Tail Risk under Price Limits

Sekyung Oh

Konkuk University, Department of Business Administration, 120 Neungdong-ro, Gwangjin-gu, Seoul, 05029, Korea, E-mail: skoh@konkuk.ac.kr

Kinam Park

Korea Asset Pricing Co., Samhwan Bldg. 4th fl., 88 Yulgok-ro, Jongno-gu, Seoul, 03131, Korea, E-mail: knpark@koreaap.com

Hyukdo Kee

Konkuk University, Department of Business Administration, 120 Neungdong-ro, Gwangjin-gu, Seoul, 05029, Korea, E-mail: hdkee@konkuk.ac.kr

Abstract

The goal of this paper is to investigate the dynamics of tail risk when the price limits exist in the stock markets. We present the expected value of tail risk under the price limits based on which we analyze the effect of the price limits on tail risk for the Korean stock markets where the price limits exist and are eased gradually. The main results are: First, tail risk is seriously underestimated in the stock markets with a price limit system. In particular, tail risk cannot be used as a meaningful risk indicator if the price limits are less than 15%. Second, tail risk is highly predictive of stock returns in the Korean stock markets if the price limits are higher than 15%. Third, tail risk is a significant risk factor in determining asset price if the price limits are higher than 15%. Lastly, tail risk has the predictive power from 6 to 12 months in advance as a systemic risk indicator related to the Korean economy if the price limits are higher than 15%.

Keywords: tail risk, price limit, systemic risk, risk price

JEL classification: G11, G12, G17

1. Introduction

Recently, there have been some attempts to extract tail risk factor as a risk indicator from the movements of individual stocks listed on the stock markets and to explain the movements of the aggregate stock markets using it. Kelly and Jiang (2014) attempt to capture common variations in the tail risk inherent in the movements of individual stocks at each point in time. They use the cross-section of crash events for individual firms to identify the common component of tail risk, assuming that firm-level tail distributions possess similar dynamics. They report that the estimated tail risk measure not only has strong explanatory power for the US aggregate stock market returns, but also predicts future extreme returns of individual stocks.

The goal of this paper is to investigate the dynamics of tail risk when the price limits exist in the stock markets. Since the price limits hinder the stock prices from moving up and down freely reflecting the market information, the effects of time-varying tail risk in stock markets with the price limits would be different from those when there are no price limit systems. To this end, we consider the Korea stock markets because Korea has gradually eased the daily price limits from the minimum 2.2 % to the current 30% and it has relatively abundant liquidity and high degree of openness to foreign investors among emerging countries.

Among price stabilization mechanisms, the price limit system that directly controls the price of trades so that stock prices are traded within the upper and lower price boundaries is implemented in Japan, Korea and many emerging economies.[1] On the other hand, in the US, Germany, UK, France and many other developed countries[2], circuit breakers and volatility interruption (VI), which are volatility mitigation devices for individual stocks, are implemented instead of the price limits. The price limits are artificial control tools that place constraints on pricing mechanism of market demand and supply.

In this paper, we examine the effects of time-varying tail risk in the stock markets with the price limits, which are not analyzed in the existing studies on tail risk. We explore how the predictive power of tail risk on future stock returns has changed as the price limits have gradually eased. We derive the expected value of tail risk in the presence of the price limits, under the assumption that the returns with negative extreme events follow the same probability distribution. We prove that the expected value of tail risk in the presence of the price limits is lower than that in the absence of the price

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[1] The Taiwan Stock Exchange (TWSE) and the Shanghai Stock Exchange (SSE) in China are implementing strong price limits of 10% and the Tokyo Stock Exchange (TSE) of Japan has an average price limit of 22% under 34 different stages. On the other hand, the Korea Stock Exchange (KRX), the Thailand's Thai Stock Exchange (SET) and the Malaysian Stock Exchange (Bursa Malaysia) implement a fairly relaxed 30% floating rate system.

[2] Most European stock exchanges do not have strict static price limit systems but adopt dynamic ones such as various types of circuit breakers or volatility interruptions to curb extreme fluctuation of stock prices. For example, Euronext exchanges including Paris, Amsterdam, and Brussels allow stock trading to occur within 5 to 10 percent up or down from the reference price. However the reference price is re-adjusted at the latest traded price, so actually it is similar to the case where there is no price limits.

limits and also confirm this by the simulation using random returns. We find that tail risk has no predictive power until the price limits are relaxed to a certain level (15%), implying caution is needed when the effects of tail risk are analyzed in countries where the price limits exist. After the price limits were expanded to 30% in Korea, tail risk turns to be a priced risk factor as found in Kelly and Jiang (2014). We also find that the estimated tail risk measure not only has strong predicting power for the aggregate stock market returns, but also forecasts the systemic risk of the economy when the price limits are above 15%.

The remainder of this paper is structured as follows. Section 2 briefly reviews the relevant literature. Section 3 derives the expected value of tail risk under the price limits. Section 4 describes the data and reports the main results. Lastly, Section 5 summarizes our main findings and concludes the paper.

2. Literature review

The major flow of studies and research on systemic risk or tail risk in academia is how risk will be measured and how risk indicator adequately predicts future economic shocks. As a research on systemic risk, Allen, Bali, and Tang (2012) devise CATFIN as a measure of the risk of the financial sector as an arithmetic average of VaR by two parametric methods and one nonparametric method. They argue that CATFIN predicts the fall of the real economy better than other microeconomic indicators such as expected shortfall (Archarya, Pedersen, Philippon, and Richardson, 2017), CoVaR (Adrian and Brunnermeier, 2009), and tail risk (Kelly and Jiang, 2014). Giglio, Kelly and Pruitt (2016) go one step further and analyze 19 system risk indicators and find that these systemic risk indicators include CoVaR and marginal expected shortfall are predictive of macroeconomic shocks, but do not show cross-sectional robustness. However they suggest the systemic risk index, which is reconstructed by these indicators, well predict the economic shocks throughout the whole period, and emphasize that the systemic risk index has a strong correlation with the economic decline risk.

Measuring tail risk as a risk indicator at every point in time and analyzing it in relation to the whole stock market is very meaningful from the perspective of profit and loss management and systemic risk management. Recently, many researchers pay attention to extracting tail risk factors from the movement of individual stocks listed on the stock market and to using them to explain the movements of the whole market. Kelly and Jiang (2014), based on the commonality inherent in the tail risk of individual stocks, devise the tail risk as a risk indicator derived from the stock returns in order to overcome the difficulties in observing tail risk. They argue that if the tail distribution of individual firms has the same dynamics, then the crash event data can be useful in capturing the common factors of tail risk at each point in time. Assuming that the return of stocks exceeding the extreme negative threshold follows the same tail risk dynamics, they estimate the time-varying variable λ_t, tail risk, by applying power law estimator of Hill (1975). They show that the tail risk has strong explanatory power on U.S. stock returns, not only in terms of time series but cross-sectional area. In other words, it has strong predictive power not only on individual stocks but also on the future extreme negative returns of the entire stock market. They also confirm that this tail risk is highly correlated with the premium of the deep OTM put option, thus confirming that tail risk is related to compensation for the inherent risks in the financial product.

Many studies show the predictability of tail risk for stock returns. Bollerslev, Todorov and Xu (2015) argue that risk premium associated with the compensation demanded by investors for bearing tail risk helps predict future market returns. Whereas Kelly and Jiang (2014) capture tail risk from the fluctuation of prevailing stock markets, Bollerslev et al. (2015) derive the risk-neutral tails from the rate at which the prices of short maturity options decay for successively deeper OTM contracts and find its predictability for the aggregate market portfolio through empirical analysis. Xiong, Idzorek and Ibbotson (2014) argue that in the studies of open-ended mutual funds in the US market from 1980 to 2011, the tail risk measured by excess conditional value-at-risk (ECVaR) has low correlation with the volatility and also show the return of the open-ended fund with high tail risk overrides the return of the fund group with low tail risk.

