Startup Financing with Patent Signaling under Ambiguity

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Abstract

One of the most important issues for startup companies is to secure financing. Indeed, it is essential for startups to signal their projects' profitability to potential investors. We develop a model of single-stage startup financing with signaling under ambiguity. Nature determines the ability of a technology entrepreneur (startup), who strategically chooses a costly patent level as a signal to inform his ability to potential investors. Since the project taken by a startup may involve highly innovative technology and may not be well known to agents, they would face ambiguity about project value. To examine ambiguity effects on startup financing, we provide three different financing models in view of the degree of ambiguity: (1) no ambiguity; (2) only investors face ambiguity; (3) all agents face ambiguity. In each model, we derive perfect Bayesian equilibria and refine them into a unique equilibrium by imposing Intuitive Criterion of Cho and Kreps (1987) or its extension. We analyze the refined equilibria in perspectives of agents' equity shares, equilibrium patent levels, and his expected profit.

KEYWORDS: startup financing; signaling; perfect Bayesian equilibrium; Intuitive Criterion

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1 Introduction

One of the most important issues for startup companies is to secure financing. Due to the absence of track records, it is essential for startups to inform their projects' profitability to potential investors. In other words, startups need to reveal reliable information about their ability to attract investors in early financing stages. For technology startups, the number of filed patents can be a useful signal to access seed investors. As Graham et al. (2009) point out, technology startups tend to hold patents for competitive advantage, securing financing, and enhancing reputation. In particular, they analyze the Berkely Patent Survey, and find that it is easier for startups to be funded from external investors by holding more patents. Conti, Thursby, and Rothaermel (2013) empirically show that, in startup financing, an increase of patents level raises both the frequency and amount of investments from venture capitals. They explain these empirical facts by using signaling game where a two-dimensional signal which consists of patent level and investments from acquaintances is considered. Conti, Thursby, and Thursby (2013) employ a single-stage financing model in which the entrepreneur uses patent signals in order to inform his ability and they empirically test the model and show that a startup's patents level is endogenously determined.[1]

Another important issue for startups is concerned with ambiguity about project value, which is the uncertainty about the true success probability of a startup's project. For instance, it is relatively difficult for outside investors to have the exact information about the entrepreneur's true success probability. Even though investors believe that the entrepreneur has high success probability, they may not know what the true success probability is and regard the entrepreneur's success probability as a random variable. In this case, we say that investors face ambiguity about the entrepreneur's project value. Often times, a startup's project may be innovative and has few track records, and thus agents (i.e., the entrepreneur and investors) would make decisions without sufficient information about the entrepreneur's ability. In a different context, Rigotti et al. (2008) point out that technology startups often have ambiguous information about their own project value. To the best of our knowledge, however, there is no startup financing model which deals with ambiguity effects on startup financing.

It is practically important to examine the effects of ambiguity when a startup signals its success probability via patents to investors. One can ask the following questions: Compared with the case where agents are well informed about the project, how they make different decisions under ambiguity? How does the entrepreneur differently signal his ability to potential investors? How do investors differently require their compensation and how are equity shares differently distributed? One may conjecture that an entrepreneur acquires more patents to show the profitability of his

[1] Elitzur and Gavious (2003) and Kim and Wagman (2016) consider different kinds of signaling device in two-stage startup financing models. In Elitzur and Gavious (2003), whether an entrepreneur approaches an angel or not is a signal about his future effort level. In Kim and Wagman (2016), the entrepreneur's decision about whom he makes a contract with in the first stage is a signal to investors in the second stage.

project and investors ask more compensation under ambiguity than otherwise. However, the validity of the conjecture depends on who faces ambiguity and what the levels of underlying parameters are (see Proposition 5.2). The purpose of this paper is to analyze how a startup's patent-signaling affects a early-stage financing under ambiguity. To do this, we provide single-stage startup financing models in which an entrepreneur strategically chooses costly patent level as a signaling device to inform his ability or success probability to potential investors. Investors participate in seed investment to initiate the entrepreneur's project after observing the entrepreneur's patent level. Similar to Spence (1973), for simplicity, we assume that the entrepreneur's inborn ability is not affected by patent level.

To analyze the decision-making under ambiguity, we employ the smooth ambiguity model of Klibanoff, Marinacci, and Mukerji (2005), who represent preferences by the expected distortion of the expected utility, and consider its special case where agents are risk-neutral and ambiguity-neutral.[2] To examine the effects of ambiguity, we provide three different models in with degree of ambiguity: (1) no ambiguity; (2) only investors face ambiguity; (3) all agents face ambiguity. In the first model (Benchmark Model), the entrepreneur exactly knows his own type or true success probability, which investors cannot observe. This model reflects the case where the project involves a well-known technology. In the second model (Model I), the entrepreneur still exactly knows his own type. However, investors face ambiguity about project value, i.e., they know only the intervals which can contain the entrepreneur's types. But they cannot observe which interval contains the entrepreneur's type. This model supports the case where the project involves an intermediate-level innovation. In the third model (Model II), even the entrepreneur does not exactly know his own type. Indeed, when the project involves a high-level innovation, it is impossible for the entrepreneur to pin down his own success probability. Here the entrepreneur as well as investors face ambiguity in the same way as investors do in Model I, but the entrepreneur recognizes the interval which contains his own type.

We derive perfect Bayesian equilibria in signaling game of each startup financing model and refine them into a unique equilibrium by imposing Intuitive Criterion of Cho and Kreps (1987) or its extension. Then we analyze the refined equilibria in the perspectives of agents' equity shares, patent level acquired by the entrepreneur, and his expected profit. It is noted that, since we assume Bertrand competition in the investment market, the investor's profit is zero in each refined equilibrium. We find that the entrepreneur should acquire the most amount of patents to inform his ability to investors when only investors face ambiguity (i.e., in Model I). It is because investors respond to the entrepreneur's signaling in a more conservative way compared with when all agents resolve or face it. On the other hand, it is likely to expect that investors ask more equity share when the project is not well known to them than otherwise. However, we find that they ask a lower equity

[2]One can employ alternative ambiguity models such as the maximin expected utility model (Gilboa and Schmeidler, 1989), the multiplier model (Hansen and Sargent, 2001), and the variational preference model (Maccheroni, Marinacci, and Rustichini, 2006).

share under ambiguity if the entrepreneur's project is sufficiently promising (i.e., investors think that the project yields a sufficiently high expected gross return). Moreover, the entrepreneur can be better off due to ambiguity on the side of the investment market if the entrepreneur's project is sufficiently promising.

Our model is closely related to Conti, Thursby, and Thursby (2013), but different from theirs in three aspects. First, they assume that the entrepreneur's type is the quality of his invention, which affects project value in a deterministic way, and thus project value is perfectly revealed to investors in the separating equilibrium. However, we take the entrepreneur's success probability as his type, which makes investors face uncertainty or ambiguity about project value. Second, they do not allow for the case where agents are so unfamiliar with the project that they face ambiguity about its value, which is accommodated in our model. Third, they allow project value to be affected by patent level (i.e., the signal is productive), similar to Spence (1974). In our model, project value is assumed to be independent of patent level (i.e., the signal is unproductive), which allows us to easily obtain closed-form solutions that are not provided by Conti, Thursby, and Thursby (2013).

The rest of the paper is organized as follows. In Section 2, we introduce three models classified by the degree of ambiguity. We derive perfect Bayesian equilibria of the signaling games for each model in Section 3 and refine them in view of Intuitive Criterion of Cho and Kreps (1987) in Section 4. We characterize our refined equilibria by comparative statics in Section 5. Concluding remarks are given in Section 6. All the proofs are relegated to Appendix.

2 Model

There are two types of risk-neutral agents: a technology entrepreneur and investors. The sequence of events persists over three periods ($\tau = 0, 1, 2$). In period $\tau = 0$, nature determines success probability s of a risky project which the entrepreneur will launch. If launched at period $\tau = 1$ with investment, the entrepreneur's risky project generates random project value R, which is realized at period $\tau = 2$ such that

$$R(\omega) = \begin{cases} A & \text{if } \omega = \text{success}, \\ 0 & \text{if } \omega = \text{failure}, \end{cases}$$

where success probability is s and A is constant. Then the project's expected value is $\mathbb{E}_s[R] = sA$.[3]

Let s_H and s_L denote the true success probabilities for high type and low type of the entrepreneur, respectively. In the environment of startup financing, the project is typically innovative and has little track record. The entrepreneur and investors may face uncertainty about the probability distribution of project value, i.e., face ambiguity, which we call *value ambiguity*.[4] Facing

[3] Let $\mathbb{E}_\eta[\cdot]$ denote the expected value under probability measure η.

[4] We may take into account the case where agents may have multiple priors about the types of the entrepreneur

(value) ambiguity, they know that success probability s has distribution ν, which is assumed to be the standard uniform distribution on $[0, 1]$. Let s^* denote a threshold success probability where $s_L < s^* \leq s_H$. The entrepreneur with high ability has the true success probability $s_H \in [s^*, 1] \equiv I_H$ and with low ability has the true success probability $s_L \in (0, s^*) \equiv I_L$. Since we assume Bertrand competition in the investment market, all investors are represented by a single investor henceforth.

To analyze the effects of ambiguity on startup financing, we introduce the following three models. For reflecting ambiguity, we need to consider two possibly different type spaces of the entrepreneur which the entrepreneur and the investor conceive, denoted by T_e and T_i, respectively.

- **Benchmark Model:** No agents face ambiguity about project value. This model is similar to a standard job-market signaling model. The entrepreneur knows his own type (i.e., true success probability): s_H or s_L. The investor knows that s_H and s_L are the entrepreneur's possible types but she cannot observe whether his type is s_H or s_L. Thus we have $T_e = T_i = \{s_H, s_L\}$. The investor has prior belief μ about the entrepreneur's possible types such that $\mu(s_L) = s^* \in (0, 1)$. All agents consider the entrepreneur with true success probability s_H (s_L) as the high (low, respectively) type. This model is suitable for the case where the project accompanies low-level innovation and has sufficient track records. This model corresponds to Conti, Thursby, and Thursby (2013).

