A Theory of Collateral for the Lender of Last Resort

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We take a macroprudential approach to analyze the optimal lending policy for the central bank, focusing on externalities that policy imposes on markets. Lending against high-quality collateral protects central banks against losses, but can adversely affect liquidity creation in markets since high-quality collateral gets locked up with the central bank rather than circulating in markets. Lending against low-quality collateral creates counterparty risk but can improve liquidity in markets. We characterize the optimal policy incorporating these trade-offs. We show that, contrary to what is generally accepted, lending against high-quality collateral can have negative effects, whereas it may be optimal to lend against low-quality collateral.

Keywords: Central bank, liquidity, macroprudential policy, externality, interbank market, lending facilities

1 Introduction

In his famous 1873 book *Lombard Street*, Walter Bagehot advocates four principles for central banks when they act as the lender of last resort (LoLR): lend only to illiquid but solvent banks, at a penalty rate, against good collateral valued at pre-panic prices, and make clear in advance the readiness to lend any amount to any institution that meets the conditions for solvency and collateral. Despite having shaped central banks' policies for more than a century, these principles continue to be the subject of intense debate, in part because they still lack a rigorous theoretical foundation. Moreover, the institutional environment has changed significantly since Bagehot's time. For one thing, unlike today's central banks, which are public institutions, the Bank of England was then privately held (with some privileges from the government). Also, the financial system today is much more connected and complex than ever before, making financial stability a bigger concern for authorities and, in some countries, an explicit goal of central banks.

In this paper, we investigate the optimal collateral policy for a central bank. In contrast to the existing literature on the LoLR, which typically takes a microprudential approach focusing on individual institutions, we take a macroprudential approach to account for the externalities central bank policies can impose on private markets by affecting the pool of collateral and liquidity creation in these markets.

Lending against high-quality collateral protects the central bank from potential losses, but it can adversely affect the pool of collateral in the markets and impair their functioning because high-quality collateral gets tied up with the central bank rather than circulating and facilitating liquidity creation. In contrast, lending against low-quality collateral exposes the central bank to counterparty risk, but it improves the pool of outstanding collateral, which, in turn, promotes the functioning of markets. We characterize the optimal policy for the central bank by taking into account these trade-offs. We show that, contrary to what is generally accepted, it can be optimal for the central bank to lend against low-quality collateral since it improves liquidity and the functioning of markets.

We build a model that incorporates maturity transformation and collateral circulation. While the existing literature focuses on the role of collateral in resolving agency or information frictions related to borrowers, our focus is on its circulation in markets and the process of solving the maturity mismatch problem for both borrowers and lenders. The model has three dates, two different groups of banks, and a central bank. The first group of banks, denoted as "borrowers," has access to long-term projects at the initial date. These projects are risky but they have a positive net present value (NPV). Projects are illiquid; that is, they suffer a loss when liquidated at the interim date. Borrowers do not have funds at the initial date, but they have collateral (that can be of high or low quality), which they can use to raise funding from the second set of banks, denoted as "lenders." Lenders have assets in place but do not have access to new projects. They also have funds which they can lend against borrowers' collateral at the initial date. Lending and borrowing is short term, creating a maturity mismatch.

At the interim date, lenders can experience a liquidity shock and their existing assets generate long-term returns at the final date only if they meet these shocks. If lenders experience a liquidity shock, they need to borrow from "outsiders" in the anonymous money market, pledging the collateral they received from the borrowers at the initial date. Outsiders are unable to identify the quality of individual collateral; they only know the composition of high- and low-quality collateral in the market. Hence, they lend up to the average value of collateral in the market.

If the average quality of collateral is high enough compared with their liquidity needs, lenders can raise enough funding from outsiders. However, when the overall quality of collateral is low, lenders may not be able to borrow enough from outsiders. In that case, lenders call back the loans they provided to borrowers at the initial date, which leads to a costly liquidation of the borrowers' projects. In other words, lenders' demand for liquidity at the interim date creates the risk of early liquidation for borrowers. Borrowers will invest at the initial date only when the expected return from investing, net of any liquidation cost, is positive. That is, borrowers forgo valuable investments when the expected cost of liquidation outweighs the

return from the project.

By imposing losses on banks, which could be mere private losses, early liquidations can lead banks to bypass positive NPV projects, resulting in real negative effects. This creates a role for the LoLR to insure banks against liquidity risk, which facilitates valuable investment ex ante. However, because the central bank lends against collateral, it affects the pool of outstanding collateral.[1] This, in turn, imposes externalities on markets. A key innovation of our paper is its macroprudential approach, which takes into account the externalities induced by the central bank's lending policy. This allows us to derive results that would have been overlooked had we relied on a microprudential approach.

Consider the case where the liquidity risk is sufficiently high so that all banks refrain from investing in the first place; that is, markets are frozen. In that case, when the central bank lends against high-quality collateral, it facilitates the investment of banks borrowing from the central bank while protecting itself from potential losses. This would be optimal when the central bank is highly concerned with its losses. The optimal policy in that case is to lend *freely* against high-quality collateral, similar to Bagehot's proposal.

However, this does not imply that it is never harmful for the central bank to lend against high-quality collateral. Consider a situation where markets are stressed but still functioning. Here, banks with high-quality collateral may find liquidity risk to be too high because they are pooled with banks with low-quality collateral, and prefer to borrow from the central bank instead. While this would directly insure banks that borrow from the central bank against liquidity risk, it would also decrease the quality of collateral in markets as high-quality collateral gets pledged with the central bank. This imposes a negative externality on the banks that are left in the market by increasing their liquidity risk. We show that the negative indirect effect on markets can more than offset the positive direct effect, so that output would be greater if high-quality collateral remained in the market, where it could circulate and generate liquidity, rather than being locked up with the central bank.

[1] Martin (2004) and Mills (2006) provide models where it is optimal for the central bank to lend against collateral.

On the other hand, while lending against low-quality collateral can expose the central bank to potential losses, it would improve the quality of collateral in markets and this indirect effect can outweigh such losses. Consider the case when the central bank faces a frozen market. By lending against low-quality collateral at a large enough scale, the central bank can improve the quality of the remaining collateral in the market sufficiently so that it can revive a frozen market resulting in a discontinuous effect on output. This may require the central bank to lend beyond the point where the marginal cost of lending exceeds the marginal benefit, at which point a central bank that takes a microprudential approach would have stopped lending. These effects would be overlooked had we not taken into account the externalities imposed by the actions of the central bank in a macroprudential context.

We present several extensions of our baseline setup. We first extend the intermediation chain, so that collateral can be circulated multiple times. We show that the effects from the baseline model get amplified. In another extension, we discuss the effects of certification, where the central bank certifies collateral rather than lending directly to banks. In a third extension, we allow for the possibility of costly information generation, as in Gorton and Ordoñez (2014, 2017), where the owners of collateral can credibly convey information about the quality of their collateral to outsiders.

Our paper is related to the vast literature on LoLR, which dates back to Thornton (1802) and Bagehot (1873).[2] Goodfriend and King (1988) argue that with efficient interbank markets central banks should not lend to individual banks but instead provide sufficient liquidity via open market operations.[3] Others, however, argue that interbank markets may fail to allocate liquidity efficiently owing to frictions such as asymmetric information about banks' assets (Flannery (1996), Freixas and Jorge (2007)); market power of banks in the interbank market (Acharya, Gromb, and Yorulmazer (2012)); banks' free-riding problems (Bhattacharya and Gale (1987) and Repullo (2005)); or banks' liquidity hoarding incentives (Diamond and Rajan

[2]For surveys, see Bordo (1990), Santos (2006), Freixas et al. (1999), and Ennis (2016). For some recent empirical studies, see Drechsler et al. (2016), Acharya et al. (2017), Ackon and Ennis (2017), and Garcia-de-Andoain et al. (2016).

[3]Also see Allen, Carletti and Gale (2009).

(2011); Gale and Yorulmazer (2013); Acharya and Skeie (2011); Malherbe (2014)).

The issue of collateral policy has not been explored much in the context of central bank lending perhaps with the exception of Koulischer and Struyven (2014) and Bindseil (2013). Koulischer and Struyven (2014) show that during systemic crises central banks can improve welfare by lending against low-quality collateral when all high-quality collateral has been exhausted. Bindseil (2013) suggests widening the set of collateral that the central bank would accept during a crisis. In contrast to these papers, we show that lending against low-quality collateral can improve welfare even when high-quality collateral is still available.

Philippon and Skreta (2012) and Tirole (2012) provide models with adverse selection in markets, where they analyze optimal interventions, including asset purchases, debt guarantees, and equity injections in the presence of policy externalities. In contrast, our focus is on LoLR policies, where we have the widely accepted Bagehot principles. Furthermore, an important feature of our model is the circulation of collateral that creates liquidity in markets.[4] In this regard, our paper is related to recent literature which attempts to explain collateral circulation. Donaldson and Micheler (2018) build a model of debt circulation, where lenders receive liquidity shocks after providing funds to borrowers, as in our setup. Focusing on debt enforceability, they argue that a decrease in credit frictions can lead banks to form a credit chain, which may increase systemic risk. Gottardi, Maurin and Monnet (2019) study repo transactions and show that, in equilibrium, lenders choose to re-use collateral. This, in turn, generates a "collateral multiplier" effect whereby borrowing increases.

Finally, Gorton and Ordoñez (2017) show that opaque lending facilities that do not reveal the identities of borrowing banks can improve the average quality of assets in the banking sector and mitigate the risk of runs. They examine how externalities differ depending on whether the LoLR maintains secrecy. In contrast to them, we focus on externalities by different lending policies when the central bank transparently communicates its rules.

[4]For estimates of the "velocity of collateral" see Singh (2011) and "Collateral velocity rebounds: recent estimates and policy implications" by Manmohan Singh and Zohair Alam, *Financial Times*, July 10, 2018.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 solves for bank behavior, investigates the effects of central bank lending, and discusses the optimal LoLR policy. Section 4 presents extensions of our model and discusses additional policy insights. Section 5 concludes. All appendices are on the internet.

2 Model setup

In this section, we present our model setup. We introduce the agents, illustrate the trading of liquidity in markets and lending by the central bank. We conclude the section with a discussion on the assumptions that drive our key findings.

2.1 Agents and liquidity shocks

The model has three dates: $t = 0, 1, 2$. There are two groups of banks—each with measure 1, denoted by $i = A, B$—and a central bank that can act as the lender of last resort. We assume that all banks are risk neutral with a discount factor of 1, have access to a storage technology with a rate of return equal to 1, and consume only at $t = 2$.

The main difference between the banks in groups A and B is the timing of their liquidity availability and needs. A banks need liquidity at $t = 0$ while B banks have cash at $t = 0$ that they can lend to A banks. However, as we will explain in detail below, B banks may be hit by a liquidity shock at $t = 1$.

Additionally, banks in group A ("borrowers") are endowed with an investment opportunity that requires one unit of cash at $t = 0$. The investment generates a random return at $t = 2$: with probability p it succeeds and generates $R_S > 1$, and with probability $(1-p)$ it generates 0. We denote the expected return from the project by $R = pR_S$ and assume that the investment has a positive NPV; that is, $R > 1$. The project is illiquid and can be liquidated at $t = 1$ in a lump-sum fashion, in which case it generates $r < 1$.[5] Out of the early liquidation loss $R - r$,

[5] We assume that the liquidation value r is fixed. However, the liquidation value may depend on the number of projects being liquidated and the liquidity available within the buyers (Shleifer and Vishny (1992), Allen

a fraction $\Delta \in [0,1]$ is attributed to an actual real output loss; that is, $\Delta(R-r)$ represents output losses from liquidations, and the rest $(1-\Delta)(R-r)$ is a mere transfer within the economy.[6]

A banks start out with no cash but are endowed with one unit of collateral, which can be of two types. A fraction α of A banks have "high-quality" collateral, denoted by $j = H$, which has the high value $c_H \geq 1$. The remaining A banks have "low-quality" collateral, denoted by $j = L$, which has the low value $c_L < 1$. Since A banks do not have any cash at the outset, they need to borrow in the "interbank market" (from B banks) to finance their investment at $t = 0$. We assume that lending is short term and needs to be rolled over at $t = 1$, which is typical of models of financial intermediation that feature maturity transformation.

B banks ("lenders") are endowed with one unit of cash at $t = 0$, which they can lend to A banks in the interbank market. We assume that the interbank market is competitive such that lenders anticipate to receive an expected rate of return equal to 1 and the contracts are enforceable due to, e.g., relationships.[7] B banks have assets in place that produce a return of $R^B > 1$ at $t = 2$, but do not have any new investment opportunity at $t = 0$ or $t = 1$. At $t = 1$, with probability ρ, B banks receive a liquidity shock, which requires a cash injection ℓ, with $\ell \sim U[0,1]$.[8] B banks' long-term return R^B is realized only if they are able to meet their liquidity need ℓ at $t = 1$; otherwise the return is zero.