These studies on tail risk have mainly been limited to the developed countries. Recent studies including Lee and Yang (2017) and Straetmans and Candelon (2013) show that the pricing power of tail risk works differently in developed and emerging countries. First, Lee and Yang (2017) expand the analysis of tail risk to 46 countries including many emerging countries. They propose the co-tail risk (hybrid-tail risk) between individual stocks and market indices as a major risk indicator find that this co-tail risk has strong predictive power on future returns but stronger pricing power in developed countries than in emerging countries. They argue that different degree of regulations among countries result in the difference in the pricing power among them. However, they do not suggest the precise mechanism for that. Straetmans and Candelon (2013) find that the tail risk in emerging markets exhibits structural shifts because structural changes are more frequently happening in emerging economies and insist that tail index and extreme quantile estimation are useful tools for assessing long-term tail risk, stress testing and financial stability but one has to apply these techniques with care in the presence of breaks in the tail behaviors.

As we have said in section 1, the price limit system exists in emerging countries, Kim and Rhee (1997) argue that the price limits not only delay the market's equilibrium price discovery process but also causes volatility spillover so that it is an inefficient system that hinders autonomous price discovery activities by market demand and supply. Kim (2001) also mentions that the price limits can affect stock market volatility. Lauterbach and Ben-zion (1993) argue that circuit breakers have no effect on the long-run response although they reduce the next-day opening order imbalance and the initial price loss. Consequently, for the study of tail risk we have to consider whether there is the price limit system or any other artificial price controlling tools in stock markets.

3. Derivation of the expected value of tail risk under price limits

Kelly and Jiang (2014)'s estimation of tail risk starts from the assumptions that extreme return events falling below the extreme negative threshold (u_t), the 5th percentile of the cross-section each period, obey a power law and tail risks of individual firms share common process. It is assumed that the tail distribution at time t is a set of extreme return events that fall below the extreme negative threshold and follows the tail probability distribution as in Equation (1).

$$P\left(R_{i,t+1} < r \,\middle|\, R_{i,t+1} < u_t \; and \; \mathcal{F}_t\right) = (\frac{r}{u_t})^{-a_i/\lambda_t} \tag{1}$$

where $r < u_t < 0,\ u_t$ is extreme negative threshold and a represents constant level of individual firm's tail risk.

In the above Equation (1), λ_t is an estimate of tail risk at time $t.\ \lambda_t$, which means the probability of occurrence of extreme events, is a time-varying tail risk estimate that varies with time t (monthly). a_i/λ_t is the tail exponent, which determines the shape of the tail risk. If λ_t is large, the probability of occurrence of extreme returns is high.

Based on the assumption that stock returns have such a tail distribution, we estimate the tail risk, time-varying variable λ_t, by the following Equation (2).

$$\lambda_t^{Hill} = \frac{1}{K_t}\sum_{k=1}^{K_t} ln\frac{R_{k,t}}{u_t} \tag{2}$$

where u_t is extreme negative threshold, $R_{k,t}$ is the kth daily return that falls below an extreme negative threshold during month t, and K_t is the total number of such exceedences within month t. To estimate tail risk, we first group daily stock return data into monthly pool of stock returns, and then calculate the extreme negative threshold corresponding to the lower 5th percentile in the pool. Next, on a monthly basis, we extract the extreme negative returns $R_{k,t}$ that fall below the extreme negative threshold and calculate $ln\frac{R_{k,t}}{u_t}$ each. Finally, we estimate the tail risk measure by averaging these values on a monthly basis.

Assuming that the condition of $x < u < 0$ is satisfied and that rates of return less than the extreme negative threshold in the ex-ante follow the same probability distribution, the expected value of log return $ln\left(\frac{x_{i,t}}{u_t}\right)$ is equal to the average of the log returns of $\frac{1}{K_t}\sum_{k=1}^{K_t} ln\left(\frac{x_{i,t}}{u_t}\right)$ which is also tail risk λ_t according to Equation (2). Accordingly, we can derive the expected value of tail risk under the condition that the price limit v exists by calculating the expected value of $ln\left(\frac{x_{i,t}}{u_t}\right)$.

Under the assumptions mentioned above, the tail risk cumulative distribution function $F(x)$ and its probability density function $f(x)$ can be simply expressed as follows.

$$F(x) = Pr(X < u) \;= (\frac{x}{u})^{-\frac{a}{\lambda}} \tag{3}$$

$$f(x) = \left(-\frac{a}{\lambda}\right)\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}} \cdot \left(\frac{1}{x}\right) \tag{4}$$

Then, the expected value of tail risk $E\left(ln\left(\frac{x}{u}\right)\right)$ with the price limit v $(v < u < 0)$ is:

$$E\left[ln\left(\frac{x}{u}\right)\right] = \int_{v}^{u} ln\left(\frac{x}{u}\right)\cdot\left(-\frac{a}{\lambda}\right)\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}}\left(\frac{1}{x}\right)dx + \int_{-\propto}^{v} ln\left(\frac{x}{u}\right)\cdot\left(-\frac{a}{\lambda}\right)\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}}\left(\frac{1}{x}\right)dx \tag{5}$$

Solving Equation (5), we finally derive the expected value of tail risk, $ln\left(\frac{x}{u}\right)$, under the condition that the price limit v exists as follows.[3]

$$E\left[ln\left(\frac{x}{u}\right)\right] = \frac{\lambda}{a}\left[1 - \left(\frac{u}{v}\right)^{\frac{a}{\lambda}}\right] \tag{6}$$

Comparing the result of Equation (6) to $\frac{\lambda}{a}$, which is the expected value of tail risk when there are no price limits[4],we find the followings. First, if the price limits are tight, tail risk is underestimated as the price discovery function is distorted. Under the tight price limits, as the frequency that returns get close to or hit the lower price limit increases, the gap between the extreme negative threshold u and the lower price limit v becomes narrower. In this case, $\left(\frac{u}{v}\right)^{\frac{a}{\lambda}}$ in Equation (6) increases and the expected value of tail risk decreases. In other words, the price limit is a factor that has a negative effect on the estimate of tail risk. In the extreme cases where the price limits are extremely tight, tail risk may approach zero. Therefore, if there are price limits, a significant amount of caution is required to the interpretation of the tail risk estimates. Second, if the price limits are large enough, it will be similar to the case where there are no price limits. If the price limits are larger enough, $\left(\frac{u}{v}\right)^{\frac{a}{\lambda}}$ will have a very small value and so the expected value of tail risk gets closer to $\frac{\lambda}{a}$. It is necessary to examine through empirical analysis which level of the price limits is large enough to be similar to the case without the price limits.

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[3] Refer to the Appendix for the derivation process of expected value model of tail risk.

[4] Kelly and Jiang (2014) p. 2,847.

4. Results

4.1 Data

The sample period is 25 years and 10 months from the beginning of January 1990 to the end of October 2015.[5] The stocks to be analyzed are 1,621 KOSPI and KOSDAQ stocks listed on the Korea Exchange. We use daily stock returns modified to reflect split-offs, increases of paid-in capital, and stock dividends. Investment securities such as REITs, infrastructure funds and ship funds, finance and insurance company stocks with high leverage characteristics, preferred stocks, and financially distressed companies (such as defaults, debt restructurings, and de-listings) are removed. As the risk-free rate of return, we use the yield of 1-year Monetary Stabilization Bond (MSB) from the Economic Statistics System (ECOS) of The Bank of Korea.

Tail risk is not only predictive of stock price but also deeply connected with the soundness of the overall financial system. Therefore, as one of the indicators of systemic risk, we examined how tail risk is related to the macroeconomic soundness. Though Allen et al. (2012) used CATFIN as an indicator of the system risk of the financial sector, we use FSI (Financial Stability Index) of The Bank of Korea, which is an index to comprehensively evaluate financial instability and macroeconomic situation.[6] FSI is a kind of fear index that The Bank of Korea developed by transforming the various indices[7] representing financial stability into one index. It has a merit that it can comprehensively assess the macroeconomic situation of Korea. It is also used as an early warning indicator for capturing the systemic risk due to the accumulation of financial imbalance.

4.2 Impact of price limits on tail risk

The Korean stock markets impose the price limits to curb extreme price fluctuations in the markets. Under the price limits which impose the lower limits, the stock prices will be blocked by the lower limits and cannot fall further down under the lower limits even if there is an external shock that will have a negative impact on the stock prices. This means that the inherent risk factors are not properly reflected in the trading prices, and the estimated tail risk λ_t does not properly reflect the inherent risk characteristics. Therefore, it is necessary to analyze the effects of the price limits on tail

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[5] This includes the period during which the KOSDAQ markets opened in July 1996 and the Korean financial markets fluctuated a lot due to the Asian currency crisis.

