Insurance and Risk Selection when Insurable Asset and Income are Separable

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Abstract: This paper develops an endogenous selection model under asymmetric information, in which risk types are endogenously determined by individuals. By assuming heterogeneity in asset sensitivity that inheres in a two-argument utility function, we find that in equilibrium, an asset sensitive type of individual may invest in self-protection and become a low-risk, whereas an insensitive type never chooses to expend effort. Unlike the standard model of Rothschild and Stiglitz (1976), the present study demonstrates that the sensitive type (low-risk) demands more insurance than the insensitive type (high-risk) under advantageous selection. We also find other types of equilibrium such as adverse selection, separating equilibrium for a single premium rate, partial pooling equilibrium, and pooling equilibrium. In contrast to all previous papers, the equilibrium results obtained in this study reflect the reality that individuals make trade-offs between an income and an insurable asset.

Keywords: two-argument utility function, income, insurable asset, self-protection, asymmetric information, endogenous selection, advantageous selection, asset-sensitivity

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Ⅰ. Introduction

The relationship between risk-taking behavior and insurance purchase with asymmetric information has been widely studied from two perspectives: selection and moral hazard. Adverse selection and moral hazard promote inefficient risk sharing in insurance markets, whereas advantageous selection brings a favorable result. In this paper, we study the endogenous selection in which risk types are endogenously determined by individuals. Under asymmetric information, the insured is assumed to have private information about his/her preferences that are both relevant to the choice of insurance contract and the choice of action which is unobservable to the insurer. In regard to the hidden action, we are specifically concerned with the self-protection effort to reduce the loss probability, which depends on the heterogeneity of asset sensitivity. Asset sensitivity that inheres in utility function indicates how much an individual suffers disutility from a loss in an insurable asset.

The traditional model of insurance with asymmetric information predicts a positive correlation between risk and insurance coverage in equilibrium, whereas empirical tests have exhibited mixed results. The classic equilibrium models developed by Rothschild and Stiglitz (1976) indicate that insurers provide a menu of contracts to screen the insured so that high-risks choose full coverage and low-risks choose partial coverage under adverse selection. However, in some markets, such as automobile insurance (Richaudeau, 1999; Chiappori and Salanie, 2000), commercial fire insurance (Wang et al., 2009), health insurance (Cardon and Hendel, 2001; Cutler, Finkelstein, and McGarry, 2008), annuity (Finkelstein and Poterba, 2004), life insurance (Cawley and Philipson, 1999), long-term care insurance (Finkelstein and McGarry, 2006), and Medigap insurance (Fang et al., 2008), there was even negative correlation or insignificant evidence of positive correlation between insurance coverage and ex post loss.

De Meza and Webb (2001) tried to explain these inconsistent results by heterogeneous risk tolerance. They demonstrated that risk-averse individuals purchase insurance and make an effort to reduce the loss probability, whereas risk-neutral individuals neither purchase insurance nor expend effort. Moreover, Huang et al. (2010) proposed the heterogeneity in risk perception, and showed that a rational individual takes precautions to reduce the loss probability and purchase higher insurance coverage compared to an overconfident individual who will not make any effort. Similarly, Spinnewijn (2013) explained the negative correlation between risk and insurance coverage by heterogeneous beliefs about the risk. On the other hand, Seog (2009) decomposed the risk into a general risk and a specific risk, and showed that the relationship between risk and insurance coverage is not in one direction when insurers and insureds both have superior information about the risk. In this paper, however, we provide a theoretical basis for the existence of advantageous selection and other types of equilibrium by asset sensitivity that reflects an insured's preferences toward an income and an asset. The rationale for this approach is based on a two-argument utility function.

In fact, many theoretically interesting economic problems that reflect reality are developed by a two commodity model, capturing the trade-offs that individuals make between two different goods in decision making (Dardanoni, 1988; Picone, Uribe, and Wilson, 1998;

Eeckhoudt, Rey, and Schlesinger, 2007; Liu and Menegatti, 2019). In insurance literature, by contrast, economic models are typically oversimplified with a single argument utility function. Although this framework has the advantage in mathematical tractability, it may fail to solve the economic problems with more than one good. For example, even dealing with two sources of uncertainty, insurable risk and uninsurable background risk, Doherty and Schlesinger (1983) used a single argument utility function.

