{-\frac{1}{A}(1-\alpha)}-1}. \tag{A49}$$

Likewise, as $c_S \to 0$, we have

$$\frac{u_S(\beta+(1-\beta)K)-u_S(\beta)}{u_S(1)-u_S(\beta)} \to \frac{e^{-\frac{1}{A}(1-\beta)K}-1}{e^{-\frac{1}{A}(1-\beta)}-1}, \tag{A50}$$

$$\frac{u_S((1-\beta)K)}{u_S(1-\beta)} \to \frac{e^{-\frac{1}{A}(1-\beta)K}-1}{e^{-\frac{1}{A}(1-\beta)}-1}, \tag{A51}$$

$$\frac{u_S(K)-u_S(\beta K)}{u_S(\beta K+(1-\beta))-u_S(\beta K)} \to \frac{e^{-\frac{1}{A}(1-\beta)K}-1}{e^{-\frac{1}{A}(1-\beta)}-1}. \tag{A52}$$

For firm 2's equilibrium for Scenario 1, by applying Equations (A48), (A50), and (A52) to Equations (A5), (A6), and (A7), we obtain

$$\frac{e^{-\frac{1}{A}(1-\alpha)K}-1}{e^{-\frac{1}{A}(1-\alpha)}-1} = G(\tau(\underline{y}_2^*-\underline{\theta}_2^*)), \tag{A53}$$

$$\frac{e^{-\frac{1}{A}(1-\beta)K}-1}{e^{-\frac{1}{A}(1-\beta)}-1} = \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2-\underline{x}_{2,1}^*))[1-G(\tau(\underline{y}_2^*-\theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2-\underline{x}_{2,1}^*))d\theta_2, \tag{A54}$$

$$\frac{e^{-\frac{1}{A}(1-\beta)K}-1}{e^{-\frac{1}{A}(1-\beta)}-1} = \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2-\underline{x}_{2,K}^*))[1-G(\tau(\underline{y}_2^*-\theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2-\underline{x}_{2,K}^*))d\theta_2. \tag{A55}$$

For firm 2's equilibrium for Scenario 2, the same argument holds by Equations (A49), (A51), and (A52), and thus, we have

$$\frac{e^{-\frac{1}{A}(1-\alpha)K}-1}{e^{-\frac{1}{A}(1-\alpha)}-1} = G(\tau(\overline{y}_2^*-\overline{\theta}_2^*)),$$

$$\frac{e^{-\frac{1}{A}(1-\beta)K}-1}{e^{-\frac{1}{A}(1-\beta)}-1} = \int_{\overline{\theta}_2^*}^{\overline{\theta}_2^{**}} \sigma f(\sigma(\theta_2-\overline{x}_{2,0}^*))[1-G(\tau(\overline{y}_2^*-\theta_2))]d\theta_2 + \int_{\overline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2-\overline{x}_{2,0}^*))d\theta_2,$$

$$\frac{e^{-\frac{1}{A}(1-\beta)K}-1}{e^{-\frac{1}{A}(1-\beta)}-1} = \int_{\overline{\theta}_2^*}^{\overline{\theta}_2^{**}} \sigma f(\sigma(\theta_2-\overline{x}_{2,K}^*))[1-G(\tau(\overline{y}_2^*-\theta_2))]d\theta_2 + \int_{\overline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2-\overline{x}_{2,K}^*))d\theta_2.$$

Then, we obtain $\overline{\theta}_2^* = \underline{\theta}_2^*$ and $\overline{\theta}_2^{**} = \underline{\theta}_2^{**}$, because the above equations for Scenario 2 are the same as Equations (A53), (A54), (A55) for Scenario 1. Therefore, $\mathbf{CE}_L \to 0$ and $\mathbf{CE}_S \to 0$ as $c_L \to 0$ and $c_S \to 0$.