[6] FSI is calculated by standardizing and averaging the selected indicators. It has a value between 0 (minimum) and 100 (maximum), so that the larger FSI is, the higher is the degree of instability (Lee, Ryu and Tsomocos, 2013, p.759).

[7] FSI is composed of the indices that represent banks (delinquency rate, etc.), stocks, foreign exchange and bond markets (stock price and exchange rate volatility, interest rate spreads, etc.), external transactions and external payments (current account balance, CDS premium, etc.), real economy (GDP growth rate, inflation rate, etc.), and household and corporate economic conditions (consumer survey index, business survey index, etc.) (Financial Stability Report, The Bank of Korea, April 2012, p.148).

risk before examining the predictive power of tail risk for stock prices.

<Table 1> shows the changes in the price limit system of the Korean stock markets. Before the end of March 1994, there was a strong price limit system with an average of about 4.6% but it has gradually expanded to the current 30% limit.[8] As a result, the liquidity and the price discovery function of the markets have improved. Since the Korean stock markets have been under the price limit system for a long period of time, we expect that the existence of the price limits would have a significant impact on the characteristics of tail risk.

[Table 1 is about here.]

<Figure 1> shows a graph of tail risk (λ_t), extreme negative threshold (u_t) and lower price limits during the sample period. We see that extreme negative thresholds and tail risks have been frequently curbed by the price limits until 1998 when the price limits were expanded to 15%, albeit the gradual expansion of the price limits. Prior to 1998, extreme negative thresholds approached the lower price limits many times, but after the price limits were expanded to 15% in December 1998, only once when the global financial crisis occurred.[9]

[Figure 1 is about here.]

We find that tail risk is underestimated when the price limits exist from Equation (6). Now we examine this through simulation, assuming the various levels of the price limits. We randomly extracted 10,000 returns, and then re-calculated returns by applying the price limits of 8%, 10%, 12%, 15% and 20%, respectively to create the conditions similar to the actual stock markets when the price limits exist. We also include the case when no price limit is applied to compare with. In addition, we do the same simulations assuming the volatility of stock prices increases by 10 and 20% arbitrarily to see the effects of the volatility of stock prices on tail risk. <Table 2> shows descriptive statistics, extreme negative threshold (u_t) and tail risk (λ_t) of the newly calculated returns.

[Table 2 is about here.]

In the case of 8% upper and lower price limits as in Panel 1, maximum and minimum values are equal to the price limits (8% up and down). As volatility increases, extreme negative threshold (u_t) gradually approaches the lower price limit (-6.56%, -7.22% and -7.88%) and tail risk (λ_t) drops sharply (0.1426, 0.0870, and 0.0151). If extreme negative

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[8] In the Korean stock markets, the price limits of the KOSPI and the KOSDAQ markets have been different. However, since there is not much difference in the price limits between the two markets, we explain only the price limits of KOSPI markets.

[9] In October 2008 when the global financial crisis occurred, extreme negative threshold reached -14.75%, which was close to the lower price limit -15% at that time.

threshold is close to the lower price limit of -8%, the interval between $R_{k,t}$ and u_t in Equation (2) becomes smaller, so $ln\frac{R_{k,t}}{u_t}$ and tail risk also becomes smaller. In other words, even if volatility of stock prices increases, tail risk decreases because of the strict price limits, so that tail risk is not working adequately as a risk indicator in this case. Even in the case of 12% price limits, tail risk decreases (0.2092, 0.2043, and 0.1955) as volatility increases. In the case of Panel 4, where the price limits of 15% are applied, tail risk (0.2051) is clearly larger than that when the price limits are 8% (0.1426) but it rarely changes (0.2051, 0.2046, and 0.2037) even if volatility increases. In Panel 5, where the price limits of 20% are applied, maximum and minimum values are below the price limits of 20%, extreme negative threshold gradually increases but tail risk does not change at all (0.2094, 0.2094, and 0.2094). Panel 5 shows that almost all of the distortions caused by the price limits have disappeared like in Panel 6 where there are no price limits.

Through the simulation analysis using random returns, we find the followings. First, if the price limits are as strict as 8%, tail risk can be seriously underestimated (0.1426 vs. 0.2068). Therefore, for countries where the price limit system exists, tail risk is underestimated so much that it may not be used as a risk indicator. Second, tail risk is not underestimated when the price limit is over 15%, and it does not change even if stock price volatility increases. Third, we find that we need to divide the sample period into the two sub periods when the price limits were below 15% and when they were above 15% because the dynamics of tail risk would be different between the two.

4.3 Characteristics of tail risk

<Table 3> shows descriptive statistics and correlation coefficients of extreme negative threshold and tail risk calculated using the stock price data for 25 years and 10 months from January 1990 to October 2015. Panel A shows the results during the whole sample period, whereas Panel B is when the price limits were below 15% and Panel C is when the price limits were above 15%.

[Table 3 is about here.]

Panel C shows that extreme negative threshold slightly increases but the average value of tail risk almost doubles (0.1823 and 0.3402), compared to Panel B. Panel C also shows that the correlation coefficient of extreme negative threshold and tail risk is very high (0.86), compared to Panel B (0.36). The results also confirm the previous finding that the characteristics of tail risk are very different, depending on whether strong price limits are applied or not.

We perform tests for equality on the means and the standard deviations of tail risk between Panel B and Panel C, finding the means and the standard deviations of tail risk between those two periods are significantly different at 1% level. We also perform an AR(1) regression analysis to measure the persistence of tail risk. Monthly AR(1) coefficients of tail risk are high in all sections, with 0.9671 in Panel A, 0.9009 in Panel B, and 0.9791 in Panel C. The high persistence of tail risk means that tail risk is not an indicator for short-term fluctuations, which means that it should be analyzed from a mid- to long-term perspective rather than a short-term risk indicator.

4.3 Predictive power of tail risk

Now, we examine whether tail risk (λ) with the above characteristic has a predictive power for future aggregate stock market returns. The estimated regression equation used for this estimation is shown in Equation (7).

$$INDEX_{t+\tau} = \alpha + \beta\lambda_t + \varepsilon_t, \quad \tau = 6, 12, 24, 36, 48 \text{ months} \tag{7}$$

where $INDEX_{t+\tau}$ is the log expected return of the stock index from time t to time $t+\tau$ and $INDEX$ is the market-weighted stock price index. Panel A of <Table 4> shows the results based on Equation (7).

Furthermore, we add volatility ($stdev$) to Equation (7) to control its effect in predicting future stock market returns.

$$INDEX_{t+\tau} = \alpha + \beta\lambda_t + stdev_t + \varepsilon_t, \quad \tau = 6, 12, 24, 36, 48 \text{ months} \tag{8}$$

where $INDEX_{t+\tau}$ is the same as above. Panel B of <Table 4> shows the results based on Equation (8).

[Table 4 is about here.]

Panel A shows the followings. In Whole Period (whole sample period), tail risk had a high predictive power only for short-term (6 and 12 months) stock market returns. In Period 1, where strict price limits were applied, the predictive power of tail risk was not significant or very weak in other periods except for 48 months. However, there was a statistical significance in Period 2 where the price limits were more than 15%. Panel B shows the results when standard deviation of $INDEX$ is added as a control variable. In period 2, the predictive power of tail risk is very high for all intervals but in Period 1, its predictive power is still low for 24 and 36 months, although it generally increases compared to Panel A. This suggests that tail risk may be a significant predictor for future stock market returns when the price limits are above 15%, but it may not be when the price limits are below 15%.

4.4 Tail risk and asset prices

In this section, we examine how tail risk can be risk priced in circumstances where the price limits have gradually eased. First, we examine the difference in average returns according to tail beta, the sensitivity of the individual stocks to tail risk. To estimate the monthly tail beta of stock i (tail β_i), we run a regression Equation (9) using the monthly return of stock i (r_i) and tail risk (λ_t) for the preceding 60 months.

$$r_{i,t} = \alpha_i + \beta_i \lambda_t + \varepsilon_t \tag{9}$$

We sort all stocks into five quintile portfolios (Low, 2, 3, 4, High) every month by the estimated tail beta. After calculating the equal-weighted returns for these portfolios, average monthly returns for the holding periods of 1, 6, and 12 months were obtained. In addition, we constructed a mimicking portfolio by purchasing High portfolio with high tail beta and selling Low portfolio with low tail beta at the same time.