Rey (2003), meanwhile, introduced a two-argument utility function and further considered the variation of the marginal utility of the insurable asset with respect to the uninsurable asset. However, the analysis still ignores an important feature of an insurance contract. Lee (2007) is the first to point out this feature that insurance premium is in practice paid out of one's income, and indemnity is made against an insured asset. In this respect, even when considering a single insurable asset using the simplest model, it is plausible to assume that utility depends on income and an insurable asset. Lee (2007) demonstrated that the demand for insurance under this assumption depends largely on an income and preferences regardless of the magnitude of the loading factor. However, he only considered the optimality of full insurance in a market that is free from asymmetric information. Adding to Lee (2007), we are concerned with insurance demand and an incentive to spend on self-protection when an insured holds private information about his/her preferences toward an income and an asset.

Following Lee (2007), we adopt a two-argument utility function $U(C, A)$, where C and A respectively indicate the composite good (income) and an insurable asset or, technically, the benefit generated by the asset. Note that income and the asset are two different goods, of which consumption is not of the same dimension. This is because insurable assets such as a house, property, and health, are irreplaceable goods. In addition, income and the asset are not easily exchangeable in dollars. For example, a house is not easily and quickly converted into income, and the benefit as a shelter cannot be readily substituted by that of composite goods. Moreover, health is neither easily convertible into, nor directly replaceable by income. This realism justifies the need for a two-argument utility function.

As we assume that utility depends on an income and an asset, the decision to purchase an additional insurance coverage depends on the preferences toward these two goods. This is because the increase in insurance premium reduces an income rather than an asset, and the additional indemnity increases the consumption of the benefit generated by the asset if a loss occurs. This trade-off between income and asset can be represented by the marginal rate of substitution between an insurance premium and indemnity.

Moreover, decomposing wealth into income and the asset is especially important in the sense that individuals are heterogeneous in asset sensitivity – sensitivity to the change in the asset value, as they have their own valuation on the asset. For instance, if one places higher value on health, he or she will be more sensitive to the change in health status. In this paper, we define an asset sensitive type of individual as one who has relatively greater marginal utility with respect to the asset. Furthermore, we assume that individuals can be divided into two types according to their asset sensitivity: the sensitive type (s) and the insensitive type (t). Then, if a loss occurs, the sensitive type suffers greater disutility than the insensitive type. Therefore, individuals with the same initial income and asset but different asset sensitivity will make different choices of self-protection and insurance coverage. Our model indicates that the

sensitive type is more likely to invest in self-protection and may become a low-risk in equilibrium, whereas the insensitive type never chooses to expend effort. On the other hand, the choice of insurance coverage largely depends on the preferences toward income and asset. If the sensitive type expends effort and at the same time has more incentive to sacrifice an income to recover a loss in an asset by means of insurance, an equilibrium with advantageous selection may occur in the market. Moreover, the existence of equilibrium depends on the proportion of the sensitive type in the market. According to this proportion, separating equilibrium or pooling equilibrium can occur.

By assuming heterogeneity in asset sensitivity, we find five Nash equilibrium configurations in the insurance market. Contrary to the results reported by Rothschild and Stiglitz (1976), our model indicates that advantageous selection and even pooling equilibrium can occur. In a separating equilibrium with advantageous (adverse) selection, the sensitive type of insured invests in self-protection and demands more (less) insurance. In a partial pooling equilibrium, the insensitive type mixes between two contracts, and sensitive type chooses the contract with higher coverage out of the two contracts. In a pooling equilibrium, on the other hand, both types of insureds choose the same contract. Furthermore, the possible equilibrium includes the case, in which both types of insured do not invest in self-protection, but the sensitive type purchases higher coverage. Unlike Huang et al. (2010) and Spinnewijn (2013), who introduce irrational (optimistic) individuals into their models, we provide such results under full rationality. Most importantly, in contrast to all previous studies, we propose a model reflecting the reality, which captures the trade-offs that individuals make between an income and an asset in decision making. Moreover, this study demonstrates that, even under the equilibrium concept of Rothschild and Stiglitz's (1976), a complete risk pooling can occur in the competitive insurance market where two different types of insured exist.