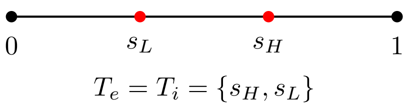

Figure 1: Type space of each agent in the benchmark model

- **Model I:** Only the investor faces ambiguity about project value. As in Benchmark Model, the entrepreneur still knows his own type. However, the investor only knows that his true success probability belongs to either I_H or I_L but she cannot not observe whether it belongs to I_H or I_L. In this case, the investor regards I_H and I_L as the entrepreneur's possible types, and therefore $T_e = \{s_H, s_L\}$ and $T_i = \{I_H, I_L\}$. The investor has prior belief μ about the entrepreneur's possible types (i.e., I_H, I_L) such that $\mu(I_L) = s^* \in (0, 1)$. The entrepreneur considers himself as the high (low) type if his true success probability is s_H (s_L, respectively). On the other hand, the investor considers the entrepreneur as the high (low) type if she believes that his true success probability belongs to I_H (I_L, respectively). This model involves the case where the project takes intermediate-level innovation.

and thus be faced ambiguity, which we call *type ambiguity*. For the convenience of analysis, however, we exclude type ambiguity. In our model, all the investors have a unique prior about the types of the entrepreneur.

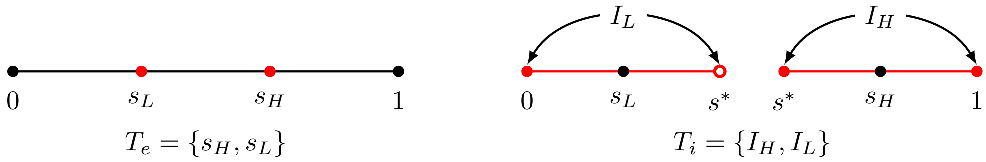

Figure 2: Type space of each agent in Model I

- **Model II:** All agents (i.e., the entrepreneur and the investor) face ambiguity about project value. Here, even the entrepreneur does not have the exact information about his own true success probability. He only knows the interval which contains his own true success probability. As in Model I, the investor only knows that his true success probability belongs to either I_H or I_L. Thus he regards I_H and I_L as the entrepreneur's possible types. But she cannot not observe whether it belongs to I_H or I_L. Consequently, we have $T_e = T_i = \{I_H, I_L\}$. Therefore there is a symmetric ambiguity between the entrepreneur and the investor. As in Model I, the investor has prior belief μ about the entrepreneur's possible types such that $\mu(I_L) = s^* \in (0,1)$. All agents consider the entrepreneur with type I_H (I_L) as the high (low, respectively) type. This model reflects the case where the project is so innovative that even the entrepreneur faces the lack of information about his true success probability.

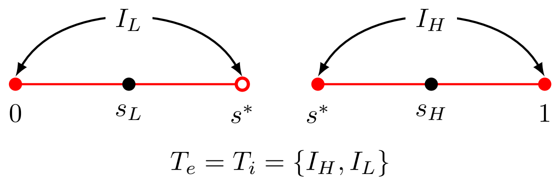

Figure 3: Type space of each agent in Model II

It is assumed that the type spaces (i.e., T_e and T_i) of the entrepreneur, which the entrepreneur and the investor conceive, are common knowledge in each model.

Ambiguity about project value plays an important role in making decisions for startup financing. To analyze decision making under ambiguity, we adopt the smooth ambiguity model and assume ambiguity-neutrality as a special case for simplicity.[5] Facing ambiguity about project value,

[5] In Klibanoff et al. (2005), a smooth ambiguity preference $\succcurlyeq$ is represented by function V:

$$V(f) = \int_{\Delta} \phi\left(\int_S u(f(\omega))d\pi(\omega)\right) d\mu(\pi) \equiv \mathbb{E}_\mu\left[\phi\left(\mathbb{E}_\pi[u(f)]\right)\right],$$

where u is a continuous strictly increasing utility function, ϕ is a continuous strictly increasing distortion function, Δ is a set of probability measure π's, and μ is a countably additive probability measure over Δ. In particular, preference $\succcurlyeq$

ambiguity-neutral agents think of the high type's success probability as $\nu(I_H) = \frac{1+s^*}{2}$ and the low type's as $\nu(I_L) = \frac{s^*}{2}$. Thus, as threshold probability s^* is higher, the investor with ambiguity expects higher project value, in which sense we say that s^* represents *market evaluation* about the project.

In order to attract the investor, the entrepreneur strategically determines patent level $\psi \in [0, \infty)$ to show his ability after learning his own type. For simplicity, we assume that his true success probability is independent of patent level.[6] We also assume that the high type can acquire patents more efficiently than the low type. In particular, the entrepreneur's cost function of acquiring patent is

$$c(s, \psi) = \begin{cases} c_H \psi & \text{if } s \in I_H, \\ c_L \psi & \text{if } s \in I_L, \end{cases}$$

where $c_L > c_H$. Note that the cost is constant in each interval of success probability, which is for the convenience of analysis.

2.1 Startup Financing

To launch the project, the entrepreneur needs seed investment K dollars in period $\tau = 1$. The investor, who has observed entrepreneur's patent level ψ, approaches the entrepreneur and offers investment K for her own share $\beta \in (0, 1)$ (i.e., the entrepreneur's share is $\theta \equiv 1 - \beta$). The investor does not directly observe whether the entrepreneur's type is high or low, but she updates her beliefs about the entrepreneur's type after observing ψ. In period $\tau = 2$, project value R is realized. Only if the project succeeds, the entrepreneur and the investor are paid proportional to their equity shares. The sequence of events is illustrated in Figure 4 below.

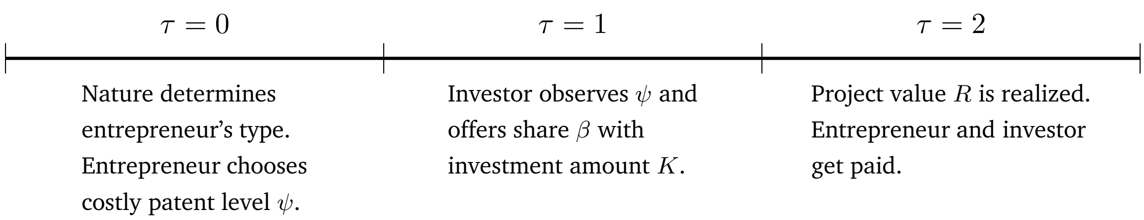

Figure 4: Sequence of events

is *ambiguity neutral* if ϕ is linear (see Corollary of Klibanoff et al. (2005)). Moreover, in the case of risk-neutrality and ambiguity-neutrality as in our model, both u and ϕ are linear functions. In our context, we represent the preferences of an agent over R by

$$V(R) = \int_{[0,1]} [\mathbb{E}_s[R]] ds = \mathbb{E}_\nu[\mathbb{E}_s[R]],$$

where ν is the uniform probability measure of s on $[0, 1]$.

[6]The 'dependent' case can be easily accommodated in our model without changing main results.

2.2 Profits of Agents

Now we define the (expected) profit functions of agents. Since all the agents are risk-neutral, their utilities are defined by retained cash-out shares. Thus type t entrepreneur's *ex post* profit is

$$u(t, \psi, \beta, \omega) = (1 - \beta)R(\omega) - c_t\psi, \, \forall \, t \in T_e.$$

Type t entrepreneur's *interim* expected profit is[7]

$$U\left(t, \psi, \beta\right) = (1 - \beta)\mathbb{E}_\nu[\mathbb{E}_s[R]|t] - c_t\psi = (1 - \beta)\mathbb{E}_\nu\left[s|\, t\right]A - c_t\psi, \, \forall \, t \in T_e.$$

The investor's *ex post* profit is

$$v\left(\beta, \omega\right) = \beta R(\omega) - K.$$

After observing the entrepreneur's patent level, the investor's *interim* expected profit is[8]

$$\tilde{V}\left(t, \beta\right) = \beta\mathbb{E}_\nu[\mathbb{E}_s[R]|t] - K = \beta\mathbb{E}_\nu\left[s\,|t\right]A - K, \;\forall \, t \in T_i.$$

Thus the investor's (ex ante) expected profit is

$$V(\psi, \beta) = \sum_{t \in T_i} \mu(t|\psi)\tilde{V}\left(t, \beta\right).$$

To ensure the participation of the investor, the lowest project value in the interim stage should be greater than the investment. Therefore it is assumed that

$$\min\left\{s_L A, \, \frac{1}{2}s^* A\right\} > K. \tag{2.1}$$

3 Equilibrium

As mentioned in Section 2, we consider three different financing models in view of the degree of ambiguity: (1) no ambiguity; (2) only the investor faces ambiguity; (3) all agents face ambiguity. Adopting perfect Bayesian equilibrium (PBE) as a solution concept in the signaling game between the entrepreneur and the investor, we derive separating and pooling equilibria in each model.

3.1 Benchmark Model: No Ambiguity

In this model, no agents face ambiguity about project value. Similar to the job-market signaling game of Spence (1973), we can easily find separating and pooling equilibria.

[7] Here, "interim" means that the entrepreneur knows his own type.

[8] Here, "interim" means that the investor observes the entrepreneur's type.

3.1.1 Separating Equilibrium

In separating equilibrium, the entrepreneur's type is perfectly revealed to the investor. If the investor observes the entrepreneur's patent level ψ_H^0 (ψ_L^0), then she believes that the entrepreneur's type is s_H (s_L) and offers her share β_H^0 (β_L^0, respectively). Having zero expected profit, the investor solves

$$\begin{aligned} V\left(\psi_H^0, \beta_H^0\right) &= \beta_H^0 s_H A - K = 0, \\ V\left(\psi_L^0, \beta_L^0\right) &= \beta_L^0 s_L A - K = 0, \end{aligned} \tag{3.1}$$

which imply

$$\beta_H^0 = \frac{K}{s_H A} \quad \text{and} \quad \beta_L^0 = \frac{K}{s_L A}. \tag{3.2}$$

We know that $0 < \beta_H^0 < \beta_L^0 < 1$ by (2.1) and (3.2). The entrepreneur's shares for type s_H and type s_L are

$$\begin{aligned} \theta_H^0 &\equiv 1 - \beta_H^0 = 1 - \frac{K}{s_H A}, \\ \theta_L^0 &\equiv 1 - \beta_L^0 = 1 - \frac{K}{s_L A}. \end{aligned} \tag{3.3}$$

Note that $0 < \theta_L^0 < \theta_H^0 < 1$ by (2.1) and (3.3).