If the liquidity shock materializes at $t = 1$, B banks need to borrow from investors in the anonymous money market, which we denote as "outsiders". Hence, we can interpret ρ as the probability of a systemic stress event in the interbank market, where B banks cannot satisfy their liquidity demand and need to borrow from outsiders whom they do not have an

and Gale (1994, 1998), and Choi, Eisenbach and Yorulmazer (2016), to cite a few). We analyze this case in Appendix C, where we introduce cash-in-the-market pricing.

[6]The transfer could include legal costs, brokers' fees, or a lower price paid by the buyers in the secondary market due to their opportunistic behavior or limited cash holdings.

[7]We provide a discussion of relationships in the interbank market in Appendix E.

[8]For simplicity, we use a uniform distribution for liquidity shocks. Our results go through with general p.d.f.s (see Appendix F). We further assume that all B banks experience the same liquidity shock, which simplifies the analysis and allows us to rule out interbank lending among B banks, without affecting our main results. See Appendix H for the case with idiosyncratic liquidity shocks for B banks.

established relationship with.

While contracts are enforceable among banks in the interbank market, e.g., due to relationships, this is not the case with outsiders, where banks' borrowing capacity is limited by the value of their collateral. Hence, although collateral is actually not needed in the interbank market at $t = 0$, B banks still demand collateral that they can reuse in case they have to borrow from outsiders at $t = 1$. Note that this arrangement insures B banks against liquidity shocks at $t = 1$, which, in turn, reduces A banks' early liquidation costs. If B banks can meet their liquidity shock by borrowing from outsiders, they roll over the loan. Otherwise, they call back the loan and A banks need to liquidate their projects early resulting in losses.

We assume the outsiders cannot identify the type of collateral.[9] The only information outsiders have is the fractions of high- and low-quality collateral held by banks. Thus, the average collateral value in the market at $t = 1$, denoted by c_1, is the maximum amount banks can borrow from outsiders. Note that c_1 is equal to $c_0 \equiv \alpha c_H + (1 - \alpha)c_L$ without any LoLR intervention, and is affected by the central bank's lending and collateral policies as analyzed in the next section. We assume that $c_0 < 1$ since otherwise the model would be trivial, i.e., B banks would always be able to borrow enough to satisfy their liquidity need ℓ at $t = 1$. This also implies that A banks cannot borrow the required funds of 1 unit from outsiders neither to take their investment at $t = 0$, nor to pay back to B banks at $t = 1$.

In contrast to outsiders, we assume that banks as well as the central bank are able to identify the type of collateral.[10] As a result, the central bank will have an informational advantage over outsiders but not over other banks.

Lastly, we make the following parametric assumptions. We assume that c_H is high enough ($c_H \geq 1$), while c_L is low enough ($c_L < 1$). Since $c_H \geq 1$, lending against high-quality col-

[9] One such example is the General Collateral Financing (GCF) repo market, a major segment of the U.S. tri-party repo market, where trades are anonymous and lenders do not designate specific securities as collateral when lending, but later receive any "general" collateral that meets a "rule set" (Copeland, Duffie, Martin, and McLaughlin 2012).

[10] This may come from relationships in the interbank market and central bank's supervisory role, respectively. Additionally, the central bank can examine the collateral ex post and impose a penalty on banks that do not report truthfully, which would induce banks to report the true quality ex ante. See Section 4.4 for a more in-depth discussion of the features that put central banks in a special position to act as LoLR.

lateral does not expose the lender to credit losses when the investment matures at $t = 2$, while lending against low-quality collateral exposes the lender to credit losses at $t = 2$, since $c_L < 1$ and with probability $1 - p$ the bank's project has the low return 0. We also assume that $c_L + r > 1$, which implies that a lender who grants a short-term loan to A banks at time $t = 0$ is not exposed to any credit or liquidity risk, since it can always recover one unit by calling back the loan at $t = 1$. We summarize these conditions in the following assumption:

Assumption: $1 - r < c_L < c_0 < 1 \leqslant c_H$.

2.2 Lender of last resort

The central bank can lend to A banks at $t = 0$. We focus on central bank lending at $t = 0$ because we want to understand how the central bank's collateral policy affects liquidity in the markets at $t = 0$ and $t = 1$ by changing the pool of collateral.[11]

At $t = 0$, the central bank announces its policy, which specifies the maximum amount x_j it is willing to lend against collateral of type $j \in \{H, L\}$.[12] The objective of the central bank is to maximize the aggregate return from the banks' projects net of early liquidation losses (denoted by output Y) minus costs associated with its LoLR activity. The banks that borrow from the central bank pay the "fair" rate at $t = 2$; that is, they pay one unit in expectation for the unit borrowed. Note that banks with eligible collateral always prefer to borrow from the central bank because this eliminates the interim liquidity risk. When the central bank pre-specifies the amount x_j it is willing to lend, we assume that it allocates x_j randomly among banks seeking loans if there is excess demand.[13]

[11]Note that one can interpret $t = 0$ as the "ex-post" stage after the onset of the crisis, and thus, $t = 0$ lending as an ex-post intervention. Also, in a dynamic setting, the distinction between ex-ante and ex-post lending becomes blurred, because any action by the central bank will have an impact in future periods. See our multi-period intermediation chain extension in Section 4.1 and in Appendix A. We discuss the effect of ex-post lending at $t = 1$ by the central bank in Appendix H and show that our main results go through.

[12]Note that banks in our model are identical in terms of their solvency, and the only difference is the type of collateral they have. See Choi (2014) for the macroprudential effect of policy interventions across banks with heterogeneous solvency.

[13]Note that in our model there is no penalty rate or stigma associated with borrowing from the central

2.3 Timeline

The timeline of our model, also summarized in Figure 1, is as follows.

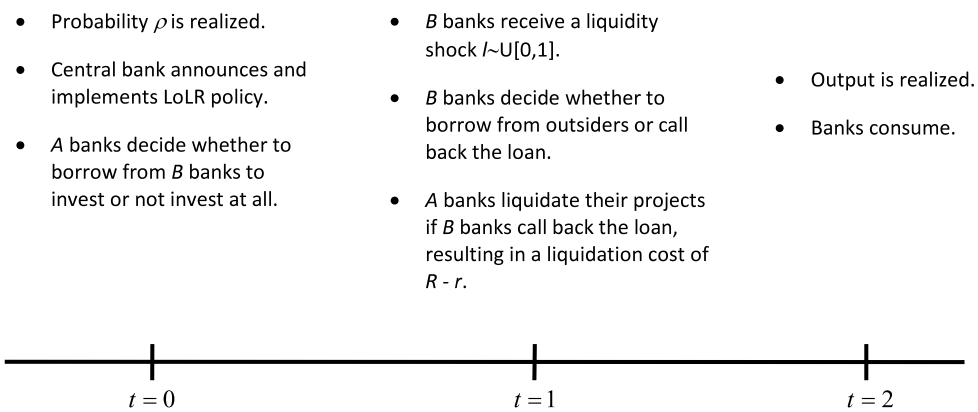

Figure 1: Timeline

At $t = 0$, ρ is realized. Based on ρ, the central bank chooses its lending policy (x_H, x_L), which affects the average collateral value c_1 in the market. After observing (ρ, c_1), remaining A banks decide whether to borrow from B banks in the interbank market or not invest at all, to maximize their expected payoff at $t = 2$.

At $t = 1$, with probability ρ, B banks are hit with the liquidity shock ℓ in which case they borrow from outsiders if possible, or call back their loans to A banks.[14]

At $t = 2$, output is realized, and banks consume. Figure 2 summarizes the flows of liquidity between agents at $t = 0$ and $t = 1$.

banks. In Appendix G, we introduce a penalty rate and show how central banks can use it to allocate its funds. We also discuss various arrangements, including auctions, that central banks often use to address excess demand.

[14]We assume that, when indifferent, B banks borrow from outsiders instead of calling back their loans.

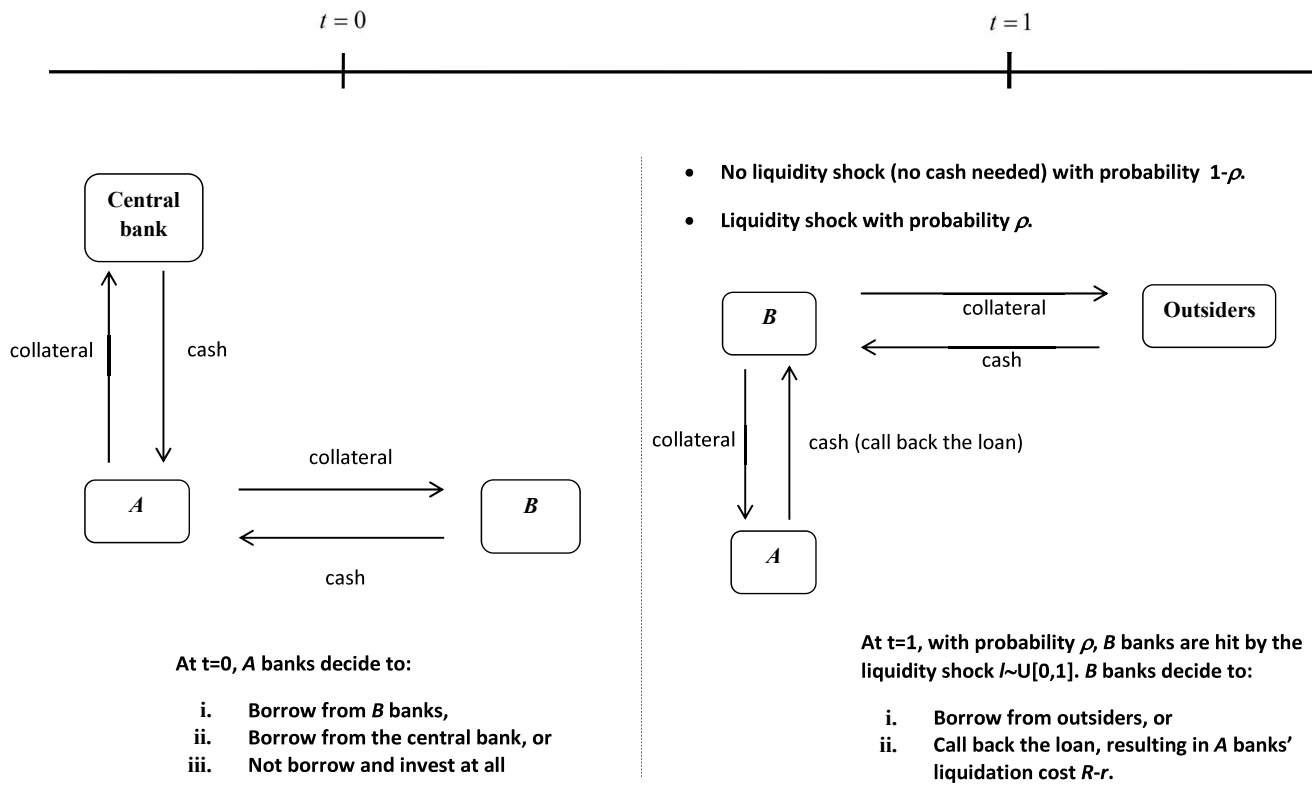

Figure 2: Flows between agents at $t = 0$ and $t = 1$

2.4 Discussion of assumptions

In this section, we discuss the three main assumptions we consider in our model: i) outsiders cannot distinguish the type of collateral; ii) when B banks get hit by a liquidity shock, they cannot secure the required liquidity in the interbank market and need to borrow from outsiders pledging collateral; and iii) lending and borrowing is short-term.

Information friction: We assume that outsiders cannot distinguish the type of collateral, and lend less if they are concerned about the quality of collateral pledged by borrowers. While we attempt to keep our model simple to focus on the main effect, our setup can be interpreted

as follows. Suppose we have two states of the economy: a "normal" state and a "distress" state, where the state of the economy is public information. In the normal state, all collateral has the high value c_H so that B banks can always borrow the amount they need from outsiders. However, collateral becomes "information sensitive" when the economy switches to the distress state (Dang, Gorton and Holmstrom (2015, 2019)), in which case a fraction α of the collateral is worth c_H and a fraction $1-\alpha$ is worth c_L. Outsiders cannot distinguish the collateral type but they know that on average it is worth $c_0 = \alpha c_H + (1-\alpha)c_L$. Our benchmark setup can be interpreted as the case where the economy already switched to the distress state.

This setup finds support in the literature. Gorton and Metrick (2009) analyze debt capacity in repo transactions and document the stark change in the debt capacity of structured products before and after the crisis. These assets had 100% debt capacity in repo transactions prior to crisis, which fell to around 55% during the crisis.[15]

When assets are information sensitive, any information about their quality can have significant effects on debt capacity and the functioning of markets (Dang, Gorton and Holmstrom (2015, 2019)). The actions of the central bank can, on their own, generate information about these assets and affect the markets, which is the main focus of our paper. Of course, the holders of collateral could also have incentives to generate information about the quality of collateral, further hampering market functionality. We extend our model to introduce private information generation in Section 4.3, and show that the actions of the central bank can strengthen those incentives, amplifying the effects we identify.