The remainder of this paper is organized as follows. Section 2 outlines the model, and Section 3 describes the market equilibrium. Section 4 discusses the differentiation of our results from the literature, and Section 5 concludes the paper. Some proofs and figures are provided in the appendix.

Ⅱ. Model

Each individual is an expected utility maximizer with an endowment income of y and an insurable asset of w. Note that an income and an asset are different kinds of goods that are not easily exchangeable in dollars. An individual faces a fixed loss of $D \leq w$, and there is no uncertainty about income. A loss occurs with probability $p(\varepsilon)$, where $p'(\varepsilon) < 0$ with self-protection effort $\varepsilon \geq 0$. An individual can affect the probability of loss by spending ε at the expense of $c(\varepsilon)$ in utility units, where $c'(\varepsilon) > 0$ and $c(0) = 0$. Without loss of generality, we assume that individuals face a binary choice about whether to make an effort, i.e., $\varepsilon \in \{0, e\}$, where $e > 0$. Let us denote p_e and p_0 to be the loss probabilities with and without effort, i.e., $p(e) = p_e$ and $p(0) = p_0$, where $p_e < p_0$. Since the investment of self-protection cannot be observed by insurers, the insurers provide a menu of insurance contracts (Q, I) to screen the individuals, where Q and I denote an insurance premium and indemnity,

respectively, where $0 \leq I \leq D$.[1] We consider a competitive insurance market, in which no insurance company can make a positive profit.

Following Lee (2007), we adopt a two-argument utility function $U(C, A)$, where C and A indicate, respectively, the consumption of composite good and the benefit generated by the asset. $U(\cdot,\cdot)$ is assumed to be continuous, increasing, and concave in both arguments. Decomposing the wealth into an income and an asset is especially important in the sense that individuals are heterogeneous in asset sensitivity – sensitivity to the change in the asset value. Asset sensitivity is difficult to be expressed using a single argument utility function because asset preference is not distinguishable from income preference. It is thus important to consider a two-argument utility function. In this paper, we define the degree of asset sensitivity as the marginal utility with respect to the asset, i.e., $U_A(C, A)$, where the subscript in the utility function denotes partial derivative such as $U_A(C, A) = \partial U(C, A)/\partial A$. Individuals with relatively greater $U_A(C, A)$ will suffer *ceteris paribus* a larger disutility when a loss occurs. Thus, we regard an individual as more asset sensitive if he/she has greater $U_A(C, A)$. Now, we assume that individuals can be divided into two types according to their asset sensitivity: the sensitive type (s) and the insensitive type (t). Then, $U_A^s(C, A) > U_A^t(C, A)$, for all (C, A) in our support, where superscripts in the utility function denote the types of individuals.[2] We do not impose any restriction on U_{CA}, the cross second derivative of utility. That is, U_{CA} can have any sign, but we assume that the sign of U_{CA} is the same across the types of individuals.

In a two-argument utility framework with the univariate insurable risk in asset, we can define the Arrow-Pratt concept of risk aversion with respect to the asset.[3] Then, type s with greater $U_A(C, A)$ can be less risk averse in terms of asset. Note that asset sensitive types are not necessarily less risk averse with respect to asset, because, by definition, we need $U_A^s(C, A) > U_A^t(C, A)$ to hold only for each (C, A) in our support. However, we can generally demonstrate the cases, in which those who are more asset sensitive are less risk averse with respect to asset. Even though the form of risk aversion in the present model is similar to that in the standard model, the implication of risk aversion can be quite different. We describe this in Section 4.

Now, we consider the individual's preferences toward income and asset. As we adopt a two-argument utility function, individuals will allocate their limited resources according to their preferences by means of insurance. This is because the insurance premium and indemnity each have an effect on different arguments of the utility function. In other words, the insurance premium decreases the income and indemnity is made against an insured asset if a loss occurs.

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[1] After purchasing an insurance contract (Q, I), the individual chooses whether or not to expend effort e. Since an insurer cannot observe the individual's action, he will never provide full or over insurance at the insurance premium rate of p_e. In this study, we do not allow over insurance, regardless of the unit price of insurance, as commonly assumed in the literature.