The investor takes patent level ψ_H^0 satisfying the following incentive compatibility constraints as a cutoff patent level for the high type:

$$\begin{aligned} U(s_H, \psi, \beta_H^0) = \theta_H^0 s_H A - c_H \psi &\geq \theta_L^0 s_H A = U(s_H, 0, \beta_L^0), \\ U(s_L, 0, \beta_L^0) = \theta_L^0 s_L A &\geq \theta_H^0 s_L A - c_L \psi = U(s_L, \psi, \beta_H^0). \end{aligned} \tag{3.4}$$

Then it is standard to establish the following result (hence its proof is omitted).

Proposition 3.1. *There are separating perfect Bayesian equilibria, in one of which the patent levels of type s_L and type s_H entrepreneurs are*

$$\psi_L^0 = 0, \quad \textit{and} \quad \psi_H^0 \in \left[\psi^0, \frac{(s_H - s_L) K}{s_L c_H}\right],$$

with

$$\psi^0 \equiv \frac{(s_H - s_L) K}{s_H c_L},$$

and the investor's posterior belief and offered share are

$$\mu(s_L|\psi) = \begin{cases} 1 & \textit{if } \psi < \psi_H^0, \\ 0 & \textit{if } \psi \geq \psi_H^0, \end{cases} \quad \textit{and} \quad \tilde{\beta}(\psi) = \begin{cases} \beta_L^0 & \textit{if } \psi < \psi_H^0, \\ \beta_H^0 & \textit{if } \psi \geq \psi_H^0, \end{cases} \tag{3.5}$$

with

$$\beta_H^0 = \frac{K}{s_H A} \quad \textit{and} \quad \beta_L^0 = \frac{K}{s_L A}.$$

3.1.2 Pooling Equilibrium

In pooling equilibrium, the investor cannot distinguish the entrepreneur's both types who acquire patent level ψ_P^0. Thus keeping her prior belief, she offers her share β_P^* to the both types. With zero expected profit, she solves

$$V(\psi_P^0, \beta_P^0) = [\mu(s_H)s_H + \mu(s_L)s_L]\beta_P^0 A - K = 0,$$

which implies

$$\beta_P^0 = \frac{K}{[(1-s^*)s_H + s^* s_L]A}. \tag{3.6}$$

Note that $\beta_P^0 \in (0,1)$ by (2.1). Then the entrepreneur's share is

$$\theta_P^0 \equiv 1 - \beta_P^0 = 1 - \frac{K}{[(1-s^*)s_H + s^* s_L]A}. \tag{3.7}$$

Clearly, θ_P^0 belong to $(0,1)$ by (2.1). Note that $\theta_P^0 \neq (1-s^*)\,\theta_H^0 + s^*\theta_L^0$.[9]

The investor picks patent level ψ_P^0 satisfying the following incentive compatibility constraints to pool the entrepreneur's types:

$$\begin{aligned} U(s_H, \psi, \beta_P^0) &= \theta_P^0 s_H A - c_H \psi \geq \theta_L^0 s_H A = U(s_H, 0, \beta_L^0), \\ U(s_L, \psi, \beta_P^0) &= \theta_P^0 s_L A - c_L \psi \geq \theta_L^0 s_L A = U(s_L, 0, \beta_L^0). \end{aligned} \tag{3.8}$$

Then it is standard to obtain the following result (hence its proof is omitted).

Proposition 3.2. *There are pooling perfect Bayesian equilibria, in one of which the entrepreneur's patent level is*

$$\psi_P^0 \in \left[0, \frac{(1-s^*)^2(s_H - s_L)K}{[(1-s^*)s_H + s^* s_L]c_L}\right],$$

and the investor's posterior belief and offered share are

$$\mu(s_L|\psi) = \begin{cases} 1 & \text{if } \psi \neq \psi_P^0, \\ \mu(s_L) & \text{if } \psi = \psi_P^0, \end{cases} \quad \text{and} \quad \tilde{\beta}(\psi) = \begin{cases} \beta_L^0 & \text{if } \psi \neq \psi_P^0, \\ \beta_P^0 & \text{if } \psi = \psi_P^0, \end{cases} \tag{3.9}$$

with

$$\beta_P^0 = \frac{K}{[(1-s^*)s_H + s^* s_L]A} \quad \text{and} \quad \beta_L^0 = \frac{K}{s_L A}.$$

3.2 Model I: Only Investor Faces Ambiguity

In this model, only the investor faces ambiguity about project value. Being ambiguity-neutral, the investor believes that the high (low) type's success probability is $\nu(I_H) = \frac{1+s^*}{2}$ $\left(\nu(I_L) = \frac{s^*}{2}\right.$, respectively$\left.\right)$.

[9] This is in contrast to that of a standard job-market signaling game.

3.2.1 Separating Equilibrium

In separating equilibrium, if the investor observes the entrepreneur's patent level ψ_H^* (ψ_L^0), then she believes that the entrepreneur's type belongs to I_H (I_L) and offers her share β_H^* (β_L^*, respectively). With zero expected profit, she solves

$$\begin{aligned} V(\psi_H^*, \beta_H^*) &= \beta_H^* \, \nu(I_H)A - K = 0, \\ V(\psi_L^*, \beta_L^*) &= \beta_L^* \, \nu(I_L)A - K = 0, \end{aligned} \tag{3.10}$$

which imply

$$\beta_H^* = \frac{K}{\nu(I_H)A} \quad \text{and} \quad \beta_L^* = \frac{K}{\nu(I_L)A} \tag{3.11}$$

where $0 < \beta_H^* < \beta_L^* < 1$. The entrepreneur's shares for type s_H and type s_L are

$$\begin{aligned} \theta_H^* &\equiv 1 - \beta_H^* = 1 - \frac{K}{\nu(I_H)A}, \\ \theta_L^* &\equiv 1 - \beta_L^* = 1 - \frac{K}{\nu(I_L)A}, \end{aligned} \tag{3.12}$$

where $0 < \theta_L^* < \theta_H^* < 1$.

The investor takes patent level ψ_H^* satisfying the following incentive compatibility constraints as a cutoff patent level for the high type:

$$\begin{aligned} U(s, \psi, \beta_H^*) = \theta_H^* sA - c_H\psi &\geq \theta_L^* sA = U(s, 0, \beta_L^*), && \forall\, s \in I_H, \\ U(s, \psi, \beta_L^*) = \theta_L^* sA &\geq \theta_H^* sA - c_L\psi = U(s, 0, \beta_H^*), && \forall\, s \in I_L, \end{aligned} \tag{3.13}$$

which is equivalent to

$$\begin{aligned} U(s^*, \psi, \beta_H^*) = \theta_H^* s^* A - c_H\psi &> \theta_L^* s^* A = U(s^*, 0, \beta_L^*), \\ U(s^*, \psi, \beta_L^*) = \theta_L^* s^* A &\geq \theta_H^* s^* A - c_L\psi = U(s^*, 0, \beta_H^*). \end{aligned}$$

Then we obtain the following result.

Proposition 3.3. *There are separating perfect Bayesian equilibria, in one of which the patent levels of type s_L and type s_H entrepreneurs are*[10]

$$\psi_L^* = 0, \quad \textit{and} \quad \psi_H^* \in \left[\psi^*, \frac{[\nu(I_H) - \nu(I_L)]s^* K}{\nu(I_L)\nu(I_H)c_H}\right) = \left[\psi^*, \frac{K}{\nu(I_H)c_H}\right),$$

where

$$\psi^* \equiv \frac{[\nu(I_H) - \nu(I_L)]s^* K}{\nu(I_L)\nu(I_H)c_L} = \frac{K}{\nu(I_H)c_L},$$

and the investor's posterior belief and offered share are

$$\mu\left(I_L|\psi\right) = \begin{cases} 1 & \textit{if } \ \psi < \psi_H^*, \\ 0 & \textit{if } \ \psi \geq \psi_H^*, \end{cases} \qquad \textit{and} \quad \tilde{\beta}(\psi) = \begin{cases} \beta_L^* & \textit{if } \ \psi < \psi_H^*, \\ \beta_H^* & \textit{if } \ \psi \geq \psi_H^*, \end{cases} \tag{3.14}$$

[10] Recall that $\nu(I_H) = (1 + s^*)/2$ and $\nu(I_L) = s^*/2$.

with
$$\beta_H^* = \frac{K}{\nu(I_H)A} \quad \textit{and} \quad \beta_L^* = \frac{K}{\nu(I_L)A}.$$

In Figure 5, we illustrate the separating equilibrium of Proposition 3.3. The left shaded area indicates all possible indifference curves for $s \in I_L$, who acquires no patent and receives equity share θ_L^*. Similarly, the right one depicts all possible indifference curves for $s \in I_H$, who acquires no patent and receives equity share θ_L^*. The investor picks a cutoff patent level ψ_H^* in the red line for the high type, which contains patent levels satisfying incentive compatibility constraints (3.13). That is, she treats the entrepreneur who acquires patents at least ψ_H^* as the high type and offers her share β_H^* (i.e., the entrepreneur share $\theta_H^* = 1 - \beta_H^*$) and who acquires them less than ψ_H^* as the low type and offers her share β_L^* (i.e., the entrepreneur share $\theta_L^* = 1 - \beta_L^*$). This strategy is described by the blue lines. Therefore, the high type's equilibrium patent level is ψ_H^* while the low type's is zero.