<Table 5> shows the results for the whole sample period and two separate sub periods. For the whole sample period and Period 1 in which the strict price limits were applied, we find monthly returns of High-Low portfolios are all negative. For Period 2 in which the price limits were 15% or more, however, we find that the average monthly return increases as tail beta increases. The returns of High-Low portfolio are all positive regardless of the holding periods and the return for one month holding period is the highest (1.474%). Only for Period 2, <Table 5> generally shows the larger tail beta, the greater the average returns of stock portfolio.

[Table 5 is about here.]

When the price limit is more than 15%, the results are consistent with the trade-off relationship of risk-return. Stocks with high tail beta are sensitive to tail risk, and thus are deeply discounted when tail risk is high and have high expected returns. On the other hand, stocks with low tail beta are well hedged against tail risk, so even if tail risk of the markets rises, their stock prices are not significantly discounted and their expected returns should be low. Stocks with high tail risk expect high returns as a reward. The results show that excess returns do exist in a portfolio with a high sensitivity to tail risk, implying that tail risk can be a useful risk indicator. However, the results show that tail risk cannot be used as a risk indicator if the price limit system is as tight as 15% or less.

Next, we examine whether significant alphas (α) exist for the 5 quintile portfolios in the CAPM and Fama and French (1993) 3-factor model. Alphas (α) for the one-month holding period on the 5 quintile portfolios are calculated by Equation (10) and Equation (11) respectively.

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_p\left(r_{M,t} - r_{f,t}\right) + \varepsilon_t \tag{10}$$

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_{1p}\left(r_{M,t} - r_{f,t}\right) + \beta_{2p} SMB_t + \beta_{3p} HML_t + \varepsilon_t \tag{11}$$

where $r_{p,t}$ is the return of portfolio p, $r_{M,t}$ is $INDEX$'s return, $r_{f,t}$ is the yield of 1-year Monetary Stabilization

Bond issued by The Bank of Korea, SMB_t is size factor, and HML_t is the book to market value factor. All are measured at time t.

In <Table 6>, we can see that monthly alphas in CAPM and FF 3-factor model for the equal-weighted portfolio tend to increase as tail beta increases. In the High-Low mimicking portfolio for Period 2 when the price limits were 15% or above, CAPM alpha is 1.134% and FF 3-factor alpha is 1.132%, which are significant at 1% level for the one month holding period. Especially, for Period 2, CAPM alpha and FF 3-factor alpha are about 2 times and 3 times bigger than those (0.532% and 0.381%) of Period 1 when the strict price limits were applied. We find that the larger tail beta, the greater the alpha in each model for Period 2. The results also confirm that tail risk cannot be risk priced appropriately in countries where the price limit system is tight.

[Table 6 is about here.]

4.5 Tail risk as a systemic risk indicator

In this section, we examine how tail risk is related with the macroeconomic stability. To test the predictive power of tail risk for macroeconomic downturn as a systemic risk indicator, we use Equation (12) which is similar to the predictive regression model of Allen et al. (2012). They use CFNAI[10], the US macroeconomic indicator, as a dependent variable in their study. We use FSI (Financial Stability Index)[11] published by The Bank of Korea as a national activity index which is known to be useful in early detection of systemic risk. The predictive power of tail risk is measured by using FSI_{t+n} as a dependent variable in the following regression.

$$FSI_{t+n} = \alpha + \gamma TAIL_t + \beta X_t + \sum_{i=1}^{12} \mu_i FSI_{t-i+1} + \varepsilon_{t+n} \tag{12}$$

where FSI_{t+n} is the Financial Stability Index n months later at time t, $TAIL$ is the tail risk derived from the stock markets, and X indicates the five control variables. These control variables are; (i) Default spread (DEF) which is the difference between 5-year Korea Treasury Bond (KTB) yield and 5-year BBB-rated corporate bond yield. (ii) Term spread (TERM) which is the difference between 5-year KTB yield and 1-month KTB yield. (iii) Relative short-term interest rate (RREL) which is the difference between 1-month KTB yield and 12-month moving average of 1-month KTB yields. (iv) Equity risk premium (ERP) which is the difference between 1-month holding period return for

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[10] CFNAI is the Chicago Fed National Activity Index published by the Federal Reserve Bank of Chicago and is a unique indicator of the overall economic activity of the United States as a whole and is a weighted average of 85 economic activity indicators classified into four broad categories: production and income, employment and unemployment, personal consumption and housing, sales and order and inventory.

[11] Because FSI is available from January 2008, we use 95 months data of FSI from January 2008 to October 2015.

KOSPI and 1-month KTB yield. (v) Monthly volatility (MVOL) of the daily equity risk premium. These control variables are associated with the stock and the bond markets and included to control financial market components of FSI. Since the control variables have a big difference in scale from the other variables, we adjust their scales by multiplying the control variables by 100. Lastly, we include 12 lags of FSI as independent variables. Since the current FSI data is only available from January 2008 and the price limit of more than 15% is applied for the period, it is not necessary to divide the whole period into two sub-periods in this analysis.

<Table 7> shows the results that analyze the predictive power of tail risk for FSI. We find that tail risk has the predictive power for FSI from 6 months to 12 months in advance. In particular, tail risk has significant coefficients for FSIs of 6 months, 10 months, 11 months, and 12 months in advance at 95% confidence level.[1 2] The tail risk derived from the stock price fluctuations of individual firms well predicts the Financial Stability Index which indicates the economic conditions of the real economy and financial stability.

[Table 7 is about here.]

4.6 Robustness tests

In order to check whether tail risk is a major risk factor for stock returns, we conducted a two-step cross-sectional regression analysis of Fama and MacBeth (1973) on double-sorted portfolios by volatility and tail beta. First, we sort the stocks into five portfolios according to the volatility of individual stocks for the latest 60 months, and then sort each portfolio again into five portfolios according to the sensitivity to tail risk, tail beta, to make 25 (5x5 matrix) double-sorted portfolios. In the first-step of the time-series regression analysis, ERP, SMB, HML, volatility (VOL), and tail risk (TAIL) factors of the 25 portfolios are calculated using Equation (13). In the second step of the cross-sectional regression analysis using Equation (14), the average value of risk premium (gamma) is calculated. First panel is about the whole sample period, Period 1 is the period when the strict price limits were applied and Period 2 is the period when the price limits were more than 15%.

Step 1: time-series regression

$$\begin{aligned} r_{p,t} - r_{f,t} = \alpha_p + \beta_{ERP,p}\left(r_{m,t} - r_{f,t}\right) + \beta_{SMB,p}SMB_t + \beta_{HML,p}HML_t + \beta_{TAIL,p}TAIL_t \\ +\beta_{VOL,p}VOL_t + \varepsilon_{p,t} \end{aligned} \tag{13}$$

[1 2] Similar results are obtained when cyclical fluctuation values of Composite Leading Indicator (CLI) are added to Equation (12) to control economic leading factors of FSI.

Step 2: cross-sectional regression

$$r_p - r_f = \alpha + \gamma_{ERP}\beta_{ERP,p} + \gamma_{SMB}\beta_{SMB,p} + \gamma_{HML}\beta_{HML,p} + \gamma_{TAIL}\beta_{TAIL,p} + \gamma_{VOL}\beta_{VOL,p} + \varepsilon_p \qquad (14)$$

where $r_{p,t}$ is return of double-sorted portfolio by tail beta and volatility at time t, $\beta_{ERP,p}$ is portfolio p's beta to ERP factor, $\beta_{SMB,p}$ is portfolio p's beta to SMB factor, $\beta_{HML,p}$ is portfolio p's beta to HML factor, $\beta_{VOL,p}$ is portfolio p's beta to volatility factor, and $\beta_{TAIL,p}$ is portfolio p's beta to tail risk factor, respectively. 4-year rolling beta and equal-weighted average returns are applied.

[Table 8 is about here.]