[2] Our support is the set of (C, A), where $y - \bar{Q} \leq C \leq y$ and $w - D \leq A \leq w$. Note that $\bar{Q}$ denotes the maximum premium, i.e., $\bar{Q} = p_o D$.

[3] Note that the implication of risk aversion in a two-argument utility framework with bivariate risks can be more complicated, considering the correlation between the risks (see Courbage, 2001). In the present model, however, there exists uncertainty only in the asset, and we simply consider Arrow-Pratt concepts of relative and absolute risk aversion with respect to asset, which are represented by $-A\frac{U_{AA}}{U_A}$ and $-\frac{U_{AA}}{U_A}$, respectively.

Then, given insurance contract (Q, I), the individual of type i's preferences toward income and asset can be represented by the marginal rate of substitution between the insurance premium and indemnity, denoted by $\mathrm{MRS}^i(Q, I; p) \equiv \frac{pU_A^i(y-Q,w-D+I)}{(1-p)U_C^i(y-Q,w)+pU_C^i(y-Q,w-D+I)}$, where p is the probability of loss. $\mathrm{MRS}^i(Q, I; p)$ is a kind of MRS, which represents the preferences for an additional insurance coverage, because it captures the trade-offs between the benefit of indemnity and the cost of insurance premium.[4] As described later in this section, $\mathrm{MRS}^i(Q, I; p(\varepsilon))$ corresponds to the slope of an indifference curve of type i in the (Q, I) plane, when the insured expends effort of ε, for a given insurance contract (Q, I). However, as we do not put restrictions on U_{CA}, and U_C can differ across the types, the relationship between $\mathrm{MRS}^s(Q, I; p)$ and $\mathrm{MRS}^t(Q, I; p)$ is not clearly decisive. Thus, we further assume that $\frac{pU_A^s(y-Q,w-D+I)}{(1-p)U_C^s(y-Q,w)+pU_C^s(y-Q,w-D+I)} > \frac{pU_A^t(y-Q,w-D+I)}{(1-p)U_C^t(y-Q,w)+pU_C^t(y-Q,w-D+I)}$ for all (C, A) in our support.[5] What this assumption means is that, given the same insurance contract, type *s* is asset sensitive enough to value an additional insurance coverage more than type *t*, when each type chooses the same level of effort.

Then, let us describe the optimal level of effort for an individual of type i. For a given insurance contract (Q, I), if an individual expends effort $\varepsilon \in \{0, e\}$, the expected utility of type i can be represented by a two-argument utility function as follows:

$$EU^i(Q, I; \varepsilon) = p(\varepsilon)U^i(y - Q, w - D + I) + \big(1 - p(\varepsilon)\big)U^i(y - Q, w) - c(\varepsilon), \quad i = \{s, t\}. \tag{1}$$

Let us assume that an individual's outside option provides no insurance. Moreover, no firms will provide the contracts with an inordinately favorable premium that generates an overall negative profit. Therefore, we only consider the acceptable insurance contracts from the perspective of both insurers and the insured. Because the demand for insurance depends largely on the preferences toward income and asset, we suppose that the insurers have sufficient contract space for the voluntary participation of the insured.[6] After purchasing the acceptable insurance contract (Q, I), the individual will decide whether or not to make an effort e to

[4] MRS refers to the marginal rate of substitution between two goods, x and y. Originally in economics, $MRS(x, y) = U_x/U_y$, where $U_x = \frac{\partial U(x,y)}{\partial x}$ and $U_y = \frac{\partial U(x,y)}{\partial y}$. Similarly to the original MRS, $\mathrm{MRS}^i(Q, I; p)$ is the rate at which an individual can give up some amount of expected consumption of composite good by premium payments in exchange for the expected loss recovery in asset by receiving the indemnity, while maintaining the same level of expected utility. When full insurance is provided, $\mathrm{MRS}^i(Q, D; p) = p\frac{U_A^i(y-Q,w)}{U_C^i(y-Q,w)} = p \cdot \mathrm{MRS}(y - Q, w)$.