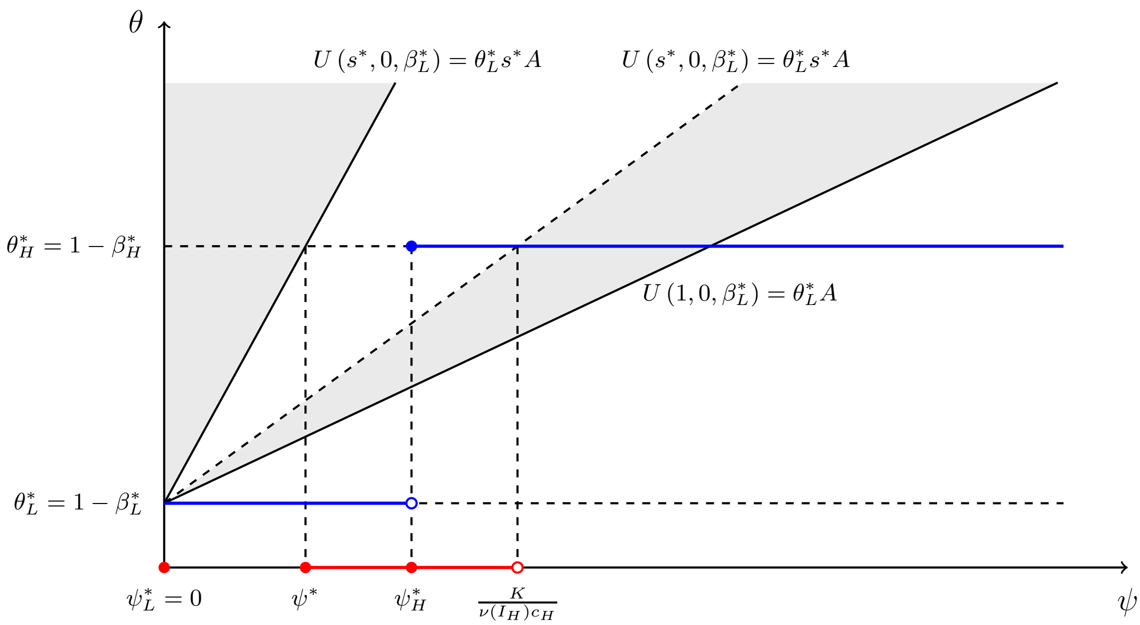

Figure 5: Patents level of the separating equilibria in Model I

3.2.2 Pooling Equilibrium

In pooling equilibrium, the investor cannot distinguish the entrepreneur's both types who acquire patent level ψ_P^*. Thus keeping her prior belief, she offers her share β_P^* to the both types. With zero expected profit, she solves

$$V(\psi_P^*, \beta_P^*) = [\mu(I_H)\nu(I_H) + \mu(I_L)\nu(I_L)]\, \beta_P^* A - K = 0,$$

which yields

$$\beta_P^* = \frac{2K}{A} \in (0,1). \tag{3.15}$$

Then the entrepreneur's equity share is

$$\theta_P^* \equiv 1 - \beta_P^* = 1 - \frac{2K}{A} \in (0,1). \tag{3.16}$$

Note that $\theta_P^* \neq (1-s^*)\,\theta_H^* + s^*\theta_L^*$.

The investor picks patent level ψ_P^* satisfying the following incentive compatibility constraints to pool the entrepreneur's types:

$$\begin{aligned} U(s,\psi,\beta_P^*) &= \theta_P^* sA - c_H\psi \geq \theta_L^* sA = U(s,0,\beta_L^*), \;\; \forall\, s \in I_H, \\ U(s,\psi,\beta_P^*) &= \theta_P^* sA - c_L\psi \geq \theta_L^* sA = U(s,0,\beta_L^*), \;\; \forall\, s \in I_L, \end{aligned} \tag{3.17}$$

which is equivalent to

$$\begin{aligned} U(s^*,\psi,\beta_P^*) = \theta_P^* s^* A - c_H\psi > \theta_L^* s^* A = U(s^*,0,\beta_L^*), \\ U(0,\psi,\beta_P^*) = -c_L\psi \geq 0 = U(0,0,\beta_L^*). \end{aligned}$$

Then we obtain a unique pooling equilibrium as follows.

Proposition 3.4. *There is a unique pooling perfect Bayesian equilibrium, where the entrepreneur's patent level is zero (i.e., $\psi_P^* = 0$), and the investor's posterior belief and offered share are given by*

$$\mu\,(I_L|\psi) = \begin{cases} 1 & \text{if } \psi \neq \psi_P^*, \\ \mu(I_L) & \text{if } \psi = \psi_P^*, \end{cases} \quad \text{and} \quad \tilde{\beta}(\psi) = \begin{cases} \beta_L^* & \text{if } \psi \neq \psi_P^*, \\ \beta_P^* & \text{if } \psi = \psi_P^*, \end{cases} \tag{3.18}$$

with

$$\beta_P^* = \frac{2K}{A} \quad \text{and} \quad \beta_L^* = \frac{K}{\nu(I_L)A}.$$

In Figure 6, the pooling equilibrium of Proposition 3.4 is illustrated. Both shaded areas are identical to them in Figure 5. When both types receive θ_P^*, zero patent level is a unique one which satisfies incentive compatibility constraints (3.17). The investor's strategy is described by the blue line. It follows that both types do not file patents at all in the pooling equilibrium.

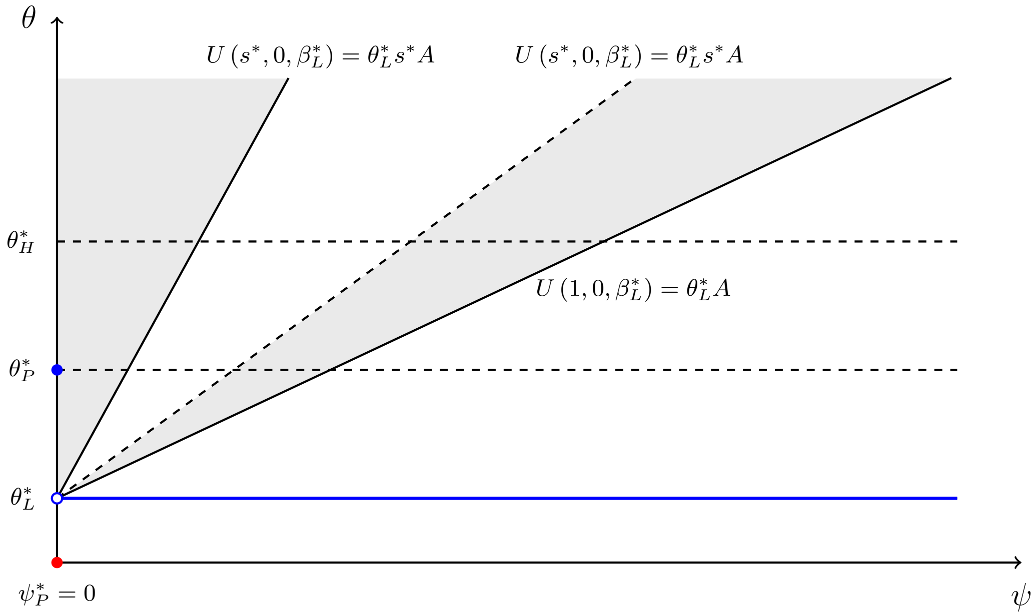

Figure 6: Patents level of the pooling equilibrium in Model I

3.3 Model II: All Agents Face Ambiguity

In this model, all agents (i.e., the entrepreneur and the investor) face ambiguity about project value. This model is similar to the benchmark model in that both the entrepreneur and the investor consider the same beliefs about the high type and the low type. Indeed, being ambiguity-neutral, all agents expect that the high (low) type's success probability is $\nu(I_H) = \frac{1+s^*}{2}$ $\left(\nu(I_L) = \frac{s^*}{2}, \text{respectively}\right)$. Since $\nu(I_H)$ $\left(\nu(I_L)\right)$ here plays the role of s_H (s_L, respectively) in the benchmark model, this model is analytically equivalent to the benchmark model.

3.3.1 Separating Equilibrium

In separating equilibrium, if the investor observes the entrepreneur's patent level ψ_H^{**} (ψ_L^{**}), then she believes that the entrepreneur's true success probability belongs to I_H (I_L) and offers her share β_H^{**} (β_L^{**}, respectively). Since she takes into account the same expected success probabilities (i.e., $\nu(I_H)$ and $\nu(I_L)$) as in Model I, the zero expected profit condition implies that

$$(\beta_H^{**}, \theta_H^{**}) = (\beta_H^*, \theta_H^*) \quad \text{and} \quad (\beta_L^{**}, \theta_L^{**}) = (\beta_L^*, \theta_L^*). \tag{3.19}$$

The investor takes patent level ψ_H^* satisfying the following incentive compatibility constraints as a cutoff patent level for the high type:

$$\begin{aligned} U(I_H, \psi, \beta_H^*) = \theta_H^* \nu(I_H)A - c_H\psi \geq \theta_L^* \nu(I_H)A = U(I_H, 0, \beta_L^*), \\ U(I_L, 0, \beta_L^*) = \theta_L^* \nu(I_L)A \geq \theta_H^* \nu(I_L)A - c_L\psi = U(I_L, \psi, \beta_H^*). \end{aligned} \tag{3.20}$$

Similar to Proposition 3.1, one can show the following result (hence its proof is omitted).