In <Table 8>, we can see that tail risk along with ERP and SMB is a priced risk factor with the statistical significance at 5% level in determining portfolio returns for Period 2. In the whole sample period, ERP and SMB are statistically significant, but tail risk and other factors are not significant. And also tail risk is not significant in Period 1 when the strict price limit was applied. These results suggest that tail risk is a major risk factor under the current price limit system but it was not during the previous periods when the strict price limit system was applied in the Korean stock markets. Tail risk behavior is very different under the strict price limit system.

5. Conclusion

We derive the expected value of tail risk when the price limits exist and find that tail risk can be seriously underestimated in the stock markets with a price limit system. Through the simulation analysis using random returns, we find that tail risk cannot be used as a meaningful risk indicator as long as the price limits are less than 15%. We also find that tail risk can be a significant predictor for future stock market returns especially for 12 and 36 month holding periods when the price limits are above 15%, but it may not be when the price limits are below 15%.

Next, we investigate whether tail risk can be a risk priced factor in the stock markets with a price limit system. First, we divide all the stocks in the sample into five quintile portfolios by tail beta and find the larger tail beta, the greater the average returns of stock portfolio when the price limits are 15% or above. In the High-Low mimicking portfolio, excess monthly returns are present for all the holding periods but highest (1.474%) for one month holding period. Second, we use CAPM and Fama-French 3-factor model (1993) to see that monthly alpha values increase as tail beta increases. In the High-Low mimicking portfolio, CAPM alpha is 1.134% and FF 3-factor alpha is 1.132%, which are significant at 1% level for the one month holding period on the equal-weighted portfolio. Lastly, by Fama-MacBeth two-step cross-sectional regression analysis on the portfolios double-sorted by tail beta and volatility, we also find that tail risk is a main determinant of stock portfolio returns. After all, tail risk plays a major role as a risk factor in asset pricing as far as the price limits are above 15% and its behavior is very different in the markets where the strict price limit system is applied as in most Asian countries.

Lastly, we also examine the possibility of tail risk as a systemic risk indicator in Korea. We find that tail risk has the predictive power for Financial Stability Index from 6 to 12 months in advance if the price limits are 15% or above.

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Appendix

Derivation of the expected value of tail risk under price limits

Kelly and Jiang (2014) presented a tail risk and tail risk probability distribution as follows.

$$P\left(R_{i,t+1} < r \middle| R_{i,t+1} < u_t \mid \mathcal{F}_t\right) = (\frac{r}{u_t})^{-a_i/\lambda_t} \tag{A-1}$$

$$\lambda_t^{Hill} = \frac{1}{K_t}\sum_{k=1}^{K_t} ln\frac{R_{k,t}}{u_t} \tag{A-2}$$

Assuming that the tail risk follows the probability distribution of Equation (A-1), the cumulative distribution function $F(x)$ of tail risk and its probability density function $f(x)$ can be simply expressed by the formula as Equation (A-3) and Equation (A-4).

$$F(x) = Pr(X < u) = (\frac{x}{u})^{-\frac{a}{\lambda}} \tag{A-3}$$

$$\begin{aligned} f(x) &= F'(x) = (-\frac{a}{\lambda})(\frac{x}{u})^{-\frac{a}{\lambda}-1}\cdot\frac{1}{u} \\ &= (-\frac{a}{\lambda})\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}}\cdot\left(\frac{1}{x}\right) \end{aligned} \tag{A-4}$$

Therefore, the expected value of the tail risk$E\left(ln\left(\frac{x}{u}\right)\right)$ can be derived as follows.

$$E\left(ln\left(\frac{x}{u}\right)\right) = \int ln\left(\frac{x}{u}\right)\cdot\left(-\frac{a}{\lambda}\right)\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}}\left(\frac{1}{x}\right)dx \tag{A-5}$$

When there are price limits, the Equation (A-5) can be divided into two sections by the lower price limit v as follows. $v < u < 0$

$$E\left(ln\left(\frac{x}{u}\right)\right) = \int_v^u ln\left(\frac{x}{u}\right)\cdot\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}}\left(\frac{1}{x}\right)\left(-\frac{a}{\lambda}\right)dx + \int_{-\infty}^v ln\left(\frac{x}{u}\right)\cdot\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}}\left(\frac{1}{x}\right)\left(-\frac{a}{\lambda}\right)dx \tag{A-6}$$

The second part of integrals among the right side of Equation (A-6) can be summarized as follows.

$$\begin{aligned} &\int_{-\infty}^{v} ln\left(\frac{v}{u}\right)\cdot\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}}\left(\frac{1}{x}\right)\left(-\frac{a}{\lambda}\right)dx \qquad \because x \text{ is fixed to } v\text{due to lower price limit } v \\ &= ln\left(\frac{v}{u}\right)\cdot\int_{-\infty}^{v}\left(-\frac{a}{\lambda}\right)\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}}\left(\frac{1}{x}\right)dx \\ &= ln\left(\frac{v}{u}\right)\cdot\left[\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}}\right]_{-\infty}^{v} \\ &= ln\left(\frac{v}{u}\right)\cdot\left\{\left(\frac{v}{u}\right)^{-\frac{a}{\lambda}} - \left(\frac{-\infty}{u}\right)^{-\frac{a}{\lambda}}\right\} \end{aligned}$$

$$= ln\left(\frac{v}{u}\right)\cdot\left(\frac{v}{u}\right)^{-\frac{a}{\lambda}} \qquad \because \ \text{since } u < 0 \ \text{and} - \frac{a}{\lambda} < 0, \ \left(\frac{-\infty}{u}\right)^{-\frac{a}{\lambda}} \cong 0 \qquad \text{(A-7)}$$

Here, if $ln\left(\frac{x}{u}\right)$ is replaced by y, $ln\left(\frac{x}{u}\right) = y$, $\left(\frac{x}{u}\right) = e^{y}$, $\frac{dy}{dx} = \frac{d}{dx} ln\left(\frac{x}{u}\right) = \frac{1}{x}$ accordingly. And $y = ln\left(\frac{x}{u}\right) = ln\left(\frac{u}{u}\right) = 0$ when $x = u$, and $y = ln\left(\frac{v}{u}\right)$ when $x = v$ at the boundary of the integration interval. Using this information and a theorem of integration by parts for interval $[a, b]$, namely$\int_{a}^{b} fg'\,dx = [fg]_{a}^{b} - \int_{a}^{b} f'g\,dx$, the first part of integrals among the right side of Equation (A-6) can be restated for y as follows.

$$\int_{v}^{u} ln\left(\frac{x}{u}\right)\cdot\left(\frac{x}{u}\right)^{-\frac{a}{\lambda}}\left(-\frac{a}{\lambda}\right)\left(\frac{1}{x}\right)dx$$

$$= \int_{ln\left(\frac{v}{u}\right)}^{0} y \cdot e^{-\frac{a}{\lambda}y} \cdot \left(-\frac{a}{\lambda}\right)dy$$

$$= \left[ye^{-\frac{a}{\lambda}y}\right]_{ln\left(\frac{v}{u}\right)}^{0} - \int_{ln\left(\frac{v}{u}\right)}^{0} 1 \cdot e^{-\frac{a}{\lambda}y}\,dy$$

$$= \left[ye^{-\frac{a}{\lambda}y}\right]_{ln\left(\frac{v}{u}\right)}^{0} - \left[-\frac{\lambda}{a}e^{-\frac{a}{\lambda}y}\right]_{ln\left(\frac{v}{u}\right)}^{0}$$

$$= -ln\left(\frac{v}{u}\right)e^{ln\left(\frac{v}{u}\right)^{-\frac{a}{\lambda}}} - \left\{\left(-\frac{\lambda}{a}\right) + \ \frac{\lambda}{a}e^{ln\left(\frac{v}{u}\right)^{-\frac{a}{\lambda}}}\right\}$$

$$= -ln\left(\frac{v}{u}\right)\left(\frac{v}{u}\right)^{-\frac{a}{\lambda}} + \left(\frac{\lambda}{a}\right) - \frac{\lambda}{a}\left(\frac{v}{u}\right)^{-\frac{a}{\lambda}} \qquad \text{(A-8)}$$

Combining the Equation (A-7) and Equation (A-8) the expected value of tail risk is as follows.

$$E\left(ln\left(\frac{x}{u}\right)\right) = -ln\left(\frac{v}{u}\right)\left(\frac{v}{u}\right)^{-\frac{a}{\lambda}} + \left(\frac{\lambda}{a}\right) - \frac{\lambda}{a}\left(\frac{v}{u}\right)^{-\frac{a}{\lambda}} + ln\left(\frac{v}{u}\right)\cdot\left(\frac{v}{u}\right)^{-\frac{a}{\lambda}}$$

$$= \left(\frac{\lambda}{a}\right) - \frac{\lambda}{a}\left(\frac{v}{u}\right)^{-\frac{a}{\lambda}}$$

$$= \frac{\lambda}{a}\left(1 - \left(\frac{u}{v}\right)^{\frac{a}{\lambda}}\right) \qquad \text{(A-9)}$$

Therefore, the expected value of the tail risk $E\left(ln\left(\frac{x}{u}\right)\right)$ under price limits (v) is $\frac{\lambda}{a}\left(1 - \left(\frac{u}{v}\right)^{\frac{a}{\lambda}}\right)$.