Borrowing from outsiders: We assume that with probability ρ, B banks need to borrow from outsiders. Hence, ρ captures the probability that banks cannot satisfy their liquidity needs in the interbank market and are forced to borrow from outsiders. This suggests that the liquidity shock we model is a systemic shock that affects all (or a majority of) banks. In an extension, we provide a microfoundation for ρ building on a longer intermediation chain. In this case, ρ denotes the probability of a bank not being able to find another bank to borrow

[15] In the extreme case of subprime related assets, the debt capacity reached 0, that is, they were not accepted as collateral in repo transactions. See Figure 2 in Gorton and Metrick (2009).

from in the interbank market (see Section 4.1).

Short-term debt: We assume that A banks use short-term debt which exposes them to early liquidation risk.[16] This could be eliminated if they were to use long-term debt. However, A banks prefer short-term debt over long-term debt when this lowers their borrowing cost, which is the case when the liquidity risk for B banks is sufficiently high. If A banks borrow from B banks using long-term debt, B banks lose their long-term return R^B if they are hit by a liquidity shock at $t = 1$ and cannot meet their liquidity demand. Since B banks need to break even in expectation, they would pass this cost on to A banks, raising the borrowing cost for them. Therefore, if B banks' loss when they cannot meet their liquidity demand is larger than A banks' loss when they liquidate their project, that is, $R^B > R - r$, then A banks prefer short-term debt since it lowers their borrowing cost.

3 Analysis

We solve the model backwards. We first analyze banks' decision given the LoLR policy. Next, we examine the effect of different LoLR policies on output Y. Finally, we discuss the optimal policy conditional on the LoLR objective.

3.1 Bank behavior

We begin by analyzing banks' optimal decisions given the LoLR policy. As discussed, B banks are always willing to lend to A banks at $t = 0$, since $c_L + r > 1$ and they can recover their loan by refusing to renew it at $t = 1$. However, when a B bank does not renew its loan, this will lead to the early liquidation of A banks' investment, resulting in a loss of $R - r$ to A banks.

With probability $1 - \rho$, there is no liquidity shock at $t = 1$. However, with probability ρ, B banks experience a liquidity shock and need to borrow from outsiders. Recall that outsiders

[16] See Calomiris and Kahn (1991) and Diamond and Rajan (2001) for the disciplining effect of short-term debt on banks, and Donaldson and Micheler (2018) for the choice of debt maturity when lenders are exposed to liquidity risk. Other papers that analyze the debt maturity structure include Flannery (1986, 1994), Diamond (1991), and Brunnermeier and Oehmke (2013).

only observe the average collateral valuation c_1, which is the maximum amount they are willing to lend. Hence, B banks can meet their liquidity needs only if $c_1 \geq \ell$. When c_1 is less than ℓ, B banks will not be able to meet their liquidity needs and will call back their loans. This occurs with probability $1 - c_1$ since $\ell \sim U[0, 1]$.

Hence, the expected payoff for an A bank from investing, denoted by Π, can be written as:

$$\Pi = (1-\rho)R + \rho\left[c_1 R + (1-c_1)\, r\right] - 1 = R - 1 - \underbrace{\rho\,(1-c_1)\,(R-r)}_{\text{liquidity risk}},$$

because with probability $\rho(1-c_1)$, lenders cannot meet their liquidity needs and call back the loan, resulting in a liquidation loss of $(R-r)$ at $t = 1$. Given that lending is competitive with the risk-free rate equal to 1, A banks will pay the expected funding cost equal to 1.

A banks will not invest if Π is negative, that is, when

$$\rho > \rho^* = \frac{R-1}{(1-c_1)(R-r)}, \tag{1}$$

which gives the following proposition.

Proposition 1 (No investment): *A banks invest if and only if $\rho \leqslant \rho^*$.*

Note that ρ^* is increasing in c_1 so that loans are rolled over more often for higher average values of collateral, which in turn induces more investment ex ante. Hence, higher quality collateral reduces liquidity risk for both lenders and borrowers and facilitates investment.

3.2 LoLR and impact on output

Next, we investigate how central bank policies affect aggregate liquidity and output in the economy. While lending to banks can expose the central bank to losses, to clearly show the positive effects of central bank policies on the functioning of markets and output generated by banks, we postpone the discussion of such losses to Section 3.3.

As described in Section 2, the central bank lends to eligible A banks at $t = 0$ after the probability ρ is realized. Recall that A banks will lose $R - r$ on their investment if it is liquidated early at $t = 1$, leading to an output loss of $\Delta(R - r)$.

As a benchmark, we start by defining output in our economy without any LoLR action. We then analyze how output responds to different LoLR policies. With no LoLR action, output, denoted by Y, can be written as

$$Y = [\{(R-1) - \rho(1-c_0)(R-r)\Delta\} \times \mathbb{1}_{\rho \leqslant \rho^*}] + R^B,$$

where $\mathbb{1}_{\rho \leqslant \rho^*}$ is an indicator function that takes the value 1 when $\rho \leqslant \rho^*$ and 0 otherwise. Note that for $\rho \leqslant \rho^*$, A banks invest and are exposed to early liquidation risk that occurs with probability $\rho(1 - c_0)$, while for $\rho > \rho^*$ they do not invest at all. The long-term return of B banks' investments R^B is trivial because B banks can always meet their liquidity needs either by borrowing from outsiders or by calling back their loans to A banks.

We first analyze the "marginal" effect of the central bank policy on output, without entailing a regime switch from investment to no-investment and vice versa; we then analyze the effects of the central bank policy when it does trigger a regime switch—that is, when it restores a frozen market or causes a functioning market to freeze.

For $\rho \leqslant \rho^*$, all A banks invest. Suppose that the central bank lends to a measure x_H of A banks with high-quality collateral. Suppose also, for now, that x_H is small enough so that we can focus on the marginal effect. The central bank's policy will affect the pool of collateral in the market, and the average value c_1 will be

$$c_1^H = \frac{(\alpha - x_H)c_H + (1-\alpha)c_L}{1 - x_H}.$$

In that case, output can be written as

$$Y_H = x_H\,(R-1) + (1-x_H)\left[(R-1) - \rho(1-c_1^H)(R-r)\Delta\right] + R^B, \tag{2}$$

because banks receiving funding from the central bank are not exposed to any liquidity risk, while the other banks, a fraction $1-x_H$, experience an increase in their liquidity risk exposure to $\rho\left(1-c_1^H\right)$. Central bank's loans have two effects that go in opposite directions. On the one hand, they eliminate the liquidity risk for x_H banks, increasing output as captured by the first term in equation (2). On the other hand, they impair the quality of collateral in the market, which increases liquidity risk for the banks remaining in the market and indirectly decrease output, as captured by the second term in (2). In particular, we have

$$\begin{aligned}\frac{\partial Y_H}{\partial x_H} &= \rho(R-r)\Delta\left[(1-c_1^H)+(1-x_H)\frac{\partial c_1^H}{\partial x_H}\right]\\ &= \rho(R-r)\Delta(1-x_H)(1-c_H).\end{aligned}$$

Note that $\frac{\partial Y_H}{\partial x_H} \leq 0$ because $c_H \geq 1$ — even though lending against high-quality collateral eliminates the liquidity risk for banks that borrow from the central bank and directly increases output, the negative externality on the rest of the banks remaining in the market more than offsets the direct effect. In other words, high-quality collateral would generate more liquidity and output if it remained and circulated in the market than being locked up with the central bank.

However, when the central bank lends against low-quality collateral to a measure x_L of banks, the average quality of collateral in the market improves to $c_1^L = \frac{\alpha c_H + (1-\alpha-x_L)c_L}{1-x_L}$. Output can be written as

$$Y_L = x_L(R-1) + (1-x_L)\left[(R-1) - \rho(1-c_1^L)(R-r)\Delta\right] + R^B, \tag{3}$$

so that

$$\frac{\partial Y_L}{\partial x_L} = \rho(R-r)\Delta(1-x_L)(1-c_L) > 0.$$

In sum, when $\rho \leqslant \rho^*$ such that all banks invest, lending against high-quality collateral de-

creases output, whereas lending against low-quality collateral has a positive effect on output.

We now analyze the second case, where $\rho > \rho^*$. In this case, A banks do not invest and the central bank stimulates only the investment of those banks it lends to, that is, $\frac{\partial Y_H}{\partial x_H} = \frac{\partial Y_L}{\partial x_L} = R - 1$. Assuming that central bank lending is small enough and does not lead to a regime switch, we can summarize our results in the following proposition, which is illustrated in Figure 3.

Proposition 2 *For $\rho > \rho^*$, central bank lending against high- and low-quality collateral has the same effect on output, $\frac{\partial Y}{\partial x_H} = \frac{\partial Y}{\partial x_L} = R - 1$. For $\rho \leqslant \rho^*$, central bank lending against high- and low-quality collateral has the following effects on output: $\frac{\partial Y}{\partial x_H} = \rho(1 - c_H)(R - r)\Delta \leq 0$ and $\frac{\partial Y}{\partial x_L} = \rho(1 - c_L)(R - r)\Delta > 0$, respectively.*

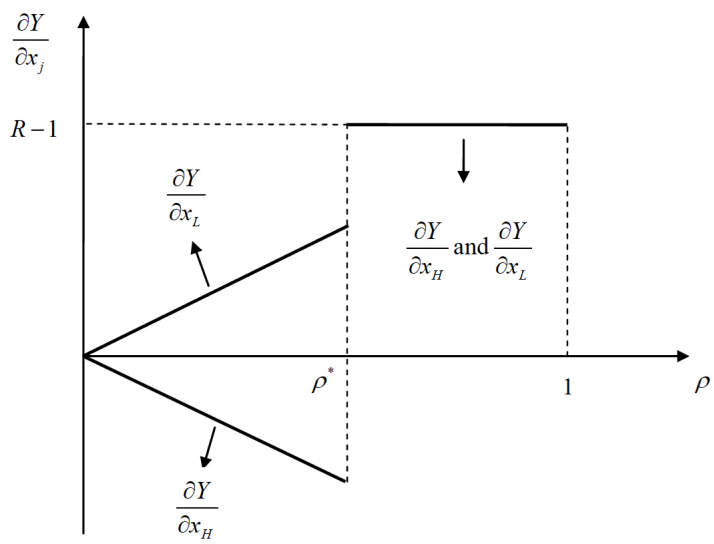

Figure 3: The effect of LoLR on output, by collateral type. When $\rho > \rho^*$, no A bank invests, and different central bank policies have the same positive effect on output. When $\rho \leqslant \rho^*$, output increases (decreases) when the central bank lends against low-quality (high-quality) collateral.

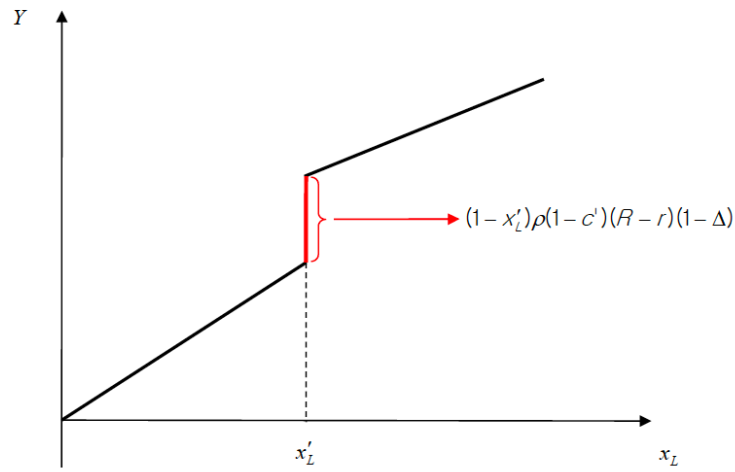

Figure 4: Market jump-start. All A banks start to borrow and invest if the central bank lends to x_L' or more banks with low-quality collateral. Output jumps upward.

So far, we focused on the effect of "marginal" lending, where central bank lending does not lead to a regime switch. Next, we analyze how it can lead to such a switch. First, we show that improvements in the average quality of collateral in markets can restore an impaired interbank market. Suppose $\rho > \rho^*$, so that no A bank invests. If the average quality of collateral increases to a level above the threshold c', where $\rho(1-c')(R-r) = R-1$, liquidity risk is sufficiently mitigated and all banks will invest. The central bank can achieve this by lending to, and taking out of the market, a minimum measure x_L' of A banks with low-quality collateral, where

$$x_L' = \frac{(R-1) - \rho(1-c_0)(R-r)}{(R-1) - \rho(1-c_L)(R-r)}. \tag{4}$$

Hence, when the central bank lends to a sufficiently large proportion of banks with low-quality collateral, the economy switches from a no-investment regime to an investment regime. Figure 4 illustrates this result. As the central bank increases its lending to banks with low-quality collateral, it increases output by inducing the investment of the direct borrowers, but

at the same time, the quality of the collateral in the market improves. If the central bank takes out x'_L of the low-quality collateral, the average quality of the collateral in the market improves sufficiently to trigger the investment of all the remaining A banks. At that point, output jumps upward by $(1 - x'_L)\rho(1 - c')(R - r)(1 - \Delta)$.