[5] Note that MRS is invariant to affine transformation. Given that $\mathrm{MRS}^s(Q, I; p) > \mathrm{MRS}^t(Q, I; p)$, we rule out the cases where utility function of type *s* and that of type *t* represent the same preference.

[6] For this to hold, we assume that an individual has small enough $U_C(C, A)$ relative to $U_A(C, A)$. This is because, if disutility from the decrease in income is too large after paying the insurance premium, the individual will not purchase the insurance contract. Since the optimal insurance coverage depends on MRS, full insurance is not always optimal even if the premium is actuarially fair. See the appendix for more details.

maximize her expected utility. The increase in expected utility from investment in self-protection is as follows:

$$\Delta^i(Q,I) = EU^i(Q,I;e) - EU^i(Q,I;0) = (p_e - p_0)[U^i(y-Q, w-D+I) - U^i(y-Q,w)] - c(e),\ i = \{s,t\}. \tag{2}$$

An individual of type *i* will invest in self-protection if and only if $\Delta^i(Q,I) \geq 0$. We intend to demonstrate that there may be an asset sensitivity threshold where no investment occurs. Here we exclude full coverage because an individual with full insurance has no incentive to invest in self-protection. For each acceptable partial insurance contract (Q,I), it is obvious that $\Delta^s(Q,I) > \Delta^t(Q,I)$:

$$U^s(y-Q,w) - U^s(y-Q,w-D+I) > U^t(y-Q,w) - U^t(y-Q,w-D+I). \tag{3}$$

Equation (3) follows because $U_A^s(C,A) > U_A^t(C,A)$, for all (C,A). Therefore, when we consider appropriate risk reduction technology, there exists an asset sensitivity threshold below which investment in self-protection does not occur. For simplicity, let us assume that $\Delta^t(Q,I) < 0$ for all acceptable insurance contracts (Q,I), so that type *t* will never choose to invest in self-protection.[7]

In the following section, we present the possible scenarios diagrammatically in the (Q,I) plane. In all of the figures, the *x*-axis represents the indemnity, whereas the *y*-axis signifies the insurance premium. Let us denote curve Z^i to be the locus of (Q,I) such that $\Delta^i(Q,I) = 0$. That is, $Z^i = \{(Q,I)|\Delta^i(Q,I) = 0\}$. However, we do not consider Z^t, because we assume that $\Delta^t(Q,I) < 0$. Now let us consider the shape of Z^s. First, if $U_{CA}^s > 0$, curve Z^s is a downward sloping locus that partitions the space into the lower region where type *s* invests in self-protection, and the upper region where he/she does not. Since we assume that $U_{CA}^s > 0$, it follows that $\frac{dQ}{dI} < 0$.[8] Secondly, if $U_{CA}^s = 0$, Z^s becomes the vertical straight line partitioning the space into the left region where type *s* expends effort, and the right region where he/she does not. Lastly, in the case of $U_{CA}^s < 0$, curve Z^s is an upward sloping locus

[7] As an extreme case, we will check the condition for $\Delta^t(Q,I) < 0$ to hold for all (Q,I). From Equation (2), an individual is more likely to invest in self-protection when he/she is provided with less indemnity. By contrast, the premium that increases the incentive to invest in self-protection depends on the sign of U_{CA}. To consider the insurance contract that gives the individual incentives to expend effort, one can refer to the next paragraph in the main body of our text, along with the footnote *8*, for the details on the locus of (Q,I) such that $\Delta^t(Q,I) = 0$. Now consider the contract $(Q',0)$ that maximizes the incentive to invest in self-protection. If $U_{CA} \geq 0$, $Q' = 0$, and if $U_{CA} < 0$, $Q' = p_0 D$. Then the type *t* will never expend effort no matter what contract is given, if $\Delta^t(Q',0) < 0$. That is, $\Delta^t(Q',0) = (p_e - p_0)[U^t(y-Q', w-D) - U^t(y-Q',w)] - c(e) < 0$. The above inequality holds if U_A^t is small enough and risk reduction technology is less efficient. Then, $\Delta^t(Q,I) < 0$ for all (Q,I).