Next, we consider the case where $c_L \to \infty$ and $c_S \to \infty$. As $c_L \to \infty$, we have $u_L(w) \to w$ for any $w \geq 0$, and thus, it holds that

$$\frac{u_L(\alpha + (1-\alpha)K) - u_L(\alpha)}{u_L(1) - u_L(\alpha)} \to K, \tag{A56}$$

$$\frac{u_L(K) - u_L(\alpha K)}{u_L(\alpha K + (1-\alpha)) - u_L(\alpha K)} \to K. \tag{A57}$$

Similarly, as $c_S \to \infty$, we have

$$\frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)} \to K, \tag{A58}$$

$$\frac{u_S((1-\beta)K)}{u_S(1-\beta)} \to K, \tag{A59}$$

$$\frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)} \to K. \tag{A60}$$

By applying Equations (A56), (A58), and (A60) to Equations (A5), (A6), and (A7), respectively, for firm 2's equilibrium for Scenario 1, we have

$$K = G(\tau(\underline{y}_2^* - \underline{\theta}_2^*)), \tag{A61}$$

$$K = \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - \underline{x}_{2,1}^*))[1 - G(\tau(\underline{y}_2^* - \theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,1}^*))d\theta_2, \tag{A62}$$

$$K = \int_{\underline{\theta}_2^*}^{\underline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - \underline{x}_{2,K}^*))[1 - G(\tau(\underline{y}_2^* - \theta_2))]d\theta_2 + \int_{\underline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,K}^*))d\theta_2. \tag{A63}$$

Likewise, by applying Equations (A57), (A59), and (A60) to the conditions for firm 2's equilibrium for Scenario 2, we have

$$K = G(\tau(\overline{y}_2^* - \overline{\theta}_2^*)),$$

$$K = \int_{\overline{\theta}_2^*}^{\overline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - \overline{x}_{2,0}^*))[1 - G(\tau(\overline{y}_2^* - \theta_2))]d\theta_2 + \int_{\overline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \overline{x}_{2,0}^*))d\theta_2,$$

$$K = \int_{\overline{\theta}_2^*}^{\overline{\theta}_2^{**}} \sigma f(\sigma(\theta_2 - \overline{x}_{2,K}^*))[1 - G(\tau(\overline{y}_2^* - \theta_2))]d\theta_2 + \int_{\overline{\theta}_2^{**}}^{\infty} \sigma f(\sigma(\theta_2 - \overline{x}_{2,K}^*))d\theta_2.$$

Then, we obtain $\overline{\theta}_2^* = \underline{\theta}_2^*$ and $\overline{\theta}_2^{**} = \underline{\theta}_2^{**}$, because the above equations are the same as Equations (A61), (A62), and (A63). Therefore, $\mathbf{CE}_L \to 0$ and $\mathbf{CE}_S \to 0$ as $c_L \to \infty$ and $c_S \to \infty$.

In case where $c_L \to 0$ and $c_S \to \infty$, for firm 2's equilibrium for Scenario 1, we apply (A48), (A58), and (A60) to Equations (A5), (A6), and (A7), respectively, to show that Equations (A5), (A6), and (A7) are equialent to Equations (A53), (A62), and (A63), respectively. Similarly, we apply (A49), (A59), and (A60) to firm 2's equilibrium conditions for Scenario 2 and have the same conditions as those of Scenario 1, except that the relevant variables for Scenario 1 (i.e., $\underline{\theta}_2^*, \underline{\theta}_2^{**}, \underline{y}_2^*, \underline{x}_{2,1}^* \underline{x}_{2,K}^*$) are replaced by those for Scenario 2 (i.e., $\overline{\theta}_2^*, \overline{\theta}_2^{**}, \overline{y}_2^*, \overline{x}_{2,0}^*, \overline{x}_{2,K}^*$). Thus, it holds that $\overline{\theta}_2^* = \underline{\theta}_2^*$ and $\overline{\theta}_2^{**} = \underline{\theta}_2^{**}$. Therefore, we have $\mathbf{CE}_L \to 0$ and $\mathbf{CE}_S \to 0$ as $c_L \to 0$ and $c_S \to \infty$.