On the contrary, a central bank policy that adversely affects the quality of collateral can lead to the breakdown of an otherwise functioning interbank market. For $\rho \leqslant \rho^*$, all banks borrow and invest. When the average quality of collateral falls below c', the economy switches to a state in which banks stop borrowing and investing. Central bank policy can lead to such a freeze when it lends to a measure x'_H (or more) of A banks with high-quality collateral, where

$$x'_H = \frac{(R-1) - \rho(1 - c_0)(R - r)}{(R-1) - \rho(1 - c_H)(R - r)}. \tag{5}$$

Figure 5 presents this result. As the central bank takes out the high-quality collateral, output decreases owing to the negative externality imposed on borrowers remaining in the market. At the threshold x'_H, the liquidity risk becomes significant enough such that the remaining A banks simply opt out from investing, and the output of remaining A banks drops to 0. We summarize these results in the following proposition.

Proposition 3 *Central bank policies can have the following effects on banks' investment decision: (i) when the central bank lends against low-quality collateral, which increases the average quality of collateral to a level above c', it can jump-start investment; and (ii) when the central bank lends against high-quality collateral, which decreases the average quality of collateral to a level below c', banks stop borrowing in the market and stop investing.*

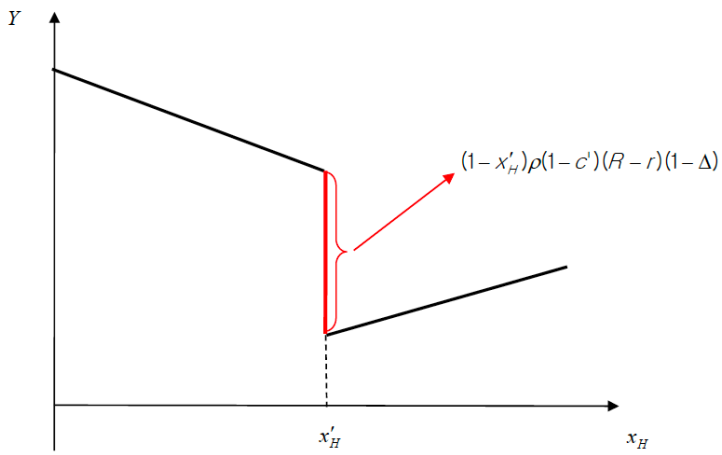

Figure 5: Market freeze. All remaining A banks stop investing if the central bank lends to x'_H or more banks with high-quality collateral. A banks' output drops to $x'_H(R-1)$.

3.3 Discussion of optimal policy

Lending directly to banks can expose the central bank to credit risk and potential losses. We refrained from adding such losses to our analysis until now to illustrate the positive implications of the LoLR policies on the functioning of markets. Building on the positive results from the previous section, we next discuss optimal LoLR policies that take into account such losses for the central bank.[17]

In performing its LoLR role, the central bank has the objective of maximizing output Y net of any costs from lending directly to banks. The central bank does not face any counterparty risk when it lends against high-quality collateral, since $c_H \geqslant 1$. However, it suffers a loss of $1 - c_L$ when it lends against low-quality collateral and the bank's project has the low return 0 at $t = 2$. Hence, when it lends to a measure x_L of banks against low-quality collateral, the resulting counterparty risk exposes the central bank to expected losses of $z = x_L(1-p)(1-c_L)$. This can be costly to the central bank, because it may affect its reputation or impair its

[17]For a detailed analysis, please see Appendix I.

independence.[18] We denote the cost to the central bank from incurring these losses by $f(z)$ and assume this function increases in z with $f(0) = 0$.

Given the cost function $f(.)$, we consider the following objective function for the central bank:

$$W = Y - f(z), \tag{6}$$

which is output Y minus the cost from the LoLR actions. Our discussion below focuses on the comparison of the marginal benefit (i.e., $\frac{\partial Y}{\partial x_j}$) versus the marginal cost (i.e., $f'(z)\frac{\partial z}{\partial x_j}$) for the case where $f(z)$ is convex.

We again distinguish functioning and frozen markets. Note that when $\rho \leqslant \rho^*$, all A banks invest. In that case, lending against high-quality collateral is never optimal since it will decrease output and possibly lead to a freeze in the market (Propositions 2 and 3). And, when lending against low-quality collateral, the central bank weighs the benefit of increased output against the costs arising from counterparty risk, where the optimal amount x_L^* is characterized by the first-order condition $\frac{\partial Y}{\partial x_L} = f'(z)\frac{\partial z}{\partial x_L}$.

When $\rho > \rho^*$, none of the A banks invests. Lending against high-quality collateral increases output by $R - 1$ per unit lent (Proposition 2) and does not expose the central bank to counterparty risk. Lending against low-quality collateral also increases output by $R - 1$ per unit lent but exposes the central bank to counterparty risk. However, lending against low-quality collateral has the benefit of improving the quality of collateral in the market, which can potentially restore investment (Proposition 3). When analyzing the optimal central bank policy, we need to take into account such discontinuous effects.

One policy, denoted as Policy H, is to (i) lend to all A banks with high-quality collateral, and then (ii) lend to the A banks with low-quality collateral according to the marginal cost-

[18] Goncharov, Ioannidou, and Schmalz (2018) provide empirical evidence of central banks' loss aversion. This is echoed by Klaas Knot (2013), Governor of the Dutch National Bank: "It shows that central banks' balance sheets are becoming more and more exposed to economic risk and political pressure. Eventually, this may result in a substantial amount of negative capital in a central bank's balance sheet. This is undesirable, because it could undermine a central bank's credibility and independence." See Section 4.5 for further discussion.

benefit analysis comparing increased output versus the costs arising from counterparty risk. Under an alternative policy, denoted by Policy L, the central bank lends only against low-quality collateral. Policy L dominates Policy H if the discrete increase in output due to revived investment is higher than the cost from the potential losses for the central bank.

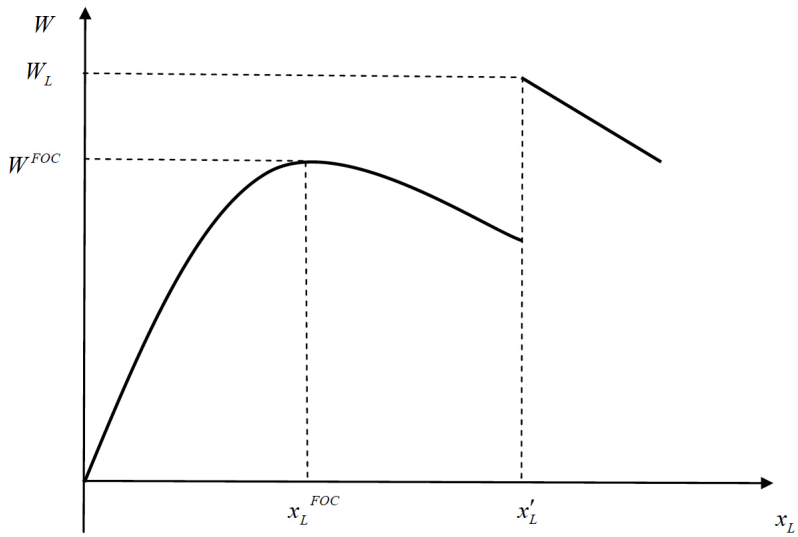

Figure 6: Optimal regime shift. The marginal increase in output is less than the marginal cost to the central bank beyond $x_L = x_L^{FOC}$. However, a regime switch from no-investment to investment arises when $x_L = x'_L$, at which point output increases in a discrete fashion. If $W_L > W^{FOC}$, the central bank should bear greater counterparty risk to restore investment than the marginal cost-benefit analysis suggests.

A unique case is depicted in Figure 6, where x_L^{FOC} satisfies the first-order condition $\frac{\partial Y}{\partial x_L} = f'(z)\frac{\partial z}{\partial x_L}$. Note that, beyond $x_L = x_L^{FOC}$, W decreases with greater x_L. However, if the central bank expands its lending to $x_L = x_L^{'}$, investment is restored and output jumps upward. When investment has been restored, if W_L is greater than the local maximum W^{FOC}, then the central bank should expose itself to more counterparty risk than the marginal cost-benefit analysis—an approach for a central bank with a microprudential view—suggests in order to restore investment. Overall, a central bank should adopt a macroprudential approach that

takes into account the effect of its actions on markets.

4 Extensions and discussion

In this section, we provide several extensions of our model to tackle important issues such as the role of a longer intermediation chain in collateral circulation and the amplification of our main results (Section 4.1); the effect of central bank's certification of collateral on the functioning of markets (Section 4.2); endogenous information generation on collateral quality by banks and how central bank lending can affect such private information generation (Section 4.3). We also discuss why central banks are uniquely positioned to act as LoLR (Section 4.4). We end the section with a review of the recent changes in central banks' collateral policies, and a discussion of the potential impact of recent liquidity regulation on the insights of our paper (Section 4.5).

4.1 Intermediation chain and collateral circulation

In the benchmark model, we kept the intermediation chain short to illustrate the main result in a simple way. We now extend the intermediation chain, where we allow for collateral to circulate among banks multiple times and show that the effects we obtain in our benchmark setup get amplified. We provide a detailed analysis in the Appendix A.

We extend the benchmark model and slightly modify our interpretation of the probability ρ as we explain below. In particular, we divide the intermediate period $t = 1$ into N equal subperiods denoted by $n = 0, 1, \ldots N - 1$ with more than N subgroups of B banks, where we denote the B banks that lent to A banks at $t = 0$ as B_0. In the benchmark model, B banks that lent to A banks at $t = 0$, denoted as B_0 banks now, received a liquidity shock with probability ρ at $t = 1$ and needed to borrow from outsiders. In this extended setup, at $n = 0$, B_0 banks get hit by a liquidity shock $\ell_0 \sim U[0, 1]$ with probability 1 and search for a B_1 bank to borrow from in the interbank market, where B_1 banks are endowed with one

unit of cash that they can lend to B_0 banks. However, with probability ρ they cannot find a match and need to borrow from outsiders. Since they can borrow up to c_1 from outsiders, with probability $\rho(1 - c_1)$ they cannot satisfy their liquidity needs and call back the loan, leading to an early liquidation of A banks' projects. Note that, as is the case in the benchmark setup, while collateral is not necessary in the interbank market, B_1 banks still take collateral from B_0 banks in anticipation of their own liquidity shock and the need to borrow from outsiders, which results in another round of circulation of collateral.

The model continues in the same fashion, where B_n banks, who lent to B_{n-1} banks in subperiod $n - 1$ if there were successful matches until then, are hit by a liquidity shock $\ell_n \sim U[0, 1]$ with probability 1 in subperiod n. They search for a B_{n+1} bank to borrow from in the interbank market using the collateral they received from a B_{n-1} bank, but with probability ρ, they are not matched with one and need to borrow from outsiders. If they cannot satisfy their liquidity needs, B_n banks call back the loan they gave to B_{n-1} banks in subperiod $n-1$, which starts a chain of loans being called back so that, ultimately, A banks' projects get liquidated.

As the intermediation chain gets longer, collateral can circulate multiple times and its capacity to generate liquidity increases. Hence, the central bank's actions that affect the average quality of collateral in markets have bigger effects on the functioning of markets, i.e., a smaller intervention can have a bigger impact on output.

4.2 Certification

We next discuss the effects of an alternative policy, where the central bank certifies the quality of collateral instead of lending directly to banks. See Appendix B for the detailed analysis.

Certification would eliminate counterparty risk because the central bank would no longer lend to banks. However, banks that are certified to have low-quality collateral would experience higher liquidity risk and may not even be able to invest. Note that certifying x_j units of collateral of type j will affect the composition of collateral for the remaining $(1 - x_j)$ un-

certified banks. The resulting effect would be the same as the effect on the composition of collateral of those banks that do not borrow from the central bank when the central bank lends directly to x_j units of banks with collateral of type j. Hence, certifying high-quality collateral would be equivalent to lending against high-quality collateral because the central bank is not exposed to any counterparty risk in both cases. However, certifying low-quality collateral and lending against low-quality collateral can have different effects. We can show that in some cases certification will be harmful and never desirable. In others, whether it dominates direct lending to banks will depend on the trade-off between reduced counterparty risk for the central bank and the increased liquidity risk and the potential forgone investment by banks certified to have low-quality collateral.

Recall that for $\rho \leqslant \rho^*$, banks are already investing and it is never optimal to lend against high-quality collateral (and therefore certify high-quality collateral). When the central bank certifies low-quality collateral, it increases the liquidity risk for the certified banks, possibly preventing them from investing, whereas the liquidity risk for the uncertified banks decreases. We can show that the positive effect is more than offset by the negative effect so that certification would not be a desirable policy, whereas it can be desirable to lend to (some) banks with low-quality collateral as we already showed.

Note that for $\rho > \rho^*$, no bank invests. When the central bank certifies x_L^C units of low-quality collateral, it can improve the average quality of collateral for uncertified banks and can jump start investment without suffering any cost from counterparty risk. However, the x_L^C units of banks that are certified to have low-quality collateral cannot invest resulting in a loss of output of $x_L^C(R-1)$. On the other hand, when the central bank lends to x_L units of banks with low-quality collateral, it can jump start investment and *all* banks invest. However, the central bank suffers a cost of $f(z)$ with $z = x_L(1-p)(1-c_L)$ from counterparty risk. Hence, the desirability of the two policies depend on the comparison between the associated costs of $x_L^C(R-1)$ and $f(z)$. For example, conservative central banks with a more steep $f(.)$ would be more averse to counterparty risk and possibly prefer certification.