Proposition 3.5. *There are separating perfect Bayesian equilibria, in one of which the patent levels of type I_L and type I_H entrepreneurs are*

$$\psi_L^{**} = 0, \quad \textit{and} \quad \psi_H^{**} \in \left[\psi^{**}, \frac{[\nu(I_H) - \nu(I_L)]K}{\nu(I_L)c_H}\right] = \left[\psi^{**}, \frac{K}{2\nu(I_L)c_H}\right],$$

with

$$\psi^{**} \equiv \frac{[\nu(I_H) - \nu(I_L)]K}{\nu(I_H)c_L} = \frac{K}{2\nu(I_H)c_L},$$

and the investor's posterior belief and offered share are

$$\mu\left(I_L|\psi\right) = \begin{cases} 1 & \textit{if } \psi < \psi_H^{**}, \\ 0 & \textit{if } \psi \geq \psi_H^{**}, \end{cases} \quad \textit{and} \quad \tilde{\beta}(\psi) = \begin{cases} \beta_L^* & \textit{if } \psi < \psi_H^{**}, \\ \beta_H^* & \textit{if } \psi \geq \psi_H^{**}, \end{cases} \tag{3.21}$$

with

$$\beta_H^* = \frac{K}{\nu(I_H)A} \quad \textit{and} \quad \beta_L^* = \frac{K}{\nu(I_L)A}.$$

3.3.2 Pooling Equilibrium

In pooling equilibrium, the investor cannot distinguish the entrepreneur's both types who acquire patent level ψ_P^{**}. Thus keeping her prior belief, she offers her share β_P^{**} to the both types. Since she takes into account the same expected success probabilities (i.e., $\nu(I_H)$ and $nu(I_L)$) as in Model I, it is obvious that $(\beta_P^{**}, \theta_P^{**}) = (\beta_P^*, \theta_P^*)$.

The investor picks patent level ψ_P^* satisfying the following incentive compatibility constraints to pool the entrepreneur's types:

$$\begin{aligned} U(I_H, \psi, \beta_P^*) = \theta_P^* \nu(I_H)A - c_H\psi > \theta_L^* \nu(I_H)A = U(I_H, 0, \beta_L^*), \\ U(I_L, \psi, \beta_P^*) = \theta_P^* \nu(I_L)A - c_L\psi \geq \theta_L^* \nu(I_L)A = U(I_L, 0, \beta_L^*). \end{aligned}$$

Similar to Proposition 3.2, one can obtain the following result (hence its proof is omitted).

Proposition 3.6. *There are pooling perfect Bayesian equilibria, in one of which the entrepreneur's patent level is*

$$\psi_P^{**} \in \left[0, \frac{[1 - 2\nu(I_L)]K}{c_L}\right] = \left[0, \frac{(1 - s^*)\,K}{c_L}\right],$$

and the investor's posterior belief and offered share are

$$\mu(I_L|\psi) = \begin{cases} 1 & \textit{if } \psi \neq \psi_P^{**}, \\ \mu(I_L) & \textit{if } \psi = \psi_P^{**}, \end{cases} \quad \textit{and} \quad \tilde{\beta}(\psi) = \begin{cases} \beta_L^* & \textit{if } \psi \neq \psi_P^{**}, \\ \beta_P^* & \textit{if } \psi = \psi_P^{**}, \end{cases} \tag{3.22}$$

with

$$\beta_P^* = \frac{2K}{A} \quad \textit{and} \quad \beta_L^* = \frac{K}{\nu(I_L)A}.$$

4 Refinements of Perfect Bayesian Equilibria

Now we refine the perfect Bayesian equilibria in Section 3 by imposing Intuitive Criterion of Cho and Kreps (1987).Let $\mathcal{T}(\psi) \subset T_i$ be the set of types of the entrepreneur who might have chosen that patent level ψ. Note that $\mathcal{T}(\psi) = T_i$ for any patent level $\psi \in [0, \infty)$. For $T' \subset \mathcal{T}(\psi)$, let $\mathrm{BR}\,(T', \psi)$ be the set of all pure-strategy best responses for the investor to patent level ψ and for beliefs $\mu(\cdot|\psi)$ such that $\mu(T'|\psi) = 1$:

$$\mathrm{BR}\left(T', \psi\right) = \bigcup_{\mu:\mu(T'|\psi)=1} \mathrm{BR}\left(\mu, \psi\right)$$

where

$$\mathrm{BR}\left(\mu, \psi\right) = \operatorname*{argmax}_{\beta} \sum_{t \in T_i} \mu(t|\psi) \tilde{V}\left(t, \psi, \beta\right).$$

Let $U^*(t)$ be the entrepreneur's expected profit of type t in equilibrium.

Recall that the entrepreneur and the investor allow for the same type space in the benchmark model, i.e., $T_e = T_i = \{s_H, s_L\}$. We employ the following Intuitive Criterion of Cho and Kreps (1987) to refine the perfect Bayesian equilibria in the benchmark model.

Definition 4.1. ***(Intuitive Criterion 1)*** *A perfect Bayesian equilibrium fails Intuitive Criterion 1 if there exists $t \in T_i \setminus J(\psi)$ with some ψ such that*

$$U^*\left(t\right) < \min_{\beta \in BR(T_i \setminus J(\psi), \psi)} U\left(t, \psi, \beta\right), \tag{4.1}$$

where

$$J(\psi) \equiv \left\{ t \in T_i \;\middle|\; U^*(t) > \max_{\beta \in BR(T_i, \psi)} U\left(t, \psi, \beta\right) \right\}. \tag{4.2}$$

We interpret Intuitive Criterion 1 in our context. Roughly speaking, the idea is that if the investor finds a type of the entrepreneur who has an incentive to send an off-the-equilibrium signal (i.e., patent level), then the equilibrium under consideration is unreasonable and fails the criterion.

The procedure starts with eliminating the types who will not benefit at best by a deviating signal. First, if the investor finds a type in T_i who cannot beat the equilibrium expected profit by deviating to off-the-equilibrium patent level ψ even when the investor offers the most favorable equity share to him, then we let $J(\psi)$ denote the set of such types. Eliminating set $J(\psi)$ from T_i, the investor restricts types to $T_i \setminus J(\psi)$. Second, if the investor finds a type in $T_i \setminus J(\psi)$ whose expected profit at off-the-equilibrium patent level ψ is higher than the equilibrium expected profit even when the investor offers the most unfavorable equity share to him, then the original equilibrium is regarded as unstable and fails to survive Intuitive Criterion 1.

In the benchmark model, it is standard to obtain a unique perfect Bayesian equilibrium by invoking Intuitive Criterion 1 (hence the proof of Proposition 4.1 is omitted).

Proposition 4.1. *In the benchmark model, there is a unique perfect Bayesian equilibrium that survives Intuitive Criterion 1 where the patent levels of type s_L and type s_H entrepreneurs are*

$$\psi_L^0 = 0 \quad \textit{and} \quad \psi_H^0 = \psi^0 = \frac{(s_H - s_L) K}{s_H c_L},$$

and the investor's posterior belief and the offered share are

$$\mu(I_L|\psi) = \begin{cases} 1 & \textit{if } \psi < \psi^0, \\ 0 & \textit{if } \psi \geq \psi^0, \end{cases} \quad \textit{and} \quad \tilde{\beta}(\psi) = \begin{cases} \beta_L^0 & \textit{if } \psi < \psi^0, \\ \beta_H^0 & \textit{if } \psi \geq \psi^0, \end{cases}$$

with

$$\beta_H^0 = \frac{K}{s_H A} \quad \textit{and} \quad \beta_L^0 = \frac{K}{s_L A}.$$

On the other hand, in Model I, the entrepreneur learns the exact type, while the investor only knows that if he is the high type, then $s_H \in I_H$ and if he is the low type, then $s_L \in I_L$. Thus the entrepreneur and the investor allows for different type spaces, i.e., $T_e = \{s_H, s_L\} \neq \{I_H, I_L\} = T_i$. For Model I, we define a variant of Intuitive Criterion 1 in the following way, which will be employed to refine the perfect Bayesian equilibria.

Definition 4.2. ***(Intuitive Criterion 2)*** *A perfect Bayesian equilibrium fails Intuitive Criterion 2 if there exists $t \in T_i \setminus J(\psi)$ with some ψ such that for some $s \in t$,*

$$U^*(s) < \min_{\beta \in BR(T_i \setminus J(\psi), \psi)} U(s, \psi, \beta), \tag{4.3}$$

where

$$J(\psi) \equiv \left\{ t \in T_i \,\middle|\, U^*(s) > \max_{\beta \in BR(T_i, \psi)} U(s, \psi, \beta), \ \forall s \in t \right\}. \tag{4.4}$$

The interpretation and idea of Intuitive Criterion 2 are similar to those of Intuitive Criterion 1. Roughly speaking, the idea is that if the investor finds a type (i.e., I_H or I_H) in T_i such that, for some success probability in the type, the entrepreneur has an incentive to send an off-the-equilibrium signal (i.e., patent level), then the equilibrium under consideration is unreasonable and fails the criterion.

The procedure starts with eliminating the types who will not benefit at best by a deviating signal. First, if the investor finds a type (i.e., I_H or I_H) in T_i such that, *for all success probabilities in the type*, the entrepreneur cannot beat the equilibrium expected payoff by acquiring off-the-equilibrium patent level ψ even when the investor offers the most favorable equity share to him, then we let $J(\psi)$ denote the set of such types. Eliminating set $J(\psi)$ from T_i, the investor restricts types to $T_i \setminus J(\psi)$. Second, if the investor finds a type in $T_i \setminus J(\psi)$ such that, *for some success probability in the type*, the entrepreneur's expected profit at off-the-equilibrium patent level ψ is higher than the equilibrium expected profit even when the investor offers the most unfavorable equity share to him, the original equilibrium is vulnerable to deviating strategy ψ and fails to survive Intuitive Criterion 2.

It is worth noting that Intuitive Criterion 2 is a generalized form of Intuitive Criterion 1 since Intuitive Criterion 2 treats a type as an interval while Intuitive Criterion 1 considers a type as a single success probability. Indeed, when T_i is the set of singletons (i.e., $I_H = \{s_H\}$ and $I_L = \{s_L\}$), Intuitive Criterion 2 reduces to Intuitive Criterion 1.

Invoking Intuitive Criterion 2 in Model 1, we obtain a unique perfect Bayesian equilibrium as follows.