[8] The slope of Z in the (Q,I) space is given by: $\frac{dQ}{dI} = \frac{U_A^i(y-Q,w-D+I)}{U_C^i(y-Q,w-D+I) - U_C^i(y-Q,w)}$.

in the (Q, I) plane. Type s invests in self-protection in the upper region of Z^s. Contrarily, he/she does not expend effort in the lower region of Z^s.

Indifference curves are drawn in (Q, I) space assuming that individuals choose optimal level of effort between 0 and e. Let us denote the indifference curves of type i as J^i, where $i = \{s, t\}$. The slope of J^i in the (Q, I) plane is given by

$$\frac{dQ}{dI} = \frac{p(\varepsilon)U_A^i(y_1,w_1)}{p(\varepsilon)U_C^i(y_1,w_1)+(1-p(\varepsilon))U_C^i(y_1,w_0)} = \frac{p(\varepsilon)U_A^i(y_1,w_1)}{E[U_C^i]} > 0,\ i = \{s, t\}. \tag{4}$$

where $y_1 = y - Q$, $w_0 = w$ and $w_1 = w - D + I$. Thus the indifference curves are increasing, and we assume that they are concave in the (Q, I) space.[9] Notice that Equation (4) coincides with $\mathrm{MRS}^i(Q, I; p(\varepsilon))$. The probability of loss for type t is always p_0, because we assume that $\Delta^t(Q, I) < 0$ for each acceptable insurance contract (Q, I). For type s, however, if $\Delta^s(Q, I) \geq 0$, he/she will expend effort, and J^s becomes flatter.[10] Thus, the indifference curves of type s are kinked where they cross Z^s. If $U_{CA}^s > 0$, the probability of loss is raised in the region above Z^s, so J^s is steeper. Likewise, if $U_{CA}^s = 0$, the indifference curves of type s are steeper in the right region of Z^s. Lastly, if $U_{CA}^s < 0$, J^s is steeper below Z^s.

Now we examine the condition for the single crossing property (SCP). In the region where type s invests in self-protection, the slope of J^s is

$$\left.\frac{dQ}{dI}\right|_{J^s,\Delta^s(Q,I)>0} = \frac{p_e U_A^s(y-Q,w-D+I)}{p_e U_C^s(y-Q,w-D+I)+(1-p_e)U_C^s(y-Q,w)}.$$

Consider the case where J^s is steeper than J^t in the relevant region where $\Delta^s(Q, I) \geq 0$:[11]

$$\frac{p_e U_A^s(y-Q,w-D+I)}{p_e U_C^s(y-Q,w-D+I)+(1-p_e)U_C^s(y-Q,w)} > \frac{p_0 U_A^t(y-Q,w-D+I)}{p_0 U_C^t(y-Q,w-D+I)+(1-p_0)U_C^t(y-Q,w)}. \tag{5}$$

Equation (5) implies that even after type s expends effort and J^s becomes flat, he/she is asset sensitive enough to have much greater U_A relative to U_C than that of type t in the relevant

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[9] It is easy to prove that the second derivative, i.e., $\frac{d^2Q}{dI^2}$, is negative if $U_{CA} \geq 0$. Thus, the indifference curves are increasing and concave if $U_{CA} \geq 0$. In the case of $U_{CA} < 0$, the sign of $\frac{d^2Q}{dI^2}$ is not determined. However, if the utility function is concave enough, or if the income and the asset are almost independent of each other, $\frac{d^2Q}{dI^2}$ is negative. For simplicity, we assume that $\frac{d^2Q}{dI^2} < 0$ when $U_{CA} < 0$, so that the indifference curves are increasing and concave, regardless of U_{CA}.

[10] It is easy to prove that $\frac{d}{dp}(\frac{dQ}{dI}) > 0$, regardless of the interaction between income and asset.