In sum, whether certification dominates lending directly to banks depends on the costs associated with counterparty risk for the central bank and the forgone investment by banks that are certified to have low-quality collateral.[19]

4.3 Information on collateral quality

We now allow collateral holders to generate information endogenously. See Appendix D for the detailed analysis. Suppose that the true quality of collateral can be revealed to outsiders if its holder chooses to generate costly information, as in Dang, Gorton and Holmstrom (2015, 2019) and Gorton and Ordoñez (2014, 2017). Otherwise, outsiders would simply attribute the average quality to any collateral as in our original setup. Suppose that A banks can generate information about their collateral by incurring a cost of δ at $t = 0$.

Here, an A bank with high-quality collateral will incur the cost to separate itself from the A banks with low-quality collateral if the benefit from eliminating liquidity risk is greater than the cost of generating information. Hence, information generation arises endogenously only when the bank with high-quality collateral faces high liquidity risk, i.e., as the probability ρ gets higher or the average collateral quality c_1 deteriorates (see Proposition 4 in Appendix D). When information is generated, the liquidity risk for the A banks with low-quality collateral increases because their collateral is no longer pooled with high-quality collateral. As a result, these banks may not invest at all if their liquidity risk becomes sufficiently high.

This implies that a (partial) market breakdown can arise even when information is generated endogenously. As the central bank takes out high-quality collateral from the market, the liquidity risk for those banks left in the market increases, which can eventually trigger information generation of other A banks with high-quality collateral. This increases the liquidity risk for A banks with low-quality collateral and can result in a freeze in an otherwise functioning interbank market, with no investment by A banks with low-quality collateral. On the

[19]Note that private securitization would have similar effects as certification by the central bank. Tranching would create senior tranches that have lower risk than the initial collateral pool, but also create junior tranches that are riskier. It hence generates similar information as certification by separating the pool of collateral.

other hand, lending against low-quality collateral can improve the overall quality of collateral in the market and reduce incentives for information generation.

4.4 Central bank as LoLR

Central banks have certain features that put them in a special position to act as lenders of last resort.[20] For instance, central banks can acquire information on banks' assets through their supervisory activities that is not publicly available to market participants. In his testimony at the U.S. House of Representatives,[21] Stephen Cecchetti talks about this issue: "Operating as the lender of last resort requires two pieces of information: (1) a determination of an institution's solvency, and (2) a valuation of the collateral that is being posted to back the loan. Supervisors, with their intimate knowledge of the bank's operations, are the officials expected to have both of these." William Buiter makes a similar point in a Treasury Committee hearing, pointing to the separation of responsibilities between the UK Treasury, the Bank of England, and the FSA during the crisis: "The notion that the institution that has the knowledge of the individual banks that may or may not be in trouble would be a different institution from the one that has the money, the resources, to act upon the observation that a particular bank needs lender of last resort support is risky. It is possible, if you are lucky, to manage it, but it is an invitation to disaster, to delay, and to wrong decisions."[22] Furthermore, even if the central bank could not evaluate the quality of collateral that banks pledge exactly, it could examine it ex post as a regulator/supervisor and impose a penalty on banks that do not report truthfully. This would induce banks to report the true quality ex ante.

Another advantage of central banks is that they can lend longer term because they have a longer horizon than private agents that may experience liquidity shocks in the short run. Lastly, central banks can conduct asset purchases and lending on a larger scale than private

[20] See Repullo (2000), Kahn and Santos (2005, 2006), and Ponce (2010) for papers that analyze who should act as the LoLR in the presence of asymmetric information and differences in policy objectives among regulators.

[21] Available at https://financialservices.house.gov/uploadedfiles/hhrg-115-ba15-wstate-scecchetti-20170912.pdf

[22] Source: "The Run on the Rock," report for the House of Commons Treasury Committee, page 105.

agents. This gives them a wedge to deal with market failures arising from moral hazard and adverse selection problems (Flannery 1996).[23] It can also have a significant impact on prices, as we show in the cash-in-the market extension of our model in Appendix C.

4.5 Policy discussion

In this section, we provide a discussion of the recent changes in the collateral policies of central banks, and the potential impact of the Basel III liquidity regulation on the insights of our paper.[24]

Historically, central banks' collateral policies followed Bagehot's insight and were quite restrictive, accepting only "good collateral". However, as documented in Bank for International Settlements (2013), the recent crisis prompted central banks to modify their collateral policies, generally broadening them to accept more asset types. This is also evident in Lee and Sarkar (2017), which compares the counterparty and collateral policies of the lending facilities of the Federal Reserve (Fed), Bank of England (BoE), Bank of Japan (BoJ) and the European Central Bank (ECB). Compared to the Fed, the other three central banks allow for a broader set of counterparties, where BoJ is the least restrictive of the four.[25] In general, the Fed accepts only Treasury and agency debt and agency mortgage-backed securities as collateral for open market operations (OMOs), but accepts a broader range of collateral at the discount window, including domestic and foreign securities and loans. The BoE has two lending facilities, and accepts only high-quality sovereign debt as collateral for its facility with the monetary policy role, while it accepts a broader set of investment-grade securities and whole loans for its facility with the financial stability role. The BoJ and ECB maintain

[23] For example, when private agents cannot lend against the entire portfolio, but only a fraction of it, they worry that the borrower will pledge the assets with lower quality. Hence, they may shy away from lending, whereas an agent with sufficient financial capacity, like the central bank, can lend against the entire portfolio.

[24] Singh (2013) argues that the unconventional monetary policies pursued by central banks that remove good collateral from markets to their balance sheet, together with demands from new regulations, significantly changed the collateral landscape.

[25] As the crisis of 2007-08 spread beyond banks, the Fed created temporary lending facilities for non-banks such as securities broker-dealers but the Dodd-Frank Act constrains its ability to invoke these special authorities.

a single collateral list for their OMOs and lending facilities, and accept a range of collateral including investment-grade fixed income securities and loans to the private and public sector.

The expansion of the list of acceptable assets in central banks' collateral policies is broadly in line with the insight from our model that it is not optimal to restrict central banks to accept only high-quality collateral. However, in contrast to the insights of our model existing collateral policies are not dependent on the liquidity conditions in the interbank market or advocate for a prioritization of some assets over others.

Finally, in recognition of the liquidity problems that emerged during the crisis, regulators introduced liquidity requirements, which became part of bank regulation worldwide. In particular, Basel III introduced two requirements: the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR).[26] The LCR, which is already in full implementation, requires banks to hold a sufficient reserve of high-quality liquid assets (HQLA) to allow them to survive a period of significant liquidity stress lasting 30 days. We abstained from the potential effects of liquidity requirements in the current paper. However, given that liquidity requirements put more constraints on bank liquidity and increase their demand for HQLA, they will likely amplify the effects of central bank collateral policies we obtain in our paper. Furthermore, given that banks are more likely to reach the liquidity threshold during stress periods, the liquidity regulation will likely result in more disruptions and amplification effects unless banks are able to utilize their liquidity buffers on these occasions. Regulators seem to recognize the potential adverse affects of the new regulation and allow banks to run down their liquid asset buffers during stress periods (see BIS (2013), paragraphs 17 and 18). However, the potential stigma associated with that utilization may limit the effectiveness of this clause.

5 Conclusion

In this paper, we provided a macroprudential approach to the analysis of optimal lending and collateral policy for the lender of last resort, an issue that dates back to Thornton (1802)

[26] For more details, see Basel Committee for Bank Supervision (2010, 2013).

and Bagehot (1873). The main idea is that the central bank policy imposes externalities by affecting the pool of collateral and liquidity creation in markets. By lending to individual banks, the central bank insures them against liquidity risk and spurs investment. While lending against high-quality collateral has the advantage of protecting the central bank against potential losses, it has an adverse impact on the pool of collateral in the markets, which can offset the positive effect generated by banks borrowing from the central bank. Furthermore, when the pool of collateral deteriorates sufficiently, it can trigger a freeze in the very markets the central bank tries to ensure functionality.

On the other hand, lending against low-quality collateral can expose the central bank to potential losses, but it improves the pool of outstanding collateral. This reduces liquidity risk for banks that borrow in the markets and has a positive effect on output. Further, when markets are frozen, by lending to a sufficiently large number of banks against low-quality collateral, the central bank can jump start investment. This requires the central bank to go beyond the marginal cost-benefit analysis of a microprudential approach and consider a macroprudential approach, taking into account the externalities that central bank lending imposes on markets and the discontinuous effects this lending may engender. While we limit our focus to central banks' LoLR policy, this framework can be used more generally to consider, for example, the implications for the implementation of monetary policy.

We abstained from considering potential moral hazard problems induced by LoLR policies. For example, if the central bank accepts low-quality collateral, it could create incentives for banks to generate and hold low-quality collateral. The central bank could address this problem by using penalty rates. However, ex-post, the penalty rate could lead to another type of moral hazard since it reduces banks' return conditional on success when banks have to exert costly effort to have a high return from their projects. This is an important topic for future research in the design of the framework for LoLR actions of central banks.

Finally, our theory provides some novel insights for empirical research. Previous empirical studies of central banks' LoLR policies mostly examine "direct" effects, that is, the impact on

banks that borrow from the central bank when the collateral policies change (e.g., Crosignani et al. (2019) and Drechsler et al. (2016)). Our model focuses, instead, on the "indirect" effects, that is, the impact on market liquidity of central banks' collateral policies. To test these effects, one could examine how market liquidity responds as the central bank changes its collateral framework, or takes out significant amount of some collateral assets. Certain jurisdictions like the European Union and the U.S. provide interesting opportunities to analyze these issues (Bank for International Settlements (2013), Bindseil et al. (2017)). For example, in the context of the GCF repo, it would be interesting to find out whether the Fed's decision to take out significant amount of Treasuries impacted the borrowing capacity and liquidity in general repo markets. ECB's changes in collateral policies over the years also provide opportunities to investigate the insights of our model.[27]

[27]The changes sometimes imposed differential impacts across the EU countries by virtue of differences in the established policies in these countries. For instance, the ECB moved to a single list of collateral during the period of 2005-2007. As a result, equities stopped being eligible as collateral in Spain, Netherlands, or Portugal, while credit claims that had been eligible only in certain jurisdictions became eligible throughout the euro area. See Bindseil et al. (2017) for more details.

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Online Appendix for A Theory of Collateral for the LoLR

Dong Beom Choi, João Santos, and Tanju Yorulmazer

A: Intermediation chain and collateral circulation

In this section, we extend the intermediation chain, where we allow for the collateral to circulate among banks multiple times and show that the effects we obtain in our benchmark setup become amplified.

We extend the benchmark model and slightly modify our interpretation of the probability ρ as we explain below. In particular, we divide the intermediate period $t = 1$ into N equal subperiods denoted by $n = 0, 1, \ldots N - 1$ with more than N subgroups of B banks, where we denote the B banks that lent to A banks at $t = 0$ as B_0. In the benchmark model, B banks that lent to A banks at $t = 0$, denoted as B_0 banks now, received a liquidity shock with probability ρ at $t = 1$ and needed to borrow from outsiders. In this extended setup, at $n = 0$, B_0 banks get hit by a liquidity shock $\ell_0 \sim U[0, 1]$ with probability 1 and search for a B_1 bank to borrow from in the interbank market, where B_1 banks are endowed with one unit of cash that they can lend to B_0 banks. However, with probability ρ they cannot find a match and need to borrow from outsiders. Since they can borrow up to c_1 from outsiders, with probability $\rho(1-c_1)$ they cannot satisfy their liquidity need and call back the loan, which leads to an early liquidation of A banks' projects. Note that, as is the case in the benchmark setup, while collateral is not necessary in the interbank market, B_1 banks still take collateral from B_0 banks in anticipation of their own liquidity shock and the need to borrow from outsiders, which results in another round of collateral circulation.

The model continues in the same fashion, where B_n banks, who lent to B_{n-1} banks in subperiod $n - 1$ if there were successful matches until then, are hit by a liquidity shock $\ell_n \sim U[0, 1]$ with probability 1 in subperiod n. They search for a B_{n+1} bank to borrow from in the interbank market using the collateral they received from a B_{n-1} bank, but with

probability ρ, they are not matched with one and need to borrow from outsiders. If they cannot satisfy their liquidity needs, B_n banks call back the loan they gave to B_{n-1} banks in subperiod $n-1$, which starts a chain of loans being called back so that, ultimately, A banks' projects get liquidated. Note that if a B_n bank is not matched with a B_{n+1} bank and needs to borrow from the outsiders, the intermediation chain breaks at subperiod n and, from then on, all the B banks in subgroups $m > n$ are self sufficient since they have enough cash at subperiod m to satisfy their own liquidity needs ℓ_m. Figure 7 summarizes the flows between agents.