<Table1> Changes of price limits in the Korean stock markets

This table shows the changes in the price limit system of the Korean stock markets. Price limits of KOSPI and KOSDAQ have been similarly changed together. In case of the KOSPI, there were price limits of 2.2% to 6.7% of the previous day's closing price (base price) were applied before the end of March 1994. Since then price limits has been changed to a fixed rate system, which has gradually expanded in line with the large increase in the stock market size and the gradual opening of capital markets to foreigners. 15% of price limits were applied by the end of 1998, when the IMF financial control was affected by the Asian currency crisis, and in June 2015 price limits were increased to 30%.

Panel A: KOSPI

Start date	Price limit	Remarks
Before Mar. 1994	Avg. 4.6%(2.2% ~ 6.7%)	Fixed amount base : 17 steps by the level of price
Apr. 1, 1994	6%	Fixed rate base
Nov. 24, 1996	8%	Fixed rate base
Mar. 2, 1998	12%	Fixed rate base
Dec. 7, 1998	15%	Fixed rate base
Jun. 15, 2015	30%	Fixed rate base

Panel B: KOSDAQ

Start date	Price limit	Remarks
Jul. 1, 1996 (market opened)	Avg. 5.4%	Fixed amount base : 11 steps by the level of price
Nov. 1, 1996	8%	Fixed rate base
May 25, 1998	12%	Fixed rate base
Mar. 28, 2005	15%	Fixed rate base
Jun. 15, 2015	30%	Fixed rate base

<Table 2> Simulation with random returns

This table shows the simulation results of random returns to identify whether the price limits affect extreme negative threshold and tail risk. We randomly extracted 10,000 returns, and then recalculated returns by the price limits of 8%, 10%, 12%, 15% and 20% respectively to create the conditions similar to the actual stock markets when the price limit exist. We also include the case when no price limit is applied to compare with. We do the same simulations assuming the volatility of stock prices increases by 10 and 20% arbitrarily to see the effects of the volatility of stock prices on tail risk. This table shows descriptive statistics, extreme negative threshold (u_t) and tail risk (λ_t) of the newly calculated returns.

Panel 1: price limits of 8% applied

	Random returns	10% increase of vol.	20% increase of vol.
Mean	-0.0000	-0.0000	0.0000
Max	0.0800	0.0800	0.0800
Min	-0.0800	-0.0800	-0.0800
Standard deviation	0.0383	0.0412	0.0439
Negative threshold (u_t)	-0.0656	-0.0722	-0.0788
Tail risk (λ_t)	0.1426	0.0870	0.0151

Panel 2: price limits of 10% applied

	Random returns	10% increase of vol.	20% increase of vol.
Mean	0.0010	0.0011	0.0012
Max	0.1000	0.1000	0.1000
Min	-0.1000	-0.1000	-0.1000
Standard deviation	0.0398	0.0434	0.0467
Negative threshold (u_t)	-0.0649	-0.0714	-0.0779
Tail risk (λ_t)	0.2024	0.1866	0.1619

Panel 3: price limits of 12% applied

	Random returns	10% increase of vol.	20% increase of vol.
Mean	-0.0009	-0.0010	0.0011
Max	0.1200	0.1200	0.1200
Min	-0.1200	-0.1200	-0.1200
Standard deviation	0.0397	0.0435	0.0472
Negative threshold (u_t)	-0.0668	-0.0735	-0.0802
Tail risk (λ_t)	0.2092	0.2043	0.1955

Panel 4: price limits of 15% applied

	Random returns	10% increase of vol.	20% increase of vol.
Mean	-0.0003	-0.0003	-0.0003
Max	0.1500	0.1500	0.1500
Min	-0.1500	-0.1500	-0.1500
Standard deviation	0.0406	0.0446	0.0487
Negative threshold (u_t)	-0.06770	-0.0737	-0.0804
Tail risk (λ_t)	0.2051	0.2046	0.2037

Panel 5: price limits of 20% applied

	Random returns	10% increase of vol.	20% increase of vol.
Mean	0.0003	0.0003	0.0004
Max	0.1411	0.1552	0.1693
Min	-0.1684	-0.1852	-0.2000
Standard deviation	0.0402	0.0443	0.0483
Negative threshold (u_t)	-0.0664	-0.0730	-0.0797
Tail risk (λ_t)	0.2094	0.2094	0.2094

(continued)

Panel 6: no price limit applied

	Random returns	10% increase of vol.	20% increase of vol.
Mean	-0.0002	-0.0003	0.0002
Max	0.1720	0.1892	0.2064
Min	-0.1426	-0.1569	-0.1711
Standard deviation	0.0401	0.0441	0.0491
Negative threshold (u_t)	-0.0662	-0.0728	-0.0795
Tail risk (λ_t)	0.2068	0.2068	0.2068

<Table 3> Descriptive statistics and correlations between negative threshold and tail risk

This table shows the descriptive statistics and correlation coefficients of extreme negative threshold and tail risk calculated using the stock price data for 25 years and 10 months from January 1990 to October 2015. Panel A shows the results during the whole sample period, whereas Panel B is when the strict price limits were applied and Panel C is where the price limits were above 15%. The number in parentheses of the correlation coefficient means the p-value.

	Panel A (Jan. 1990 ~ Oct. 2015)		Panel B (Jan. 1990 ~ Dec. 1998)		Panel C (Jan. 1999 ~ Oct. 2015)	
	Negative threshold	Tail risk	Negative threshold	Tail risk	Negative threshold	Tail risk
Mean	-0.0522	0.2852	-0.0453	0.1823	-0.0558	0.3402
Median	-0.0458	0.3311	-0.0418	0.1677	-0.0495	0.3558
Max	-0.0206	0.5504	-0.0206	0.4521	-0.0303	0.5504
Min	-0.1475	0.0053	-0.1065	0.0053	-0.1475	0.0126
Standard dev.	0.0206	0.1135	0.0174	0.0939	0.0212	0.0799
correlation	0.3321 (0.000)		0.3628 (0.000)		0.8602 (0.000)	

<Table 4> Predictive power of tail risk

This table shows the predictive power of tail risk for future aggregate stock market returns. Panel A shows the predictive regression analysis of $INDEX_{t+\tau} = \alpha + \beta\lambda_t + \varepsilon_t$, τ = 6, 12, 24, 36, 48 months, and in Panel B we add volatility as a control variable to control its effect in predicting future stock market returns with regression of $INDEX_{t+\tau} = \alpha + \beta\lambda_t + stdev_t + \varepsilon_t$, τ = 6, 12, 24, 36, 48 months. $INDEX_{t+\tau}$ is the log expected return of the stock index from time t to time $t+\tau$, and $INDEX$ is the market-weighted stock price index. $INDEX$ is calculated by multiplying the number of issued shares of 1,621 stocks subject to analysis by the adjusted stock prices reflecting capital changes such as capital increase and set the initial value index of January 3, 1990 at 100. $INDEX\_6M$ means the log expected return of $INDEX$ to 6 months after time t. $stdev_t$ is the monthly standard deviation of $INDEX$'s daily returns. Whole Period is the whole sample period of this study, and Period 1 is a period in which the price limits were less than 15%. On the other hand, Period 2 has a price limit of 15% or above which has little effect on stock prices. The numbers in parentheses indicate the t-value, and ***, **, * indicate statistical significance at the 1%, 5%, and 10% levels respectively.