[11] The area of interest in the (Q, I) plane is the neighborhood of the zone where indifference curves with equilibrium contracts exist. Throughout this paper, we refer to this area as the relevant region.

region.[12] By contrast, in the region where $\Delta^s(Q,I) < 0$, the slope of J^s becomes steeper as the probability of loss increases; besides this, we assume that $\mathrm{MRS}^s(Q,I;p_0) > \mathrm{MRS}^t(Q,I;p_0)$. Therefore, if Equation (5) is satisfied, SCP will hold. Now, we consider the case in which J^s is flatter than J^t in the relevant region where $\Delta^s(Q,I) \geq 0$:

$$\frac{p_e U_A^s(y-Q,w-D+I)}{p_e U_C^s(y-Q,w-D+I)+(1-p_e)U_C^s(y-Q,w)} < \frac{p_0 U_A^t(y-Q,w-D+I)}{p_0 U_C^t(y-Q,w-D+I)+(1-p_0)U_C^t(y-Q,w)}.$$

Recall that $\mathrm{MRS}^s(Q,I;p_0) > \mathrm{MRS}^t(Q,I;p_0)$ in the region where $\Delta^s(Q,I) < 0$. Then, double crossing of indifference curves is obtained by combining the two parts of the region where $\Delta^s(Q,I) \geq 0$ and $\Delta^s(Q,I) < 0$. In summary, the double crossing property (DCP) holds if $\mathrm{MRS}^s(Q,I;p_e) < \mathrm{MRS}^t(Q,I;p_0)$ in the relevant region where $\Delta^s(Q,I) \geq 0$.

Ⅲ. Market equilibrium

We consider the Nash equilibrium in the insurance market in which insurers are perfectly competitive, so that the equilibrium contracts break even. In this study, we adopt the equilibrium concept of Rothschild and Stiglitz (1976): (a) Each individual chooses at most one insurance contract from the provided menu that maximizes his/her expected utility; (b) each equilibrium contract makes nonnegative profit to an insurer; and (c) there is no other insurance contract that will make a nonnegative profit. An insurer assumes that other insurers do not change contracts after new contracts are offered. In a separating equilibrium, each type prefers his/her equilibrium contract to the contracts chosen by other types (incentive constraint). In a pooling equilibrium, by contrast, only the pooling contract is offered, and everybody chooses it. In a partial pooling equilibrium, at least some of each type of the insured purchase the pooling contract, and the rest of the insured purchase other contracts. In equilibrium, an individual purchases an insurance contract that is at least as good as no insurance (participation constraint). Moreover, an individual will invest in self-protection if the investment yields nonnegative expected utility (effort incentive). In this section, we will demonstrate that five Nash equilibrium configurations can occur in this market:

(i) a separating equilibrium with advantageous selection,

(ii) a separating equilibrium with adverse selection,

(iii) a separating equilibrium for a single premium rate,

(iv) a pooling equilibrium, and

(v) a partial pooling equilibrium.

In all the figures, the zero profit offer curves corresponding to the probability of loss p_0 and p_e are depicted as P_0 and P_e, respectively. Moreover, $\bar{P}$ is the zero profit offer curve under the pooling contract with the probability of loss $\bar{p} = \theta p_e + (1-\theta)p_0$, where θ is the

[12] Note that this condition is a weaker condition of SCP, as equation (5) should be satisfied in the relevant region, i.e., the indifference curves of type *s* and *t* cross only once, or double crossing does not apply only in the relevant region.

proportion of type *s* who invest in self-protection.

Now, let us consider the following contracts. First, consider the contract denoted by $C_A = (Q^A, I^A)$ that is located at the tangency of J^t and P_0. Then, C_A maximizes the type *t*'s expected utility under the line P_0. Similarly, consider the contract denoted by $C_B = (Q^B, I^B)$ that maximizes the expected utility of type *s* under the line P_e.[13] Moreover, let us denote $C_D = (Q^D, I^D)$ and $C_{D'} = (Q^{D'}, I^{D'})$ as contracts that are located at the intersection of J^t passing through C_A and the line P_e, where $I^D > I^{D'}$. Then, the type *t* is indifferent about purchasing insurance contracts C_A, C_D, and C_D'. That is, $EU^t(Q^A, I^A; 0) = EU^t(Q^D, I^D; 0) = EU^t(Q^{D'}, I^{D'}; 0)$. Next, let contract $C_F = (Q^F, I^F)$ denote the intersection of Z^s and the indifference curve of type *t* that is tangent to the line P_0. Then, $EU^t(Q^A, I^A; 0) = EU^t(Q^F, I^F; 0)$ and $EU^s(Q^F, I^F; e) = EU^s(Q^F, I^F; 0)$. Moreover, let us denote $C_G = (Q^G, I^G)$ as the contract located at the intersection of $\bar{P}$ and Z^s. Then, $EU^s(Q^G, I^G; e) = EU^s(Q^G, I^G; 0)$. We also consider the indifference curve of type *s* that is tangent to the line P_0. Let us denote $C_H = (Q^H, I^H)$ as a contract that is located at the tangency of the indifference curve of type *s* and P_0.[14]