In case where $c_L \to \infty$ and $c_S \to 0$, we use a similar way to derive firm 2's equilibrium conditions for Scenario 1 and 2. In particular, for Scenario 1, we use (A50), (A52), and (A56) to show that Equations (A5), (A6), and (A7) are equivalent to Equations (A61), (A54), and (A55), respectively. Also, applying (A51), (A52), and (A57) to firm 2's equilibrium conditions for Scenario 2 leads to the same conditions as those of Scenario 1, except that the relevant variables for Scenario 1 (i.e., $\underline{\theta}_2^*, \underline{\theta}_2^{**}, \underline{y}_2^*, \underline{x}_{2,1}^* \underline{x}_{2,K}^*$) are replaced by those for Scenario 2 (i.e., $\overline{\theta}_2^*, \overline{\theta}_2^{**}, \overline{y}_2^*, \overline{x}_{2,0}^*, \overline{x}_{2,K}^*$). Thus, it holds that $\overline{\theta}_2^* = \underline{\theta}_2^*$ and $\overline{\theta}_2^{**} = \underline{\theta}_2^{**}$. Therefore, we have $\mathbf{CE}_L \to 0$ and $\mathbf{CE}_S \to 0$ as $c_L \to \infty$ and $c_S \to 0$.

As an example for information precisions (i.e., τ and σ), we show that, either as $c_S \to 0$ or $c_S \to \infty$, the contagion effects (i.e., $\mathbf{CE}_L$ and $\mathbf{CE}_S$) approach zero as $\tau \to \infty$. As $\tau \to \infty$, for firm 2's equilibrium for Scenario 1, Equation (A5) implies that $\underline{y}_2^* \to \underline{\theta}_2^*$. Besides, Equations (A6) and (A7) are equivalent to

$$\begin{aligned}
\frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)} &= \int_{\underline{\theta}_2^*}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,1}^*)) d\theta_2 = F(\sigma(\underline{x}_{2,1}^* - \underline{\theta}_2^*)), \\
\frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)} &= \int_{\underline{\theta}_2^*}^{\infty} \sigma f(\sigma(\theta_2 - \underline{x}_{2,K}^*)) d\theta_2 = F(\sigma(\underline{x}_{2,K}^* - \underline{\theta}_2^*)).
\end{aligned}$$

Thus, by applying the above two equations to Equations (A8) and (A9), we have

$$\begin{aligned}
\underline{\theta}_2^* = {} & \frac{(1-\lambda)(1-\beta)}{\lambda(1-\alpha) + (1-\lambda)(1-\beta)} (1 - F(\sigma(x_1^* - \theta_1)) \frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)} \qquad \text{(A64)} \\
& + \frac{(1-\lambda)(1-\beta)}{\lambda(1-\alpha) + (1-\lambda)(1-\beta)} F(\sigma(x_1^* - \theta_1)) \frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)}, \\
\underline{\theta}_2^{**} = {} & \frac{\lambda(1-\alpha)}{\lambda(1-\alpha) + (1-\lambda)(1-\beta)} \qquad \text{(A65)} \\
& + \frac{(1-\lambda)(1-\beta)}{\lambda(1-\alpha) + (1-\lambda)(1-\beta)} (1 - F(\sigma(x_1^* - \theta_1)) F\left(F^{-1}\left(\frac{u_S(\beta + (1-\beta)K) - u_S(\beta)}{u_S(1) - u_S(\beta)} \right) - \underline{D}_2 \right) \\
& + \frac{(1-\lambda)(1-\beta)}{\lambda(1-\alpha) + (1-\lambda)(1-\beta)} F(\sigma(x_1^* - \theta_1)) F\left(F^{-1}\left(\frac{u_S(K) - u_S(\beta K)}{u_S(\beta K + (1-\beta)) - u_S(\beta K)} \right) - \underline{D}_2 \right),
\end{aligned}$$

where $\underline{D}_2 = \sigma(\underline{\theta}_2^{**} - \underline{\theta}_2^*)$ holds. For firm 2's equilibrium for Scenario 2, the same argument holds, and thus, we have $\overline{y}_2^* \to \overline{\theta}_2^*$. Besides, we have