Panel A: Basic regression ($INDEX_{t+\tau} = \alpha + \beta\lambda_t + \varepsilon_t$, τ = 6, 12, 24, 36, 48 months)

Dependent variables	Whole Period (Jan. 1990~ Oct. 2015)			Period 1 (Jan. 1990 ~ Dec. 1998)			Period 2 (Jan. 1999 ~ Oct. 2015)		
	Coeff.	p-value	$adj.R^2$	Coeff.	p-value	$adj.R^2$	Coeff.	p-value	$adj.R^2$
INDEX_6M	0.4817^{***}	0.001	0.034	0.5878^{*}	0.072	0.021	0.8567^{***}	0.000	0.064
	(3.435)			(1.816)			(3.795)		
INDEX_12M	0.6310^{***}	0.002	0.028	0.9125^{*}	0.083	0.019	1.5096^{***}	0.000	0.126
	(3.087)			(1.750)			(5.337)		
INDEX_24M	-0.0483	0.845	-0.003	-0.2282	0.741	-0.008	0.9823^{***}	0.000	0.062
	(-0.196)			(-0.332)			(3.583)		
INDEX_36M	0.1194	0.644	-0.003	-0.28765	0.658	-0.008	1.5390^{***}	0.000	0.113
	(0.463)			(-0.444)			(4.712)		
INDEX_48M	0.2827	0.241	0.002	1.3813^{**}	0.015	0.046	0.8285^{**}	0.012	0.034
	(1.175)			(2.482)			(2.543)		

Panel B: Regression controlled by standard deviation of $INDEX$ ($INDEX_{t+\tau} = \alpha + \beta\lambda_t + stdev_t + \varepsilon_t$, τ = 6, 12, 24, 36, 48 months)

Dependent variables	Whole Period (Jan. 1990~ Oct. 2015)			Period 1 (Jan. 1990 ~ Dec. 1998)			Period 2 (Jan. 1999 ~ Oct. 2015)		
	Coeff.	p-value	$adj.R^2$	Coeff.	p-value	$adj.R^2$	Coeff.	p-value	$adj.R^2$
INDEX_6M	0.4467^{***}	0.000	0.049	0.6623^{***}	0.005	0.108	1.0107^{***}	0.000	0.086
	(4.114)			(2.850)			(4.376)		
INDEX_12M	0.6676^{***}	0.000	0.064	1.2957^{***}	0.000	0.273	1.7141^{***}	0.000	0.163
	(4.284)			(3.839)			(5.943)		
INDEX_24M	0.2497	0.178	0.050	0.7621^{*}	0.079	0.303	1.1921^{***}	0.000	0.085
	(1.352)			(1.772)			(4.158)		
INDEX_36M	0.1926	0.329	0.002	0.2562	0.585	0.073	1.9503^{***}	0.000	0.165
	(0.979)			(0.548)			(5.721)		
INDEX_48M	0.3518^{*}	0.053	0.020	1.6037^{***}	0.000	0.243	1.0465^{***}	0.003	0.060
	(1.944)			(4.302)			(3.021)		

<Table 5> Average monthly returns of portfolios sorted by tail beta

This table shows the difference in the average monthly returns according to the tail beta, the sensitivity of the individual stocks to tail risk. To estimate the monthly tail beta of stock *i* (tail β_i), we run a regression of $r_{i,t} = \alpha_i + \beta_i \lambda_t + \varepsilon_t$ using the monthly return of stock *i* (r_i) and tail risk (λ_t) for the preceding 60 months. We sort all stocks into five quintile portfolios (Low, 2, 3, 4, High) every month by the estimated tail beta. After calculating the equal-weighted returns for these portfolios, average monthly returns for the holding periods of 1, 6, and 12 months were obtained. In addition, we constructed a mimicking portfolio by purchasing High portfolio with high tail beta and selling Low portfolio with low tail beta at the same time. First panel, Whole Period, is about the whole sample period of this study, and Period 1 is a period in which the price limits were less than 15%. On the other hand, Period 2 has the price limit of 15% or more.

Whole Period (Jan. 1990 ~ Oct. 2015)

	Low	2	3	4	High	High-Low
For 1 m	2.739%	2.099%	1.562%	1.626%	2.090%	-0.649%
For 6 m	2.593%	2.054%	1.574%	1.637%	2.110%	-0.484%
For 12 m	2.439%	1.953%	1.518%	1.575%	1.997%	-0.441%

Period 1 (Jan. 1990 ~ Dec. 1998)

	Low	2	3	4	High	High-Low
For 1 m	2.675%	1.086%	0.171%	-0.047%	0.275%	-2.400%
For 6 m	2.664%	0.718%	-0.262%	-0.664%	-0.492%	-3.156%
For 12 m	2.947%	0.838%	-0.128%	-0.765%	-0.849%	-3.795%

Period 2 (Jan. 1999~ Oct. 2015)

	Low	2	3	4	High	High-Low
For 1 m	1.427%	2.065%	2.260%	2.381%	2.900%	1.474%
For 6 m	1.376%	2.027%	2.208%	2.314%	2.789%	1.412%
For 12 m	1.346%	1.938%	2.102%	2.199%	2.636%	1.289%

<Table 6> Alphas (α) of portfolio sorted by tail beta

This table shows the result of alphas (α) by CAPM and Fama and French (1993) 3-factor model for both the equal-weighted portfolios sorted by tail beta (tail β). The analytical regression equation used in this analysis is as follows.

CAPM model : $$r_{p,t} - r_{f,t} = \alpha_p + \beta_p(r_{M,t} - r_{f,t}) + \varepsilon_t$$
Fama and French 3-factor model : $$r_{p,t} - r_{f,t} = \alpha_p + \beta_{1p}(r_{M,t} - r_{f,t}) + \beta_{2p}SMB_t + \beta_{3p}HML_t + \varepsilon_t$$

where $r_{p,t}$ is the return of portfolio p, $r_{M,t}$ is $INDEX$'s return, $r_{f,t}$ is the yield of 1-year Monetary Stabilization Bond issued by The Bank of Korea, SMB_t is size factor, and HML_t is the book to market value factor. All are measured at time t. First panel is about the whole sample period of this study, and Period 1 is a period in which the price limits were less than 15%. On the other hand, Period 2 has the price limit of 15% or more. The numbers in parentheses mean t-value, ***, **, * indicate statistical significance at 1%, 5% and 10% respectively.

		Low	2	3	4	High	High-Low
Whole Period	CAPM alpha	1.778%***	1.848%***	1.309%***	1.390%***	1.910%***	0.132%
		(2.626)	(5.471)	(7.670)	(6.338)	(6.018)	
	FF3 alpha	1.209%**	1.626%***	1.370%***	1.548%***	2.047%***	0.838%
		(2.046)	(5.198)	(8.140)	(7.781)	(6.770)	
Period 1	CAPM alpha	1.996%**	1.605%***	1.297%***	1.658%***	2.528%***	0.532%
		(2.313)	(3.189)	(3.423)	(2.995)	(2.916)	
	FF3 alpha	1.972%**	1.743%***	1.392%***	1.715%***	2.353%***	0.381%
		(2.293)	(4.685)	(3.854)	(3.074)	(2,810)	
Period 2	CAPM alpha	0.789%***	1.345%***	1.488%***	1.549%***	1.923%***	1.134%
		(3.387)	(7.342)	(8.844)	(9.287)	(8.772)	
	FF3 alpha	0.769%***	1.319%***	1.501%***	1.573%***	1.901%***	1.132%
		(3.303)	(7.194)	(8.848)	(9.443)	(8.890)	

<Table 7> Predictive power of tail risk for FSI

This table shows the result of analyzing the predictive power of tail risk for macroeconomic downturn. To test the possibility of tail risk as a systemic risk indicator, we use the following regression equation which is similar to the predictive regression model of Allen et al. (2012). They use CFNAI, the US macroeconomic indicator, as a dependent variable in their study. We use FSI (Financial Stability Index) published by The Bank of Korea as a national activity index which is known to be useful in early detection of systemic risk. The predictive power of tail risk is measured by using $FSI_{\ t+n}$ as a dependent variable in the following regression. Because FSI is available from January 2008, we use 94 months data of FSI from January 2008 to October 2015. Since the current FSI data is only available from January 2008 and the price limit of more than 15% is applied for the period, it is not necessary to divide the whole period into two sub-periods in this case.