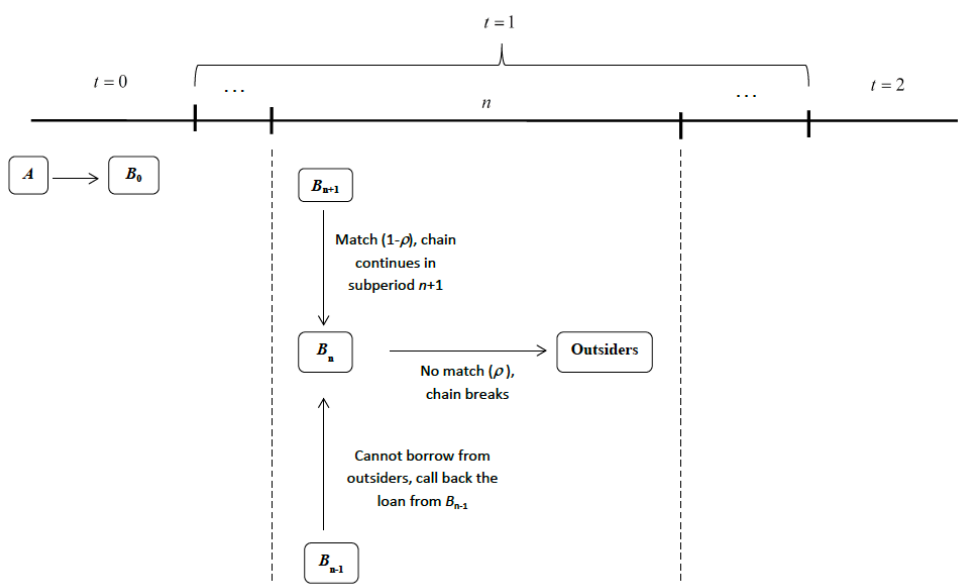

Figure 7: Flows between agents at subperiod n of $t = 1$.

With this extended intermediation chain, we can derive the probability of A banks' projects being liquidated as follows. At $n = 0$, the probability that B_0 banks cannot satisfy their liquidity need is $\rho(1 - c_1)$. The probability that B_0 borrows from B_1 but B_1 cannot satisfy its liquidity need at $n = 1$ is $(1 - \rho)\rho(1 - c_1)$. Similarly, we can easily show that the ex-ante likelihood of (i) B_k banks' successfully borrowing from B_{k+1} banks for all $k \in \{0, ..., n-1\}$; but (ii) B_n banks' failure to satisfy their liquidity need, is equal to $(1-\rho)^n\rho(1-c_1)$.[28] Hence,

[28]Note that there are more subgroups of B banks than N so that the B_{N-1} banks have other B banks

as of $t = 0$, the probability of an early liquidation of A banks' project can be written as

$$\rho(1-c_1)\sum_{n=0}^{N-1}(1-\rho)^n = (1-c_1)\left(1-(1-\rho)^N\right).$$

Note that as the intermediation chain gets longer, the probability of early liquidation increases, and this probability converges to $1 - c_1$ as $N \to \infty$.

A banks will not invest when $R - 1 < (1 - c_1)\left(1 - (1 - \rho)^N\right)(R - r)$, that is, when

$$\rho > \rho_N^* = 1 - \left(1 - \frac{R-1}{(1-c_1)(R-r)}\right)^{1/N}.$$

Note that as the chain gets longer, the threshold ρ_N^* decreases, that is, $\frac{d\rho_N^*}{dN} < 0$. Hence, longer chains result in higher expected liquidation costs if A banks invest and, therefore, A banks bypass investment for a larger set of parameter values.

Next, we show that as the chain gets longer, the effect of central bank actions on output gets amplified. Suppose that the central bank lends to a measure x_H of A banks with high-quality collateral so that the average quality of collateral in the market will be c_1^H. We focus on the marginal effect.[29] In that case, output can be written as

$$Y_H = (R-1) - (1-x_H)\left[\left(1-(1-\rho)^N\right)(1-c_1^H)(R-r)\Delta\right] + NR^B. \tag{7}$$

We thus have

$$\frac{\partial Y_H}{\partial x_H} = \left(1-(1-\rho)^N\right)(R-r)\Delta(1-x_H)(1-c_H).$$

Note that $\frac{\partial Y_H}{\partial x_H} \leq 0$, since $c_H \geq 1$ as in the benchmark case. Furthermore, the negative effect $\frac{\partial Y_H}{\partial x_H}$ is stronger for longer chains. Similarly, when the central bank lends against low-quality

they can be matched with in subperiod $N - 1$. Since the B banks receive the long-term return R^B from their existing projects at $t = 2$, the B_N banks that lent to B_{N-1} banks always have enough cash to satisfy their own liquidity needs and loans are never called back after subperiod $N - 1$.

[29] We can show similar amplification effects when we analyze discontinuous effects such as a jump start or a market freeze.

collateral to a measure x_L of banks, we obtain

$$\frac{\partial Y_L}{\partial x_L} = \left(1 - (1-\rho)^N\right)(R-r)\Delta(1-x_L)(1-c_L) > 0.$$

Again, the effect $\frac{\partial Y_L}{\partial x_L}$ gets larger as the chain gets longer.

B: Certification

An alternative policy for the central bank would be to certify collateral instead of lending directly to banks. This policy would eliminate counterparty risk for the central bank. However, the banks that are certified to have low quality collateral would experience higher liquidity risk and may not even be able to invest. Hence, whether certification would be a better policy than lending directly to banks would depend on the trade-off between reduced counterparty risk for the central bank and the increased liquidity risk and the potential forgone investment by banks with low quality collateral.

Note that certifying x_j units of collateral of type j will have the same effect on the composition of the remaining collateral in the market as lending to x_j units of A banks with collateral of type j. Hence, certifying high quality collateral would be equivalent to lending against high quality collateral since the central bank is not exposed to any counterparty risk in both cases. However, certifying low quality collateral and lending against low quality collateral can have different effects. We analyze this next.

When an A bank is certified to have low quality collateral, it will not invest when

$$\rho > \rho_L^* = \frac{R-1}{(1-c_L)(R-r)}.$$

Suppose $\rho > \rho^*$ so that no bank is investing. By certifying low quality collateral, the central bank can improve the average quality of collateral for the uncertified banks and can jump start investment. In particular, when the central bank certifies x_L units of low quality collateral with $x_L \geqslant x_L'$, this would restore investment by the remaining $1 - x_L$ banks and

output can be written as:

$$Y_L^C = (1 - x_L)\left[(R-1) - \rho(1 - c_L^C)(R - r)\Delta\right] + R^B,$$

where the average quality of collateral for the uncertified banks can be written as

$$c_L^C = \frac{\alpha c_H + (1 - \alpha - x_L)c_L}{1 - x_L}.$$

We obtain

$$\frac{\partial Y_L^C}{\partial x_L} = \rho(R - r)\Delta(1 - x_L)(1 - c_L) - (R - 1).$$

Hence, the central bank certifies x_L^C units of low quality collateral given as

$$x_L^C = \begin{cases} 1 - \alpha & \text{for} \quad \rho(R - r)\Delta\alpha(1 - c_L) > (R - 1) \\ x_L' & \text{for} \quad \rho(R - r)\Delta(1 - x_L')(1 - c_L) < (R - 1) \\ 1 - \frac{R-1}{\rho(R-r)\Delta(1-c_L)} & \text{otherwise} \end{cases}.$$

Whether certifying low quality collateral will be a better policy than lending directly to banks with low quality collateral would depend on the trade-off between reduced counterparty risk for the central bank and the forgone investment by banks with low quality collateral. In particular, when the central bank lends to x_L units of banks with low quality collateral, it can jump start investment so that all banks invest but the central bank suffers a cost of $f(z)$ with $z = x_L(1-p)(1-c_L)$ from counterparty risk. On the other hand, when the central bank certifies x_L^C units of low quality collateral, it does not suffer any cost from counterparty risk but the x_L^C units of banks that are certified to have low quality collateral cannot invest resulting in a loss of output of $x_L^C(R-1)$. Whether lending directly to banks with low quality collateral or certifying low quality collateral is a better policy depends on the comparison between the associated costs $f(z)$, with $z = x_L(1-p)(1-c_L)$, and $x_L^C(R-1)$. Conservative central banks

with a more steep $f(.)$ would be more averse to counterparty risk and certification can be a better policy than lending directly to banks for them.

Now, suppose, $\rho \leqslant \rho^*$ so that banks are already investing. Recall that it is never optimal to lend against high quality collateral (and therefore certify high quality collateral) in this case. We have two cases.

Case 1: For $\rho \leqslant \rho_L^*$, even banks that are certified to have low quality collateral will continue investing. The liquidity risk for banks that are certified to have low quality collateral is $\rho(1-c_L)(R-r)$, whereas the liquidity risk for uncertified banks decrease. Hence, the output when the central bank certifies x_L units of low quality collateral can be written as:

$$Y_L^C = x_L\left[(R-1) - \rho(1-c_L)(R-r)\Delta\right] + (1-x_L)\left[(R-1) - \rho(1-c_L^C)(R-r)\Delta\right] + R^B,$$

This gives us:

$$\begin{aligned}\frac{\partial Y_L^C}{\partial x_L} &= (R-1) - \rho(1-c_L)(R-r)\Delta + \rho(R-r)\Delta(1-x_L)(1-c_L) - (R-1) \\ &= -\rho(1-c_L)(R-r)\Delta x_L < 0.\end{aligned}$$

Hence, the overall effect is a reduction in output so that certification would not be a desirable policy in this case. In contrast, as we showed, it can be desirable to lend to (some) banks with low quality collateral.

Case 2: For $\rho_L^* < \rho \leqslant \rho^*$, while the liquidity risk for uncertified banks gets reduced, banks that are certified to have low quality collateral stop investing. Output can be written as:

$$Y_L^C = (1-x_L)\left[(R-1) - \rho(1-c_L^C)(R-r)\Delta\right] + R^B,$$

so that

$$\frac{\partial Y_L^C}{\partial x_L} = \rho(R-r)\Delta(1-x_L)(1-c_L) - (R-1).$$

Hence, the central bank certifies x_L^C units of low quality collateral given as

$$x_L^C = \begin{cases} 1-\alpha & \text{for} \quad \rho(R-r)\Delta\alpha(1-c_L) > (R-1) \\ 0 & \text{for} \quad \rho(R-r)\Delta(1-c_L) < (R-1) \\ 1-\frac{R-1}{\rho(R-r)\Delta(1-c_L)} & \text{otherwise} \end{cases}.$$

Hence, for $\rho(R-r)\Delta(1-c_L) < (R-1)$, it is never optimal to certify low quality collateral, while it can still be optimal to lend against low quality collateral.

In sum, there are cases where certification would be harmful and whether it is a better policy than lending directly to banks depends on the costs associated with counterparty risk for the central bank and the forgone investment by banks that are certified to have low quality collateral.

C: Cash-in-the-market pricing

We relax our assumption of the fixed liquidation value r, and analyze the effects of cash-in-the-market pricing.[30] Suppose that the liquidation value decreases as more projects are liquidated, that is, r is increasing in the measure x_j of banks to which the central bank lends to so that $\partial r/\partial x_j > 0$. This is because banks that borrow from the central bank do not experience any liquidity risk and their projects can be held until maturity at $t = 2$.

Suppose that $\rho \leqslant \rho^*$ so that all A banks invest and the central bank lends to a measure x_L of A banks with low-quality collateral. In that case, using the output given in equation (3), we obtain

$$\frac{\partial Y_L}{\partial x_L} = \rho\Delta(R-r)\left((1-c_1^L) + (1-x_L)\frac{\partial c_1^L}{\partial x_L}\right) + \underbrace{\rho\Delta(1-x_L)(1-c_1^L)\frac{\partial r}{\partial x_L}}_{\text{price effect } (>0)}.$$

Note that the second term, which has a positive sign, results from the effect central bank

[30] See Shleifer and Vishny (1992), Allen and Gale (1994, 1998), and Choi, Eisenbach and Yorulmazer (2016), to cite only a few.

lending has on the liquidation value r, since fewer projects need to be liquidated in the secondary market now.[31] Hence, central bank lending has an additional boosting effect on output through improved prices in the secondary market.

Furthermore, central bank lending can decrease the investment threshold ρ_L^* through improved prices in the secondary market. In particular, we have $\rho_L^* = \frac{R-1}{\left(1-c_1^L\right)(R-r(x_L))}$, so that

$$\frac{\partial \rho_L^*}{\partial x_L} = \frac{1}{\left(1-c_1^L\right)(R-r)} \left(\frac{\partial c_1^L / \partial x_L}{1-c_1^L} + \underbrace{\frac{\partial r / \partial x_L}{R-r}}_{\text{price effect } (>0)} \right) > 0.$$

Note that the second term in the parentheses has a positive sign. Hence, central bank lending can have an additional effect on output through improved prices in the secondary market even when A banks were not investing in the first place, that is, in addition to the output $R-1$ generated by the A banks with low-quality collateral that borrow from the central bank, the threshold for the probability of the liquidity shock ρ_L^* below which A banks invest increases, which makes investment more likely.