$$\overline{\theta}_2^* = \frac{(1-\lambda)(1-\beta)}{\lambda(1-\alpha)+(1-\lambda)(1-\beta)}(1-F(\sigma(x_1^*-\theta_1))\frac{u_S((1-\beta)K)}{u_S(1-\beta)} \tag{A66}$$
$$+\frac{(1-\lambda)(1-\beta)}{\lambda(1-\alpha)+(1-\lambda)(1-\beta)}F(\sigma(x_1^*-\theta_1))\frac{u_S(K)-u_S(\beta K)}{u_S(\beta K+(1-\beta))-u_S(\beta K)},$$
$$\overline{\theta}_2^{**} = \frac{\lambda(1-\alpha)}{\lambda(1-\alpha)+(1-\lambda)(1-\beta)} \tag{A67}$$
$$+\frac{(1-\lambda)(1-\beta)}{\lambda(1-\alpha)+(1-\lambda)(1-\beta)}(1-F(\sigma(x_1^*-\theta_1))F\left(F^{-1}\left(\frac{u_S((1-\beta)K)}{u_S(1-\beta)}\right)-\overline{D}_2\right)$$
$$+\frac{(1-\lambda)(1-\beta)}{\lambda(1-\alpha)+(1-\lambda)(1-\beta)}F(\sigma(x_1^*-\theta_1))F\left(F^{-1}\left(\frac{u_S(K)-u_S(\beta K)}{u_S(\beta K+(1-\beta))-u_S(\beta K)}\right)-\underline{D}_2\right),$$

where $\overline{D}_2 = \sigma(\overline{\theta}_2^{**} - \overline{\theta}_2^*)$ holds.

As $c_S \to 0$, we have

$$\frac{u_S(\beta+(1-\beta)K)-u_S(\beta)}{u_S(1)-u_S(\beta)} = \frac{u_S((1-\beta)K)}{u_S(1-\beta)} = \frac{u_S(K)-u_S(\beta K)}{u_S(\beta K+(1-\beta))-u_S(\beta K)} = \frac{e^{-\frac{1}{A}(1-\beta)K}-1}{e^{-\frac{1}{A}(1-\beta)}-1},$$

as shown in Equations (A50), (A51), and (A52). By applying this to (A64) and (A66), we have $\mathbf{CE}_S = \overline{\theta}_2^* - \underline{\theta}_2^* = 0$. Likewise, by applying this to (A65) and (A67), we obtain $\mathbf{CE}_L = \overline{\theta}_2^{**} - \underline{\theta}_2^{**} = 0$, because Equations (A65) and $\underline{D}_2 = \sigma(\underline{\theta}_2^{**} - \underline{\theta}_2^*)$ become the same as Equations (A67) and $\overline{D}_2 = \sigma(\overline{\theta}_2^{**} - \overline{\theta}_2^*)$, respectively.

On the other hand, as $c_S \to \infty$, it holds that

$$\frac{u_S(\beta+(1-\beta)K)-u_S(\beta)}{u_S(1)-u_S(\beta)} = \frac{u_S((1-\beta)K)}{u_S(1-\beta)} = \frac{u_S(K)-u_S(\beta K)}{u_S(\beta K+(1-\beta))-u_S(\beta K)} = K,$$

as shown in Equations (A58), (A59), and (A60). Then, the same argument as above gives rise to $\mathbf{CE}_S = \mathbf{CE}_L = 0$.

REFERENCES

[1] Allen, F., A. Babus, and E. Carletti, 2012, "Asset Commonality, Debt Maturity and Systemic Risk," *Journal of Financial Economics*, 104, 519-534.

[2] Allen, F. and D. Gale, 2000, "Financial Contagion," *Journal of Political Economy*, 108, 1-33.

[3] Angeletos, G. M., C. Hellwig, and A. Pavan, 2007, "Dynamic Global Games of Regime Change: Learning, Multiplicity, and Timing of Attacks," *Econometrica*, 75, 711-756.

[4] Arrow, K. J., 1965, *Aspects of the Theory of Risk-Bearing*, Helsinki: Yrjo Jahnssonin Saatio.