Proposition 4.2. *In Model I, there is a unique perfect Bayesian equilibrium that survives Intuitive Criterion 2 where the patent levels of type s_L and type s_H entrepreneurs are*

$$\psi_L^* = 0 \quad \textit{and} \quad \psi_H^* = \psi^* = \frac{K}{\nu(I_H)c_L},$$

and the investor's posterior belief and the offered share are

$$\mu\left(I_L|\psi\right) = \begin{cases} 1 & \textit{if } \ \psi < \psi^*, \\ 0 & \textit{if } \ \psi \geq \psi^*, \end{cases} \qquad \textit{and} \quad \tilde{\beta}(\psi) = \begin{cases} \beta_L^* & \textit{if } \ \psi < \psi^*, \\ \beta_H^* & \textit{if } \ \psi \geq \psi^*, \end{cases}$$

with

$$\beta_H^* = \frac{K}{\nu(I_H)A} \quad \textit{and} \quad \beta_L^* = \frac{K}{\nu(I_L)A}.$$

In Model II, the entrepreneur does not know his own true success probability but only knows whether it belongs to I_H or I_L. Thus the entrepreneur behaves as if his type is $\nu(I_H)$ or $\mu(I_L)$. Since the investor faces the same ambiguity as the entrepreneur, $\nu(I_H)$ $\big(\nu(I_L)\big)$ in Model II plays the role of s_H (s_L, respectively) in the benchmark model. By invoking Intuitive Criterion 1, analogous arguments to the benchmark model leads to a unique perfect Bayesian equilibrium of Model II as follows (hence its proof is omitted).

Proposition 4.3. *In Model II, there is a unique perfect Bayesian equilibrium that survives Intuitive Criterion 1 where the patent levels of type I_L and type I_H entrepreneurs are*

$$\psi_L^{**} = 0 \quad \textit{and} \quad \psi_H^{**} = \psi^{**} = \frac{K}{2\nu(I_H)c_L},$$

and the investor's posterior belief and the offered share are

$$\mu\left(I_L|\psi\right)=\begin{cases}1 & \text{if } \psi<\psi_H^{**},\\ 0 & \text{if } \psi\geq\psi_H^{**},\end{cases}\quad \textit{and}\quad \tilde{\beta}(\psi)=\begin{cases}\beta_L^* & \text{if } \psi<\psi_H^{**},\\ \beta_H^* & \text{if } \psi\geq\psi_H^{**},\end{cases}$$

with

$$\beta_H^*=\frac{K}{\nu(I_H)A}\quad \textit{and}\quad \beta_L^*=\frac{K}{\nu(I_L)A}.$$

5 Comparative Statics

In this section, we characterize the perfect Bayesian equilibria, which survive Intuitive Criterion given in Propositions 4.1–4.3. In each model, the investor evaluates the project's gross return based on her information about the project's true success probability. In the benchmark model, the investor knows that the high (low) type entrepreneur has true success probability s_H (s_L, respectively). Facing ambiguity in Models I and II, on the other hand, the investor believes that the high (low) type has success probability $\nu(I_H)$ ($\nu(I_L)$, respectively). Thus the high (low) type's true success probability is *overvalued* by the investor if $\nu(I_H)>s_H$ ($\nu(I_L)>s_L$, respectively). We simply say that the entrepreneur's true success probability is *overvalued* if both $\nu(t)>t$ for all $t=I_H,I_L$.

The investor believes that the project's expected gross return in the benchmark model is

$$\lambda^0(t)\equiv\frac{\mathbb{E}_t[R]}{K}=\frac{tA}{K},\ \forall\, t\in T_i=\{s_H,s_L\},$$

and in both Model I and Model II is

$$\lambda(t)\equiv\frac{\mathbb{E}_\nu[\mathbb{E}_s[R]]}{K}=\frac{\nu(t)A}{K},\ \forall\, t\in T_i=\{I_H,I_L\}.$$

In the investor's standpoint, it is clear that the project has a higher expected gross return under ambiguity (i.e., in Model I and Model II) than without it (i.e., in the benchmark model) if and only if the true success probability is overvalued. Then we derive the following relationships among the entrepreneur's equity share, the project's expected gross return believed by the investor, and overvaluation. Recall that the low-type entrepreneur acquires no patent in each model. Therefore only the high-type entrepreneur is involved in Propositions 5.1–5.2, which characterize the entrepreneur's patent level.

Proposition 5.1. *The following hold.*

(1) *In the absence of ambiguity (i.e., in benchmark model), the high-type entrepreneur's patent level increases in his success probability s_H.*

(2) *In the absence of ambiguity (i.e., in benchmark model), the high-type entrepreneur's patent level decreases in the low type's success probability s_L.*

(3) *In the presence of ambiguity (i.e., in Model I and Model II), the high-type entrepreneur's patent level decreases in the project's expected gross return λ evaluated by the investor where A is fixed.*

(4) *In each model, the high-type entrepreneur's patent level decreases in the low type's marginal patent cost c_L.*

In the absence of ambiguity (i.e., in benchmark model), the refined equilibrium patent level can be rewritten as

$$\psi^0 = \frac{(\theta_H^0 - \theta_L^0)s_L A}{c_L}$$

where

$$\theta_H^0 = 1 - \frac{K}{s_H A} \quad \text{and} \quad \theta_L^0 = 1 - \frac{K}{s_L A}.$$

Note that a change in the high type's true success probability s_H only affects his equity share θ_H^0 in (3.3). As s_H is higher, the investor offers a lower her equity share and thus the high type entrepreneur takes a higher equity share. Then the high type's equilibrium patent level is higher than before.

On the other hand, a change in the low type's true success probability s_L directly affects ψ_0 as well as indirectly affects it via his equity share θ_L^0. Since an increase of s_L leads to a decrease of the investor's equity share β_L^0, the entrepreneur's equity share θ_L^0 increases. Therefore, an increase of s_L directly increases ψ^0 while indirectly decreases it via θ_L^0. However, since the former effect is dominated by the the latter one, ψ^0 decreases.

In the presence of ambiguity (i.e., in Model I and Model II), since the investor cannot observe each type's true success probability, the equity shares of the entrepreneur and the investor are not affected by the true success probabilities but affected by market evaluation s^* about project value. In Model I and Model II, the refined equilibrium patent levels can be rewritten as, respectively,

$$\psi^* = \frac{(\theta_H^* - \theta_L^*)s^* A}{c_L} \quad \text{and} \quad \psi^{**} = \frac{(\theta_H^* - \theta_L^*)s^* A}{2c_L}$$

where

$$\theta_H^* = 1 - \frac{K}{\nu(I_H)A} \quad \text{and} \quad \theta_L^* = 1 - \frac{K}{\nu(I_L)A}.$$

Note that as market evaluation s^* about project value is higher, the investor asks a lower her equity share for each type, and thus both types of the entrepreneur take a higher equity shares. Market evaluation s^* directly affects refined patent levels ψ^* and ψ^{**} as well as indirectly affects them via the high type and low type's equity shares θ_H^* and θ_L^*. An increase of s^* indirectly decreases ψ^* and ψ^{**} since an increase of the high type's equity share θ_H^* is dominated by that of the low type's equity share θ_L^*. It is clear that an increase of s^* directly increases ψ^* and ψ^{**}. However, since the indirect effect dominates the direct one, ψ^* and ψ^{**} decrease in s^*. On the other hand, an increase in investment K required to launch the project increases the equilibrium patent levels. Consequently, the high type entrepreneur should acquire a higher patent level in order to signal his ability as the investor facing ambiguity considers a lower expected gross return λ of the project.

Note that the equilibrium patent level of the high type in each model does not depend on his marginal patent cost c_H but depend on the low type's marginal patent cost c_L. As marginal cost c_L of the low-type entrepreneur increases, the high type can signal his type to the investor with a lower cost in filing patents.

Proposition 5.2. *The following hold.*

(1) The high-type entrepreneur acquires the highest patent level in Model I among all the models.

(2) The high-type entrepreneur acquires a higher level of patent in the benchmark model than in Model II if and only if

$$\nu(I_H) > \frac{s_H}{2(s_H - s_L)}. \tag{5.1}$$

In Model I, facing ambiguity with knowing that the entrepreneur resolves it , the investor allows for a more conservative cutoff patent level compared with when she also resolves it (benchmark model) or when both agents are under ambiguity (Model II). This implies that the high-type entrepreneur in Model I should acquire the highest patent level in order to signal his ability, i.e., $\psi^* > \psi^0$ and $\psi^* > \psi^{**}$.

According to the second claim of Proposition 5.2, even if ambiguity is present on the side of the investment market, the high-type entrepreneur does not always acquire more patent than in the benchmark case, when he is also under ambiguity. In this case, due to the symmetric ambiguity between agents, the high type's patent level depends on market evaluation $\nu(I_H)$ of the high type's true success probability. In particular, from Proposition 4.3, we know that the high type's patent level decreases in $\nu(I_H)$. Furthermore, if market evaluation $\nu(I_H)$ is sufficiently high such that (5.1) holds, the high type spend less money in filing patents compared with the benchmark model.

Proposition 5.3. *The following hold.*

(1) In each model, the entrepreneur's equity share increases in the project's expected gross return (λ^0 or λ) evaluated by the investor.

(2) The entrepreneur obtains a more equity share in Model I and Model II than in the benchmark model if and only if his true success probability is overvalued.

Recall that the investor's expected profit is zero in each model since the investment market is under Bertrand competition. As a consequence, the investor's equity share increases in investment amount K and decreases in project's expected value. From (3.2), (3.11), and (3.19), we know that the investor's equity share decreases in the expected gross return in each model. Therefore, as the project's expected gross return is more highly evaluated by the investor, the entrepreneur takes more equity share.

One may believe that the investor asks more equity share when she faces ambiguity than otherwise. However, the second claim of Proposition 5.3 shows that the equity share asked by the investor does not only depend on the presence of ambiguity but is determined by her belief about the success probability. In fact, the investor asks a lower equity share under ambiguity than otherwise if she believes that the entrepreneur's project yields a sufficiently high expected gross return.