D: Information on collateral quality

Here, we allow endogenous generation of information by the holders of collateral. Suppose that the true quality of collateral would be revealed to the outsiders only if information that can be credibly conveyed to them is generated, as in Gorton and Ordoñez (2014, 2017); otherwise, outsiders would simply attribute the average quality c_1 to any collateral at $t=1$ as in our original setup. In particular, at $t=0$, A banks can generate information about their collateral type by incurring a cost of δ.[32] Note that banks will never generate information when $\delta > R-1$. Next, we analyze the case with $\delta \leqslant R-1$.

[31] We can show that when the central bank lends against high-quality collateral, the price effect on output and ρ^* would be equal to $\rho\Delta(1-x_H)(1-c_1^H)\frac{\partial r}{\partial x_H}$ and $\frac{\partial r/\partial x_H}{\left(1-c_1^H\right)(R-r)^2}$, respectively.

[32] Note that information generation can take time and cannot be done immediately when facing a run at $t=1$. Furthermore, B banks would not have any incentive to generate information, since they can always satisfy their liquidity need by calling back the loan from A banks.

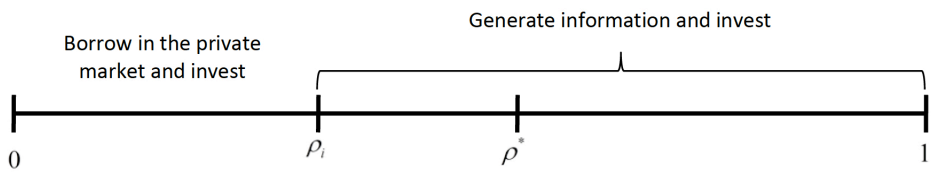

Figure 8: A banks with high-quality collateral generate information and invest when $\rho \geq \rho_i$.

An A bank with low-quality collateral will never generate information since it would only be worse off by doing so, and an A bank with high-quality collateral will incur the cost of δ to separate itself from the A banks with low-quality collateral if the benefit from lowering liquidity risk is greater than the cost of generating information. Once perceived as a borrower with high-quality collateral, the A bank can eliminate liquidity risk altogether since $c_H \geq 1$. Therefore, A banks with high-quality collateral will choose to generate information when $\rho\,(1 - c_1)\,(R - r) \geq \delta$, which can also be written as

$$\rho > \frac{\delta}{(1 - c_1)(R - r)} \equiv \rho_i. \tag{8}$$

Note that $\rho_i < \rho^*$ with $R - 1 > \delta$, which gives us the following proposition (see Figure 8).

Proposition 4 (Information generation): *For $R - 1 < \delta$, A banks will not generate information. For $R - 1 \geqslant \delta$, we have two cases: For $\rho < \rho_i$, no information is generated and A banks with both types of collateral are pooled and invest. For $\rho > \rho_i$, A banks with high-quality collateral will generate information to separate themselves.*

Hence, information generation arises endogenously as the probability ρ gets higher or the average quality c_1 of collateral deteriorates. Note that when information is generated, the liquidity risk for the A banks with low-quality collateral increases, since their collateral is no longer pooled with high-quality collateral, and these banks might not invest at all if their

liquidity risk becomes sufficiently high.

This result implies that a (partial) market breakdown can arise even when information is generated endogenously. Unless information is generated, all of the previous results on marginal central bank lending (Proposition 2) and on regime switch (Proposition 3) go through in a similar way. Note that the information generation threshold ρ_i is increasing in c_1; hence, when the central bank lends against high-quality collateral, which would lower the average quality of collateral to c_1^H and, therefore, lower ρ_i, could force banks with high-quality collateral to generate and convey information about their collateral. Hence, central bank lending can trigger information generation by banks with high-quality collateral, which, in turn, can result in a freeze in an otherwise functioning market, with no investment by banks with low-quality collateral. On the other hand, lending against low-quality collateral can improve the overall quality of collateral in the market and can reduce incentives for information generation.

E: Relations in the interbank market

In our model, we assume that lending in the interbank market can be enforced, while it is not the case in the anonymous money market and thus outsiders only lend against collateral. This could arise, for instance, if relations in the interbank market matter and outsiders do not have a relationship with banks. In this section, we discuss relationships in the interbank market and why banks have relationships with a limited number of banks.

Relationships in the interbank market can arise from "tiering," whereby some banks (tier-1 banks) access central bank liquidity and act as clearing banks for other banks (tier-2 banks). Even in large interbank markets, such tiering exists, and hence, issues of market power remain important. For example, in the U.S. federal funds market, JPMC and Bank of America are much bigger borrowers than others, and State Street and JPMC are much bigger lenders. Furthermore, many banks are connected with only one or two banks, and the average number of connections is between three and four (Bech and Atalay 2010). The UK also has a tiered system, and the volatility induced in interbank lending rates due to the cornering of collateral

and liquidity by some of the large settlement banks during 2001–2005 was one of the main rationales for the Sterling Money Market Reform in 2006. Post-reform, the Bank of England increased the number of banks allowed to participate in open market operations from 10 to more than 35 (Bank of England 2005 and Tucker 2004).

Another important feature is that lending and borrowing in the interbank market make peer monitoring among banks important (see Rochet and Tirole 1996 and Freixas and Holthausen 2005). Such monitoring can create information monopolies in interbank markets. Cocco, Gomes, and Martins (2009) report evidence of strong relationships in the Portuguese interbank market, suggesting that some banks are more important lenders and are pivotal, even in normal times. Furthermore, smaller banks, with limited access to foreign interbank markets, concentrate all of their borrowing in only a few large banks in the domestic interbank markets. They also highlight the bilateral nature of interbank lending: most of the lending volume is accounted for by "direct" loans in which loan amount and interest rate are agreed to on a one-to-one basis between the borrower and the lender, whereas other banks do not necessarily have access to the same terms. Lastly, such relationships become more important during stress periods, which is the main focus of our paper.

F: Probability distribution for liquidity shocks

In the benchmark model, we assumed that the liquidity shock ℓ has a uniform distribution. In this section, we relax this assumption. Suppose that the liquidity shock ℓ has a probability density function $g(.)$, and the c.d.f. $G(.)$. In that case, the probability of an early liquidation will be $\rho(1 - G(c_1))$, as opposed to $\rho(1 - c_1)$ for the case with the uniform distribution and the results in Propositions 1 and 2 follow immediately with this modification.

Next, we analyze the "marginal" effect of central bank lending on output as characterized in Proposition 2. In this new setup, when the central bank lends against low-quality collateral

to a measure x_L of banks, we have

$$Y_L = (R-1) - (1-x_L)\left[\rho(1-G(c_1^L))(R-r)\Delta\right] + R^B,$$

so that

$$\frac{\partial Y_L}{\partial x_L} = \rho(R-r)\Delta\left[(1-G\left(c_1^L\right)) + g\left(c_1^L\right)\frac{\alpha\left(c_H-c_L\right)}{1-x_L}\right] > 0.$$

Note that when the central bank lends against low-quality collateral, it eliminates the liquidity risk for the x_L banks it lends to and, furthermore, the quality of collateral in the market improves so that the liquidity risk for the remaining banks are mitigated. These two effects go in the same direction and increase output. Note that this result is independent of the probability distribution $g(.)$.

Similarly, when the central bank lends against high-quality collateral to a measure x_H of banks, we have

$$\frac{\partial Y_H}{\partial x_H} = \rho(R-r)\Delta\left[(1-G\left(c_1^H\right)) - g\left(c_1^H\right)\frac{(1-\alpha)\left(c_H-c_L\right)}{1-x_H}\right].$$

We can show that $\frac{\partial Y_H}{\partial x_H} \leqslant 0$ when

$$\frac{g\left(c_1^H\right)}{1-G\left(c_1^H\right)} \geqslant \frac{1-x_H}{(1-\alpha)\left(c_H-c_L\right)}. \tag{9}$$

When the central bank lends against high-quality collateral, it eliminates the liquidity risk for the x_H banks it lends to. However, the quality of collateral in the market deteriorates so that the liquidity risk for the remaining banks increases. The negative effect dominates the positive effect when the condition in (9) is satisfied, that is, when the value of the probability density $g\left(c_1^H\right)$ is sufficiently large compared with the probability of early liquidation $(1-G\left(c_1^H\right))$, in which case a small change in the overall quality of collateral affects liquidity risk significantly.

G: LoLR capacity

In the benchmark model, we assume that the central bank could arbitrarily limit the capacity of LoLR lending to x_j. Since we do not have penalty rates or stigma associated with borrowing from the central bank, we have excess demand for central bank funding. We assumed that central bank funds would be assigned randomly or on a first-come-first-served basis. Nonetheless, the central bank can achieve the desired allocation through a market clearing rate or via an auction mechanism if banks are heterogeneous. To do that, the central bank charges a rate $\gamma_j \geqslant 0$, when it lends against collateral type $j \in \{H, L\}$, over the fair rate the banks are paying in the benchmark case. We can interpret γ_j as a penalty rate.

Suppose that the long-term return of the A banks, denoted by R, is heterogeneous, such that $R \sim [R - \epsilon, R + \epsilon]$ for all A banks with high- and low-quality collateral. Since there is no adverse selection with respect to LoLR lending,[33] the marginal borrower from the central bank is the bank with the return R^* so that its benefit $\rho(1 - c_1)(R^* - r)$ from LoLR access is just equal to the penalty rate γ_j. Any bank with $R > R^*$ will borrow from the central bank since the benefit is greater for more productive banks.

This implies that starting from a sufficiently high penalty rate, the LoLR will attract only the most productive banks, and, by lowering the penalty rate slightly, it would attract the next set of productive banks. Suppose that the central bank can maximize output by lending to x_j of most productive A banks with collateral type $j = H, L$. Hence, the central bank would like to limit its lending capacity to x_j, and would also aim to clear the demand for its funds by setting γ_j so as to make the demand equal x_j. This is possible by choosing the right penalty rate, since the demand for central bank funds decreases monotonically in γ_j. In other words, as the cost of accessing the central bank funds increases, only the productive banks whose benefits from liquidity insurance are large enough will borrow from the central bank; and the central bank can achieve the desired allocation by choosing the penalty rate to clear

[33] Note that all of the borrowers would need to pay $1 + \gamma_j$ in expectation, unlike in the credit rationing model in Stiglitz and Weiss (1981).

the market.

Note that with the heterogeneous productivity of A banks, the same allocation could be achieved through an auction even without the adjustment of the penalty rate. Again, the more productive banks would bid more for central bank funds, and given the limit x_j of the facility, the marginal bank should bid the rate $\gamma_j = \rho(1 - c_1)(R^* - r)$.

H: Ex-post central bank lending

Throughout the paper, we assumed that the central bank lends at $t = 0$ after the probability ρ is realized, instead of lending at $t = 1$ after the liquidity shock ℓ. We chose this timing so that we could examine the effects of central bank lending on the functionality of markets going forward. Furthermore, as is clear from our multi-period extension in Section 4.1, in a dynamic setting, the distinction between ex-ante and ex-post lending becomes blurred, since any action by the central bank will have an effect in future periods. Nevertheless, we now discuss lending by the central bank at $t = 1$ and show that our main results would go through.

Suppose that the central bank announces its lending policy at the beginning of $t = 0$, but lends at $t = 1$ after B banks' liquidity shock ℓ is realized.[34] When B banks call back the loan from A banks, A banks get back the collateral they pledged to B banks at $t = 0$ and use it to borrow from the central bank.

We first start with the case where all B banks are hit by the same liquidity shock ℓ as in the benchmark case. This way, we do not need to consider interbank lending among B banks at $t = 1$. We analyze the case with idiosyncratic liquidity shocks later on.

We focus on the case where A banks invest at $t = 0$, since, otherwise, there are no projects to be liquidated at $t = 1$ and central bank lending would be irrelevant. We focus on the marginal effect first. We show that when the central bank lends against low-quality collateral, the effect on output is the same as in the benchmark case. However, when the central bank

[34] As we discussed, when B banks call back the loan from A banks, they are guaranteed to be paid in full. We assume that when B banks have the same payoff, they prefer to call back the loan from A banks rather than borrowing from the central bank (e.g. due to stigma).

lends against high-quality collateral, banks borrow from the central bank only when markets are frozen. Therefore, even though central bank lending lowers the quality of collateral in the markets, it does not lead to any actual effect on output, since the market is already frozen and only banks that borrow from the central bank invest.

We begin with the case where the central bank lends to a measure x_L of banks against low-quality collateral. In this case, the average quality of collateral in the market is c_1^L as in the benchmark case.

Note that with probability $1-\rho$, there is no liquidity shock. However, with probability ρ, B banks experience a liquidity shock and need to go to the outsiders for funds. For $\ell < c_0$, B banks can borrow from outsiders so that there is no early liquidation. For $c_0 < \ell < c_1^L$, a measure x_L of banks borrow from the central bank, and the rest can borrow from outsiders. However, for $\ell > c_1^L$, while a measure x_L of banks borrow from the central bank, the rest cannot borrow from outsiders so that A banks' projects are liquidated. Hence, output can be written as

$$Y_L = x_L(R-1) + (1-x_L)\left[(R-1) - \rho(1-c_1^L)(R-r)\Delta\right] + R^B,$$

the same as in equation (3) in our benchmark case.