[5] Bannier, C., 2005, "Big Elephants in Small Ponds: Do Large Traders Make Financial Markets More Aggressive?" *Journal of Monetary Economics*, 52, 1517-1531.

[6] Bolton, P. and D. S. Scharfstein, 1996, "Optimal Debt Structure and the Number of Creditors," *Journal of Political Economy*, 104, 1-25.

[7] Bruche, M., 2011, "Creditor Coordination, Liquidation Timing, and Debt Valuation," *Journal of Financial and Quantitative Analysis*, 46, 1407-1436.

[8] Calomiris, C. W. and C. M. Kahn, 1991, "The Role of Demandable Debt in Structuring Optimal Banking Arrangements," *American Economic Review*, 81, 497-513.

[9] Carlsson, H. and E. van Damme, 1993, "Global Games and Equilibrium Selection," *Econometrica*, 61, 989-1018.

[10] Corsetti, G., A. Dasgupta, S. Morris, and H. S. Shin, 2004, "Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders," *Review of Economic Studies*, 71, 87-114.

[11] Corsetti, G., P. Pesenti, and N. Roubini, 2002, "The Role of Large Players in Currency Crises," *Preventing Currency Crises in Emerging Markets*, edited by S. Edwards and J. Frankel, University of Chicago Press.

[12] Dasgupta, A., 2004, "Financial Contagion through Capital Connections: A Model of the Origin and Spread of Bank Panics," *Journal of the European Economic Association*, 2, 1049-1084.

[13] Diamond, D. W., 1991, "Debt Maturity Structure and Liquidity Risk," *Quarterly Journal of Economics*, 106, 709-737.

[14] Diamond, D. W. and R. G. Rajan, 2001, "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking," *Journal of Political Economy*, 109, 287-327.

[15] Flannery, M. J., 1986, "Asymmetric Information and Risky Debt Maturity Choice," *Journal of Finance*, 41, 19-37.

[16] Goldstein, I. and A. Pauzner, 2004, "Contagion of Self-Fulfilling Financial Crises due to Diversification of Investment Portfolios," *Journal of Economic Theory*, 119, 151-183.

[17] Manz, M., 2010, "Information-based Contagion and the Implications for Financial Fragility," *European Economic Review*, 54, 900-910.

[18] Morris, S. and H. S. Shin, 1998, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, 88, 587-597.

[19] Morris, S. and H. S. Shin, 2003, "Global Games: Theory and Applications," *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society)*, edited by M. Dewatripont, L. P. Hansen and S. J. Turnovsky, Cambridge University Press.

[20] Morris, S. and H. S. Shin, 2004, "Coordination Risk and the Price of Debt," *European Economic Review*, 48, 133-153.

[21] Nam, S. W., 1996, "The Principal Transactions Bank System in Korea and a Search for a New Bank-Business Relationship," *Financial Deregulation and Integration in East Asia*, edited by T. Ito and A. O. Krueger, University of Chicago Press.

[22] Obstfeld, M., 1996, "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review*, 40, 1037-1047.

[23] Oh, F. D., 2013, "Contagion of a Liquidity Crisis Between Two Firms," *Journal of Financial Economics*, 107, 386-400.

[24] Oh, F. D., 2015, "The Role of the Central Bank in Contagious Currency Crises," Working Paper, Korea Advanced Institute of Science and Technology.

[25] Pratt, J. W., 1964, "Risk Aversion in the Small and in the Large," *Econometrica*, 32, 122-136.

[26] Takeda, F. and K. Takeda, 2008, "How Large Creditors Affect Corporate Bond Prices through Creditor Coordination," *Journal of International Economic Studies*, 22, 31-49.

[27] Taketa, K., 2004, "A Large Speculator in Contagious Currency Crises: A Single "George Soros" Makes Countries More Vulnerable to Crises, but Mitigates Contagion," IMES Discussion Paper No. 2004-E-23, Institute for Monetary and Economic Studies, Bank of Japan.

[28] Wagner, W., 2010, "Diversification at Financial Institutions and Systemic Crises," *Journal of Financial Intermediation*, 19, 333-354.