$$FSI_{\ t+n} = \alpha + \gamma TAIL_t + \beta\, X_t + \sum_{i=1}^{12} \mu_i\, FSI_{t-i+1} + \varepsilon_{t+n}$$

where $FSI_{\ t+n}$ is the Financial Stability Index n months later at time t, $TAIL$ is the tail risk derived from the stock markets, and X indicates the five control variables. These control variables are; (i) Default spread (DEF) which is the difference between 5-year Korea Treasury Bond (KTB) yield and 5-year BBB-rated corporate bond yield. (ii) Term spread (TERM) which is the difference between 5-year KTB yield and 1-month KTB yield. (iii) Relative short-term interest rate (RREL) which is the difference between 1-month KTB yield and 12-month moving average of 1-month KTB yields. (iv) Equity risk premium (ERP) which is the difference between 1-month holding period return for KOSPI and 1-month KTB yield. (v) Monthly volatility (MVOL) of the daily equity risk premium. These control variables are associated with the stock and the bond markets and included to control financial market components of FSI. Since the control variables have a big difference in scale from the other variables, we adjust their scales by multiplying the control variables by 100. Lastly, we include 12 lags of FSI as independent variables. The dependent variable, FSI_6M, refers to the FSI after 6 months. The numbers in parentheses mean t-value, ***, **, * indicate statistical significance at 1%, 5% and 10% respectively.

Explanatory	Dependent variables											
variables	FSI_4M	FSI_5M	FSI_6M	FSI_7M	FSI_8M	FSI_9M	FSI_10M	FSI_11M	FSI_12M	FSI_13M	FSI_14M	FSI_15M
TAIL	0.0615	0.1319	0.2647^{***}	0.1609^{*}	0.1229	0.1428	0.1883^{**}	0.2111^{**}	0.2021^{**}	0.0832	-0.0353	-0.0300
	(0.626)	(1.392)	(3.225)	(1.864)	(1.421)	(1.630)	(2.084)	(2.197)	(2.120)	(0.864)	(-0.377)	(-0.324)
DEF	5.2703^{***}	4.3201^{***}	3.1817^{**}	1.3618	-0.6525	-1.6290	-0.7185	0.1386	1.6813	3.5055*	3.4898*	1.3027
	(3.284)	(2.769)	(2.295)	(0.948)	(-0.440)	(-1.066)	(-0.446)	(0.080)	(0.968)	(1.958)	(1.936)	(0.716)
TERM	-3.7479^{***}	-3.2652^{***}	-2.2897^{**}	-1.1055	0.3208	1.2621	1.2269	1.1931	0.3523	-0.4541	-0.2381	1.3176
	(-3.679)	(-3.309)	(-2.613)	(-1.216)	(0.341)	(1.301)	(1.196)	(1.084)	(0.319)	(-0.400)	(-0.210)	(1.156)
RREL	2.5830^{**}	4.0608^{***}	5.6590^{***}	6.6018^{***}	6.9650^{***}	6.0079^{***}	4.2413^{***}	3.9985^{***}	3.2655^{***}	2.5431^{**}	3.0471^{**}	4.1634^{***}
	(2.553)	(4.108)	(6.437)	(7.071)	(6.950)	(5.908)	(3.954)	(3.417)	(2.773)	(2.030)	(2.375)	(3.216)
ERP	-0.2184^{***}	-0.0962	-0.0786	-0.0766	-0.0453	0.0366	-0.0235	-0.0620	0.0437	0.0326	0.0266	0.0543
	(-2.801)	(-1.278)	(-1.211)	(-1.126)	(-0.671)	(0.539)	(-0.336)	(-0.834)	(0.595)	(0.439)	(0.366)	(0.753)
MVOL	-0.2136	-0.1500	-0.2224	-0.1718	0.0441	0.3490^{*}	0.5990^{***}	0.4136*	0.4322^{**}	0.2438	0.0631	0.0100
	(-0.956)	(-0.692)	(-1.194)	(-0.894)	(0.227)	(1.791)	(2.991)	(1.948)	(2.047)	(1.143)	(0.305)	(0.049)
Adj. R^2	0.856	0.786	0.792	0.734	0.684	0.658	0.636	0.599	0.615	0.611	0.636	0.651

<Table 8> Fama-MacBeth Test for tail risk

This table shows the results of a two-step regression analysis of Fama and MacBeth (1973). We conducted a two-step cross-sectional regression analysis of Fama and MacBeth (1973) on double-sorted portfolios by volatility and tail beta. First, we sort the stocks into five portfolios according to the volatility of individual stocks for the latest 60 months, and then sort each portfolio again into five portfolios according to the sensitivity to tail risk, tail beta, to make 25 (5x5 matrix) double-sorted portfolios. In the first-step of the time-series regression analysis, ERP, SMB, HML, volatility (VOL), and tail risk (TAIL) factors of the 25 portfolios are calculated using equation in Step 1 as follows. In the second step of the cross-sectional regression analysis using equation in Step 2, the average value of risk premium (gamma) is calculated. Whole Period is the whole sample period, Period 2 is the period of when the price limits were less than 15% and Period 2 is the period when the price limits were more than 15%. 4-year rolling beta and equal-weighted average returns are applied. The numbers in parentheses mean t-value, ***, **, * indicate statistical significance at 1%, 5% and 10%, respectively.

(Step 1) $r_{p,t} - r_{f,t} = \alpha_p + \beta_{ERP,p}\left(r_{m,t} - r_{f,t}\right) + \beta_{SMB,p}SMB_t + \beta_{HML,p}HML_t + \beta_{TAIL,p}TAIL_t$
$+\beta_{VOL,p}VOL_t + \varepsilon_{p,t}$

(Step 2) $r_p - r_f = \alpha + \gamma_{ERP}\beta_{ERP,p} + \gamma_{SMB}\beta_{SMB,p} + \gamma_{HML}\beta_{HML,p} + \gamma_{TAIL}\beta_{TAIL,p} + \gamma_{VOL}\beta_{VOL,p} + \varepsilon_p$

where $r_{p,t}$ is the return of double-sorted portfolio by tail beta and volatility at time t, $\beta_{ERP,p}$ is portfolio p's beta to ERP factor, $\beta_{SMB,p}$ is portfolio p's beta to SMB factor, $\beta_{HML,p}$ is portfolio p's beta to HML factor, $\beta_{VOL,p}$ is portfolio p's beta to volatility factor, and $\beta_{TAIL,p}$ is portfolio p's beta to tail risk factor respectively.

	Whole Period (Jan. 1990 ~ Oct. 2015)	Period 1 (Jan. 1990 ~ Dec. 1998)	Period 2 (Jan. 1999 ~ Oct. 2015)
γ_{ERP}	2.3517***	2.7057	1.6505**
	(3.213)	(0.702)	(2.602)
γ_{SMB}	0.8817**	7.5240	0.6422**
	(2.411)	(1.377)	(1.976)
γ_{HML}	0.4802	-3.7518*	0.2406
	(1.617)	(-1.966)	(1.112)
γ_{TAIL}	-1.2828	5.2847	1.5379**
	(-0.867)	(0.937)	(2.028)
γ_{VOL}	0.1403	-0.7083	0.0193
	(0.665)	(-0.993)	(0.120)
$constant$	-0.3452	3.3839	0.3096
	(-0.668)	(1.110)	(0.682)
$R-squared$	0.509	0.652	0.509

<Figure 1> Tail risk (λ_t), extreme negative threshold (u_t) and price limits (lower price limits)

This figure shows a graph of tail risk, extreme negative threshold and lower price limits during the sample period. We see the that extreme negative thresholds and tail risks have been frequently curbed by the price limits until 1998 when the price limits were expanded to 15%, albeit the gradual expansion of the price limits. Prior to 1998, extreme negative thresholds approached the lower price limits many times, but after the price limits were expanded to 15% in December 1998, only once when the global financial crisis occurred. The shaded area is the period when the price limit was less than 15%. The extreme negative threshold is the lower 5th percentile of the monthly pool of returns by month. Tail risk is a risk indicator estimated to be $\frac{1}{K_t}\sum_{k=1}^{K_t} ln\frac{R_{k,t}}{u_t}$ for the extreme returns below the extreme negative threshold. We use the right value for tail risk and the left value for extreme negative threshold and the price limits.