Next, we analyze the case when the central bank lends against high-quality collateral. When the central bank lends to a measure x_H of banks against high-quality collateral, the average quality of collateral in the market becomes c_1^H as in the benchmark case. Note that $c_1^H < c_0$, since the quality of collateral in the market deteriorates.

With probability $1-\rho$, there is no liquidity shock. However, with probability ρ, B banks are hit by the liquidity shock and need to borrow from outsiders. In that case, for $\ell < c_0$, B banks can borrow from outsiders and there is no early liquidation. For $\ell > c_0$, a measure x_H of banks with high-quality collateral borrow from the central bank but the rest cannot borrow from outsiders, since $c_1^H < c_0$ as central bank lending leads to a deterioration in the quality

of collateral in the market.

In other words, for $\ell < c_0$, B banks can borrow from outsiders, they do not call back the loan, and no one needs to borrow from the central bank. However, for $\ell > c_0$, B banks cannot borrow from outsiders; that is, the market is frozen already and the central bank, by lending against high-quality collateral, only lowers the quality of collateral in the market without having any actual effect on output.

Hence, in this case, the central bank lends against high-quality collateral only when markets are frozen and we would only observe the direct effect this has on banks that borrow from the central bank, similar to the policy suggested by Bagehot as in our benchmark case.

Idiosyncratic liquidity shocks

Suppose that B banks experience idiosyncratic liquidity shocks at $t = 1$.[35] We analyze the marginal effects when all A banks are investing at $t = 0$. We first analyze the case where the central bank lends against low-quality collateral and then the case where it lends against high-quality collateral.

When the central bank lends to a measure x_L of banks with low-quality collateral, the average quality of collateral in the market becomes c_1^L as before. Now, suppose that the central bank lends to the banks that have difficulty in borrowing in the market, that is, the banks with a liquidity shock $\ell > c_1^L$. Again, with probability $1 - \rho$, there is no liquidity shock. However, with probability ρ, B banks experience a liquidity shock and for $\ell \leqslant c_1^L$ they can borrow from outsiders. However, B banks with high liquidity shocks $\ell > c_1^L$ cannot borrow from outsiders and a measure x_L of them borrow from the central bank. Hence, a measure $(1 - c_1^L - x_L)$ cannot borrow from the central bank or from outsiders, and they call back the loan, leading to an early liquidation of A banks' projects. In this case, output can be written

[35] Note that B banks with low liquidity shocks have spare debt capacity. They can borrow from outsiders and lend the excess liquidity they have to B banks with high liquidity shocks. This would not change our results qualitatively. To keep the analysis simple, we do not focus on this case.

as

$$Y_L = (R-1) - \rho\left[1 - c_1^L - x_L\right](R-r)\Delta + R^B.$$

Hence, the effect of central bank lending on output can be written as:

$$\begin{aligned}\frac{\partial Y_L}{\partial x_L} &= \rho(R-r)\Delta\left[1 + \frac{\partial c_1^L}{\partial x_L}\right] \\ &= \rho(R-r)\Delta\left[1 + \frac{(1-\alpha)(c_H - c_L)}{(1-x_L)^2}\right].\end{aligned}$$

Note that both the direct effect through the banks that borrow from the central bank and the indirect effect, which works through improved collateral quality in markets, are positive, and are stronger in this case, compared to $\frac{\partial Y_L}{\partial x_L}$ for the benchmark case. In this case, the central bank lends only to those banks whose projects would otherwise be liquidated, whereas in the benchmark case where it lends at $t = 0$, the central bank ends up lending to banks that might have had a low liquidity shock $\ell \leqslant c_1^L$ and might have been able to borrow in the markets at $t = 1$. Hence, the positive effect of central bank lending is stronger in this case.

When the central bank lends to a measure x_H of banks with high-quality collateral, the average quality of collateral in the market is c_1^H as before. Now, suppose that the central bank lends to the banks with a liquidity shock $\ell > c_1^H$. Again, with probability $1-\rho$, there is no liquidity shock. However, with probability ρ, B banks experience a liquidity shock and for $\ell \leqslant c_1^L$ they can borrow from outsiders. However, B banks with high liquidity shocks $\ell > c_1^H$ cannot borrow from outsiders and a measure x_H of them borrow from the central bank. Hence, a measure $(1 - c_1^H - x_H)$ of loans are called back, leading to an early liquidation of A banks' projects. In this case, output can be written as

$$Y_H = (R-1) - \rho\left[1 - c_1^H - x_H\right](R-r)\Delta + R^B.$$

Hence, the effect of central bank lending on output can be written as:

$$\frac{\partial Y_H}{\partial x_H} = \rho(R-r)\Delta\left[1 - \frac{(1-\alpha)(c_H - c_L)}{(1-x_H)^2}\right].$$

Note that both the direct effect, which is positive, and the indirect effect, which is negative, are stronger in this case, compared to $\frac{\partial Y_H}{\partial x_H}$ for the benchmark case. Hence, the overall negative effect we received in the benchmark case is not guaranteed. However, a sufficient condition for $\frac{\partial Y_H}{\partial x_H} < 0$ is $c_H - c_L > \frac{1}{\alpha}$; that is, when the difference between the values of high- and low-quality collateral is sufficiently large, lending against high-quality collateral would have a negative effect on output, as in the benchmark case. Again, in this case, the central bank lends only to those banks whose projects would otherwise have been liquidated so that the positive effect of central bank lending is stronger.

I: Optimal LoLR policy

Here, we analyze the optimal LoLR policy taking into account the potential losses the central bank can experience from lending. Banks that borrow from the central bank pay 1 in expectation. However, the central bank can suffer losses when the projects of the banks it lends to have a low return and the collateral it received has a low value. This may be costly to the central bank, because it may affect its reputation or impair its independence.

The central bank does not face any counterparty risk when it lends against high-quality collateral, since $c_H \geqslant 1$. However, it suffers a loss of $1 - c_L$ when it lends against low-quality collateral and the bank's project has the low return 0 at $t = 2$. Hence, when it lends to a measure x_L of banks against low-quality collateral, the resulting counterparty risk exposes the central bank to expected losses of $z = x_L(1-p)(1-c_L)$. We denote the cost to the central bank from incurring these losses by $f(z)$ and assume this function is convex and increasing in z with $f(0) = 0$.

The central bank tries to maximize $W = Y - f(z)$, which is output Y minus any cost to

the central bank from its LoLR actions. Next, we analyze the central bank policies that would achieve its objective.

We start with the case $\rho \leqslant \rho^*$, where all A banks invest. In that case, while lending against high-quality collateral does not expose the central bank to any counterparty risk, it is never optimal because it will decrease output and possibly lead to a freeze in the market (Propositions 2 and 3). Hence, the central bank would be better off refraining from lending against high-quality collateral and rather leave it in the market, where it generates more liquidity and output.

When lending against low-quality collateral, the central bank should weigh the benefit of increased output against the costs arising from counterparty risk. Hence, the central bank lends to a measure x_L of A banks with low-quality collateral, where the first order condition

$$\frac{\partial Y}{\partial x_L} = f'(z)(1-p)(1-c_L), \tag{10}$$

which can also be written as $\frac{(R-r)\Delta}{1-p} = f'(z)$, is satisfied for $x_L < 1-\alpha$; or to all (no) A banks with low-quality collateral, that is, $x_L = 1-\alpha$ (0), if

$$\frac{(R-r)\Delta}{1-p} \geq (<) f'(z) \text{ for } x_L = 1-\alpha \text{ (0)}. \tag{11}$$

Next, we consider the case where $\rho > \rho^*$, such that none of the A banks invests. Lending against high-quality collateral increases output by $R-1$ per unit lent (Proposition 2) and does not expose the central bank to counterparty risk; hence, the net increase in output is $R-1$ per unit lent. Lending against low-quality collateral also increases output by $R-1$ per unit lent but exposes the central bank to counterparty risk entailing the cost $f(z)$. However, lending against low-quality collateral has the benefit of improving the quality of collateral in the market, which can lead to a switch from a no-investment regime to one where investment is restored (Proposition 3). When analyzing the optimal central bank policy, we need to take into account such discontinuous effects.

One policy for the central bank, denoted as Policy H, is to (i) lend to all A banks with high-quality collateral (i.e., $x_H = \alpha$), which would increase output by $\alpha(R-1)$, and then (ii) lend to the A banks with low-quality collateral according to the marginal cost-benefit analysis characterized by the first-order conditions given in (10) and (11). Note that when the central bank lends to all A banks with high-quality collateral, only A banks with low-quality collateral are left in the market, and the latter will invest only if they can borrow from the central bank; that is, the market is completely replaced by the central bank. This results in

$$W_H = (\alpha + x_L)(R-1) + R^B - f(z),$$

since only the banks that borrow from the central bank, a measure $(\alpha + x_L)$, invest. Under this policy, similar to the policy proposed by Bagehot, the central bank lends "freely" to all banks with high-quality collateral.

Under an alternative policy, denoted by Policy L, the central bank lends only against low-quality collateral. This would be optimal only if it leads to a regime switch by restoring investment in the interbank market, since, otherwise, incremental lending against high-quality collateral would generate the same increase in output ($R-1$ per unit lent) without exposing the central bank to any counterparty risk. Hence, Policy L with no regime switch would be dominated by Policy H. For the central bank to induce a regime shift, it should lend at least to a measure x'_L of banks with low-quality collateral. When the economy switches to the investment regime, output jumps upward by $(1-x'_L)\rho(1-c')(R-r)(1-\Delta)$, and further lending to A banks with low-quality collateral will generate a continuous effect, as in Figure 4.

The optimal capacity of this lending facility depends on the cost to the central bank. Suppose that the central bank just managed to jump start investment with $x_L = x'_L$. If

additional lending to A banks with low-quality collateral at that point is too costly, that is,

$$\frac{(R-r)\Delta}{1-p} < f'(z) \tag{12}$$

for $x_L = x'_L$, then the central bank should just restore investment by lending to x'_L of A banks with low-quality collateral but should not lend beyond x'_L. Otherwise, the central bank will lend to a measure $x_L > x'_L$ of A banks with low-quality collateral, where the first order condition

$$\frac{(R-r)\Delta}{1-p} = f'(z), \tag{13}$$

is satisfied for $x_L < 1-\alpha$; or it will lend to all A banks with low-quality collateral, that is, $x_L = 1-\alpha$ if

$$\frac{(R-r)\Delta}{1-p} \geqslant f'(z) \tag{14}$$

for $x_L = 1-\alpha$.[36] Policy L results in

$$W_L = (R-1) - (1-x_L)\rho(1-c_L^{CB})(R-r)\Delta + R^B - f(z),$$

where the first term is potential output and the second term represents the expected output loss, due to liquidity risk, for banks that borrow in the interbank market.

For $W_L > W_H$, the optimal policy would be Policy L, that is, to lend against low-quality collateral and induce a shift to an investment regime. For $W_L \leqslant W_H$, the optimal policy would be Policy H, that is, lending to all A banks with high-quality collateral, and then to (some of the) A banks with low-quality collateral.

Let x_L^* and x_L^{**} be the optimal levels of lending to A banks with low-quality collateral

[36] Recall from Proposition 2 that the high-quality collateral is better left in the private market than kept by the central bank, unless the interbank market is frozen. Hence, once the economy has switched to an investment regime, any lending against high-quality collateral is suboptimal.

under Policy H and Policy L, respectively; and let $f(z^*)$ and $f(z^{**})$ represent the resulting costs from counterparty risk under Policy H and Policy L, respectively. We have $W_L > W_H$ when

$$(1-\alpha-x_L^*)(R-1) > (1-x_L^{**})\rho(1-c_L^{CB})(R-r)\Delta + (f(z^{**})-f(z^*)), \tag{15}$$

where the LHS is the loss in output arising from the inability of banks that cannot borrow from the central bank to invest under Policy H; the first term on the RHS represents the output loss from liquidity risk for banks that borrow in the market under Policy L; and the second term represents the difference between the costs of counterparty risk under Policy L and Policy H. We summarize these results in the following proposition.

Proposition 5 *The optimal central bank policy can be characterized as follows:*

i) For $\rho \leqslant \rho^*$, it is never optimal to lend against high-quality collateral. The optimal level of lending against low-quality collateral x_L satisfies $\frac{\partial Y}{\partial x_L} = f'(z)\frac{\partial z}{\partial x_L}$; or $x_L = 1-\alpha$ (0) for $\frac{\partial Y}{\partial x_L} > (<) f'(z)\frac{\partial z}{\partial x_L}$.

ii) For $\rho > \rho^*$, when $W_L > W_H$, that is, when (15) holds, the central bank optimally lends against low-quality collateral and induces a switch to an investment regime. Otherwise, the central bank lends to all A banks with high-quality collateral, that is, $x_H = \alpha$, and to (some of) the A banks with low-quality collateral characterized by the FOCs in (13) and (14).

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