Proposition 5.4. *The low-type entrepreneur's expected profit is the lowest in the benchmark model and is the highest in Model II if and only if the low type's true success probability is overvalued, i.e.,* $\nu(I_L) > s_L$.

The low-type entrepreneur does not spend money on filing patents in all the refined perfect Bayesian equilibria in Propositions 4.1–4.3. Thus his expected profit only depends on his equity share and the project's expected value. Suppose that the low type's true success probability is overvalued, i.e., $\nu(I_L) > s_L$ in Model I and Model II. In the benchmark model and Model I, the low-type entrepreneur knows his true success probability s_L. The low type takes a higher equity share in Model I than in the benchmark model by (2) of Proposition 5.3. Thus, the low type's expected profit in Model I is higher than in the benchmark model.

Now we compare the low-type entrepreneur's expected profits in Model I and Model II. In both models, the investor who faces ambiguity, asks equity share β_L^* in (3.11) and thus the low-type entrepreneur takes θ_L^* in (3.12). On the other hand, the low-type entrepreneur considers a higher success probability under ambiguity than otherwise. It follows that the low type expects a higher profit in Model II than that in Model I.

Proposition 5.5. *The following hold.*

(1) The high-type entrepreneur's expected profit is higher in Model I than in the benchmark model if and only if

$$\nu(I_H) > \frac{(c_H + s_H c_L)s_H}{(s_H - s_L)\, c_H + c_L s_H}. \tag{5.2}$$

(2) The high-type entrepreneur's expected profit is higher in Model II than in the benchmark model if

$$\nu(I_H) > \max\left\{ s_H, \frac{s_H}{2(s_H - s_L)} \right\}. \tag{5.3}$$

(3) The high-type entrepreneur's expected profit is higher in Model I than in Model II if and only if

$$s_H > \nu(I_H) + \frac{c_H K}{2c_L\left[\nu(I_H)A - K\right]}. \tag{5.4}$$

Unlike the low type, the high-type entrepreneur acquires a positive level of patent to signal his ability in the separating equilibrium. Thus the high type's expected profit depends on patent cost as well as his equity share and the project's expected value. As shown in (1) of Proposition 5.2, he spends more money in filing patents in Model I than in the benchmark model. On the other hand, in the benchmark model and Model I, the high-type entrepreneur learns his true success probability s_H. Thus the difference between the high type's revenues in these two models only depends on his equity share. If market evaluation $\nu(I_H)$ of the high type's true success probability is sufficiently high such that (5.2) holds, the high type takes higher equity share in Model I than in the benchmark model enough to offset his increased patent cost, which implies (1) of Proposition 5.5.

Now suppose that market evaluation $\nu(I_H)$ is sufficiently high such that (5.3) holds. The high type pays more patent costs in Model II than in the benchmark model by (2) of Proposition 5.2. Furthermore, his expected revenue is higher in Model II than in the benchmark since (2) of Proposition 5.3 implies $\theta_H^*\nu(I_H)A > \theta_H^0 s_H A$. Thus we obtain the result (2) of Proposition 5.5.

Now we compare the high type's expected profits in Model I and Model II. From (1) of Proposition 5.2, we know that if the investor faces ambiguity, the high type always spends more money in filing patents when he resolves ambiguity than otherwise. On the other hand, his revenue may be higher or lower in Model I than in Model II depending on his true success probability s_H and market evaluation $\nu(I_H)$.[11] Suppose that the high type's success probability is sufficiently undervalued such that (5.4) holds. Then, it is clear that the his revenue exceeds in Model I than in Model II. Moreover, since the increase in patent cost is exceeded by that in the revenue, the high type without ambiguity makes a higher expected profit. Note that the right-hand side in (5.4) consists of market evaluation $\nu(I_H)$ and an additional term. Unlike the low type's expected profit, if the investor faces ambiguity, the high type's expected profit is higher without ambiguity than otherwise only when his true success probability sufficiently exceeds the expected success probability because he need to pay more patent costs to signal his type in Model I than in Model II.

6 Concluding Remarks

Agents in early-stage investment usually face ambiguity if the entrepreneur's project involves highly innovative technology, which is not well known to agents. To examine the effects of ambiguity on startup financing, we provide three different models in view of the degrees of ambiguity: (1) no ambiguity, (2) only the investor faces ambiguity, (3) all agents face ambiguity. In each model, we derive the perfect Bayesian equilibria of the signaling game and refine them into a unique equilibrium by imposing Intuitive Criterion of Cho and Kreps (1987) or its extension. We analyze the refined equilibria in perspectives of agents' equity shares and the entrepreneur's patent level and expected profit. In particular, the entrepreneur should spend the most money in filing patents to inform his ability to the investor when he solely resolves ambiguity and the investor faces it. It is because the investor allows for a more conservative cutoff patent level when she faces ambiguity but

[11]Recall that the entrepreneur takes the same equity share in Model I and Model II.

the entrepreneur does not than when both agents face ambiguity or when she does not. We also find that the investor asks a lower equity share under ambiguity than otherwise if the entrepreneur's project is expected to yield a sufficient high gross return. The entrepreneur can make a higher expected profit when the investor faces ambiguity than otherwise if market evaluation is sufficiently high.

Future research can proceed in three possible directions. First, one can investigate the case where patents are productive and add value to the project as in Conti, Thursby, and Rothaermel (2013) and Conti, Thursby, and Thursby (2013). Second, one can employ ambiguity-averse preferences. For instance, one may use a concave distortion function instead of a linear one in Klibanoff, Marinacci, and Mukerji (2005) or the maxmin expected utility of Gilboa and Schmeidler (1989). Third, it is interesting to consider asymmetric ambiguity between an entrepreneur and investors. In Model II, ambiguity faced by the investor is equivalent to that faced by the entrepreneur. One can make the entrepreneur's ambiguous information more precise than investors.

Appendix

PROOF OF PROPOSITION 3.3: If the belief of the investor is μ in (3.14), the investor's optimal offer is $\tilde{\beta}$ in (3.14). Under belief μ in (3.14) of the investor, the low type chooses patent level of zero and the high type chooses patent level ψ_H^*. For the entrepreneur's types to be separated, incentive compatibility constraints (3.13) should hold. Then we have

$$\psi_H^* \in \left[\frac{(\theta_H^* - \theta_L^*)\, s^* A}{c_L}, \frac{(\theta_H^* - \theta_L^*)\, s^* A}{c_H}\right] = \left[\psi^*, \frac{[\nu(I_H) - \nu(I_L)] s^* K}{\nu(I_L)\nu(I_H) c_H}\right). \tag{A.1}$$

Substituting θ_L^* and θ_H^* of (3.12), (A.1) can be rewritten as

$$\psi_H^* \in \left[\frac{K}{(1+s^*)\, c_L}, \frac{K}{s^* c_H}\right]. \qquad \blacksquare$$

PROOF OF PROPOSITION 3.4: If the belief of the investor is μ in (3.18), the investor's optimal offer is $\tilde{\beta}$ in (3.18). For the entrepreneur's types to be pooled, incentive compatibility constraints (3.17) should hold. Then we have

$$\psi_P^* \in \left[0, \frac{(\theta_P^* - \theta_L^*)\, s^* A}{c_L}\right]$$

for every $s_L \in I_L$, which implies $\psi_P^* = 0$. $\blacksquare$

PROOF OF PROPOSITION 4.2 We refine perfect Bayesian equilibria in Model I by imposing Intuitive Criterion 2. Recall that the entrepreneur and the investor have different type spaces such that $T_e = \{s_H, s_L\}$ and $T_i = \{I_H, I_L\}$ in Model I.

(1) Separating Equilibria

Consider the separating equilibria in Proposition 3.3.

(Case 1) Separating equilibrium with $\psi_H^* \neq \psi^*$

Consider off-the-equilibrium patent level $\psi = \psi^*$. The investor believes that the low type has no incentive to deviate from equilibrium patent level $\psi_L^* = 0$ to ψ since

$$U^*(s) = \theta_L^* sA > \theta_H^* sA - c_L\psi = \max_{\beta \in \text{BR}(T_i,\psi)} U(s,\psi,\beta)\,, \;\; \forall\, s \in I_L.$$

On the other hand, she considers that the high type can increase his expected profit by sending off-the-equilibrium message ψ since

$$U^*(s) = \theta_H^* sA - c_H\psi_H^* < \theta_H^* sA - c_H\psi = \max_{\beta \in \text{BR}(T_i,\psi)} U(s,\psi,\beta)\,, \;\; \forall\, s \in I_H.$$

Thus we have $J(\psi,\gamma) = \{I_L\}$ and $T_i \setminus J(\psi,\gamma) = \{I_H\}$.

Now we check whether inequality (4.3) holds for the high type. Since we have

$$U^*(s) = \theta_H^* sA - c_H\psi_H^* < \theta_H^* sA - c_H\psi = \min_{\beta \in \text{BR}(\{I_H\},\psi)} U(s,\psi,\beta)\,, \;\; \forall\, s \in I_H,$$

the original equilibrium with high type's patent level ψ_H^* fails Intuitive Criterion 2.

(Case 2) Separating equilibria with $\psi_H^* = \psi^*$

First, consider off-the-equilibrium patent level $\psi \in (\psi^*, \infty)$. Since

$$\begin{aligned} U^*(s) &= \theta_H^* sA - c_H\psi^* > \theta_H^* sA - c_H\psi = \max_{\beta \in \text{BR}(T_i,\psi)} U(s,\psi,\beta)\,, \;\; \forall\, s \in I_H, \\ U^*(s) &= \theta_H^* sA > \theta_H^* sA - c_L\psi = \max_{\beta \in \text{BR}(T_i,\psi)} U(s,\psi,\beta)\,, \;\; \forall\, s \in I_L, \end{aligned}$$

each type has no incentive to deviate from equilibrium patent level to ψ. Thus $J(\psi,\gamma) = T_i$ and $T_i \setminus J(\psi,\gamma) = \emptyset$. Therefore the original equilibrium with high type's patent level ψ^* survives Intuitive Criterion 2.

Second, consider off-the-equilibrium patent level $\psi \in [0, \psi^*)$. The investor considers that each type can increase expected profit since

$$\begin{aligned} U^*(s) &= \theta_H^* sA - c_H\psi^* < \theta_H^* sA - c_H\psi = \max_{\beta \in \text{BR}(T_i,\psi)} U(s,\psi,\beta)\,, \;\; \forall\, s \in I_H, \\ U^*(s) &= \theta_L^* sA < \theta_H^* sA - c_L\psi = \max_{\beta \in \text{BR}(T_i,\psi)} U(s,\psi,\beta)\,, \;\; \forall\, s \in I_L. \end{aligned}$$

Thus we have $J(\psi,\gamma) = \emptyset$ and $T_i \setminus J(\psi,\gamma) = T_i$.

Now we check whether inequality (4.3) holds for the both types. Each type obtains the minimum expected profit when the investor believes that he is the low type. Since we have

$$\begin{aligned} U^*(s) &= \theta_L^* sA > \theta_L^* sA - c_L\psi = \min_{\beta \in \text{BR}(T_i,\psi)} U(s,\psi,\beta)\,, \;\; \forall\, s \in I_L, \\ U^*(s) &= \theta_H^* sA - c_H\psi^* > \theta_L^* sA - c_H\psi = \min_{\beta \in \text{BR}(T_i,\psi)} U(s,\psi,\beta)\,, \;\; \forall\, s \in I_H, \end{aligned}$$

the original equilibrium with high type's patent level ψ^* survives Intuitive Criterion 2.

(2) Pooling Equilibrium

Consider the pooling equilibrium in Proposition 3.4, in which each type does not acquire patents, i.e., $\psi_P^* = 0$. Let

$$\psi_1 \equiv \frac{2s^* sK}{(1+s^*)\, c_L} \text{ for } s \in I_L \quad \text{and} \quad \psi_2 \equiv \frac{2s^* sK}{(1+s^*)\, c_H} \text{ for } s \in I_H$$

where ψ_1 and ψ_2 satisfy

$$\theta_H^* sA - c_L \psi_1 = \theta_P^* sA \text{ for } s \in I_L \quad \text{and} \quad \theta_H^* sA - c_H \psi_2 = \theta_P^* sA \text{ for } s \in I_H,$$

respectively. We take off-the-equilibrium message $\psi = (\psi_1 + \psi_2)/2$. Since

$$\begin{aligned} U^*(s) &= \theta_P^* sA > \theta_H^* sA - c_L \psi = \max_{\beta \in \mathrm{BR}(T_i, \psi)} U(s, \psi, \beta), \ \forall s \in I_L, \\ U^*(s) &= \theta_P^* sA < \theta_H^* sA - c_H \psi = \max_{\beta \in \mathrm{BR}(T_i, \psi)} U(s, \psi, \beta), \ \forall s \in I_H, \end{aligned}$$

the investor believes that only the high type can be better off by deviating from ψ_P^* to ψ. Thus we have $J(\psi, \gamma) = \{I_L\}$ and $T_i \setminus J(\psi, \gamma) = \{I_H\}$. Now we check whether (4.3) holds for the high type. Since we have

$$U^*(s) = \theta_P^* sA - c_H \psi_P^* < \theta_H^* sA - c_H \psi = \min_{\beta \in \mathrm{BR}(\{I_H\}, \psi)} U(s, \psi, \beta), \ \forall s \in I_H,$$

the original patent level with each type's patent level ψ_P^* fails Intuitive Criterion 2. ■

Proof of Proposition 5.1 (1) From Proposition **??**, since we have

$$\frac{\partial \psi^0}{\partial s_H} = \frac{s_L K}{s_H^2 c_L} > 0,$$

equilibrium patent level ψ^0 increases in s_H. □

(2) From Propositions 4.2 and 4.3, we have

$$\psi^* = \frac{2K}{(1+s^*)c_L} = \frac{A}{c_L \lambda(I_H)} \quad \text{and} \quad \psi^{**} = \frac{K}{(1+s^*)c_L} = \frac{A}{2c_L \lambda(I_H)},$$

both of which decrease in λ for fixed A. □

(3) From Propositions 4.1–4.3, it is clear that ψ^*, ψ^*, and ψ^{**} decrease in c_L. ■

Proof of Proposition 5.2 (1) By Propositions 4.1-4.2, we have

$$\psi^* - \psi^0 = \frac{[(1-s^*)\, s_H + (1+s^*)\, s_L]\, K}{(1+s^*)\, c_L s_H} > 0.$$

Furthermore, $\psi^* > \psi^{**}$ by Propositions 4.2-4.3. Hence the claim holds. □

(2) From Proposition 4.1 and Proposition 4.3, it follows that

$$\psi^0 - \psi^{**} = \frac{[s^* s_H - (1+s^*)\, s_L]\, K}{(1+s^*)\, c_L s_H} > 0$$

if and only if (5.1) holds. ■

PROOF OF PROPOSITION 5.3 (1) In the benchmark model, the entrepreneur's equity share is (3.3), which increases in $\lambda^0(t)$. In Model I and Model II, his equity share is (3.12), which increases in $\lambda(t)$. □

(2) From (3.3) and (3.12), we have

$$\theta_H^* - \theta_H^0 = \frac{(1+s^* - 2s_H)K}{(1+s^*)s_H A} > 0$$

if and only if $\nu(I_H) > s_H$ and

$$\theta_L^* - \theta_L^0 = \frac{(s^* - 2s_L)K}{s^* s_L A} > 0$$

if and only if $\nu(I_L) > s_L$. ■

PROOF OF PROPOSITION 5.4 The difference between the equilibrium expected utilities of the low type in Model I and in the benchmark model is

$$U^*\left(s_L, 0, \beta_L^*\right) - U^*\left(s_L, 0, \beta_L^0\right) = \frac{(s^* - 2s_L)\, K}{s^*} > 0,$$

and that in Model II and in the benchmark model is

$$U^*\left(I_L, 0, \beta_L^*\right) - U^*\left(s_L, 0, \beta_L^0\right) = \frac{1}{2}\left(s^* - 2s_L\right) A > 0,$$

if and only if $s^*/2 > s_L$. Thus the low-type entrepreneur obtains a higher expected profit in Model I and Model II than in the benchmark model if and only if $s^*/2 > s_L$.

The expected profits of the low-type entrepreneur in Model I and Model II are

$$\begin{aligned} U^*\left(s_L, 0, \beta_L^*\right) &= \theta_L^* s_L A = \frac{(s^* A - 2K)\, s_L}{s^*}, \\ U^*\left(I_L, 0, \beta_L^*\right) &= \frac{s^*}{2}\theta_L^* A = \frac{s^* A}{2} - K. \end{aligned}$$

Since we have

$$U^*\left(I_L, 0, \beta_L^*\right) - U^*\left(s_L, 0, \beta_L^*\right) = \frac{s^* - 2s_L}{As^* - 2K} 2s > 0,$$

the low type obtains a higher expected profit in Model II than in Model I if and only if $s^*/2 > s_L$. ■

PROOF OF PROPOSITION 5.5 (1) The difference between the equilibrium expected utilities of the high type in Model I and in the benchmark model is

$$\begin{aligned} & U^*\left(s_H, 0, \beta_H^*\right) - U^*\left(s_H, 0, \beta_H^0\right) \\ = \; & \frac{[(1+s^* - 2s_H)\, c_L s_H - \{(1-s^*)\, s_H + (1+s^*)\, s_L\}\, c_H]\, K}{(1+s^*)\, c_L s_H}, \end{aligned}$$

which increases in s^*. Since $U^*\left(s_H,0,\beta_H^*\right)-U^*\left(s_H,0,\beta_H^0\right)$ becomes zero if and only if

$$s^*=\frac{c_H\left(s_H+s_L\right)-\left(1-2s_H\right)c_Ls_H}{\left(s_H-s_L\right)c_H+c_Ls_H},$$

the high-type entrepreneur has a higher expected profit in Model I than in the benchmark if and only if (5.2) holds. □

(2) The difference between the equilibrium expected utilities of the high type in Model II and in the benchmark model is

$$\begin{aligned}&U^*\left(I_H,0,\beta_H^*\right)-U^*\left(s_H,0,\beta_H^0\right)\\=\;&\frac{\left(1+s^*\right)\left(1+s^*-2s_H\right)c_Ls_HA-2\left[\left(1+s^*\right)s_L-s^*s_H\right]c_HK}{2\left(1+s^*\right)c_Ls_H},\end{aligned}$$

which is greater than zero if $1+s^*-2s_H>0$ and $\left(1+s^*\right)s_L-s^*s_H<0$. Therefore, if (5.3) holds, we have $U^*\left(I_H,0,\beta_H^*\right)>U^*\left(s_H,0,\beta_H^0\right)$. □

(3) The equilibrium expected utilities of the high-type entrepreneur in Model I and Model II are

$$\begin{aligned}U^*\left(s_H,0,\beta_H^*\right)&=\theta_H^*s_HA-c_H\psi^*=s_HA-\frac{2\left(c_H+c_Ls_H\right)K}{\left(1+s^*\right)c_L},\\U^*\left(I_H,0,\beta_H^*\right)&=\left(\frac{1+s^*}{2}\right)\theta_H^*A-c_H\psi^{**}=\frac{\left(1+s^*\right)A}{2}-\frac{\left[\left(1+s^*\right)c_L+c_H\right]\left(K_1+K_2\right)}{\left(1+s^*\right)c_L}.\end{aligned}$$

We have

$$\begin{aligned}&U^*\left(s_H,0,\beta_H^*\right)-U^*\left(I_H,0,\beta_H^*\right)\\=\;&-\frac{\left(1+s^*-2s_H\right)\left[\left(1+s^*\right)A-2K\right]c_L+2c_HK}{2\left(1+s^*\right)c_L},\end{aligned}$$

which is higher than zero if and only if

$$s_H>\frac{1+s^*}{2}+\frac{c_HK}{\left[\left(1+s^*\right)A-2K\right]c_L}.$$ ■

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