In addition, we denote C^{i*} as an equilibrium contract for type *i*. For example, if type *t* chooses C_A in equilibrium, we denote the equilibrium contract for type *t* as C_A^{t*}. On the other hand, if both types choose the same contract C_G in equilibrium, we denote this contract by C_G^{p*}, where the superscript *p* denotes the pooling contract.

In the first place, we suppose that the SCP holds. In a separating equilibrium, if it exists, the following Proposition holds.

Proposition 1: *[Separating equilibrium under SCP].*

(A) If $EU^s(Q^D, I^D; e) \geq EU^s(Q^D, I^D; 0)$ and $EU^t(Q^D, I^D; 0) \geq EU^t(Q^B, I^B; 0)$, then there exists separating equilibrium with advantageous selection in which type *t* chooses C_A^{t*} and type *s* chooses C_B^{s*}. In this case, both types obtain their first-best contracts.

(B) Suppose that $EU^s(Q^D, I^D; e) > EU^s(Q, I; e)$, where the contract (Q, I) is on the pooling price line $\bar{P}$. If $EU^s(Q^D, I^D; e) \geq EU^s(Q^D, I^D; 0)$ and $EU^t(Q^D, I^D; 0) < EU^t(Q^B, I^B; 0)$, then, there exists separating equilibrium with advantageous selection in which type *t* chooses C_A^{t*} and type *s* chooses C_D^{s*}. In this case, type *t* obtains the first-best contract, whereas type *s* obtains the best fair contract such that type *t* has no incentive to buy this contract.

(C) Suppose that $EU^s(Q^H, I^H; 0) > EU^s(Q^{D'}, I^{D'}; e)$. Further, suppose that $EU^s(Q^H, I^H; 0) > EU^s(Q, I; e)$, where the contract (Q, I) is on the pooling price line $\bar{P}$.

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[13] If $EU^s(Q^B, I^B; e) < EU^s(Q^B, I^B; 0)$, we consider the most preferable indifference curve of type *s* who invests in self-protection. Let the contract $(p_e I^{thresh}, I^{thresh})$ denote the intersection of Z^s and P_e, where $I^{thresh} < D$, and replace the contract C_B by $(p_e I^{thresh}, I^{thresh})$.

[14] Recall that we do not allow over insurance in this model. Therefore, if $I^H > D$, we consider the most preferable indifference curve of type *s* that cuts $p_0 I$ at point $(p_0 D, D)$, and replace the contract C_H by $(p_0 D, D)$. Note that $I^A < I^H$. This is because type *s* has more incentive to sacrifice income to recover a loss in an asset by means of insurance than that of type *t*, as we assume that $\mathrm{MRS}^s(Q, I; p_0) > \mathrm{MRS}^t(Q, I; p_0)$.

Then, there exists separating equilibrium for a single premium rate in which type *t* chooses C_A^{t*} and type *s* chooses C_H^{s*}. In this case, both types do not invest in self-protection in equilibrium.

Proof: These propositions are depicted in Figures 1, 2, and 3, respectively. As we do not impose any limitations on the interaction between an income and an asset, we can consider all possible scenarios of equilibrium according to U_{CA}. One of the differences between the cases is the slope of Z^s, but the results of the equilibrium are similar. As we will see later in this section, the market equilibrium depends largely on asset sensitivity and $\mathrm{MRS}^i(Q, I; p(\varepsilon))$, regardless of U_{CA}. Therefore, without loss of generality, we mainly discuss the cases in which $U_{CA} > 0$. Only for Proposition 1 (A), all the cases of $U_{CA} > 0$, $U_{CA} = 0$, and $U_{CA} < 0$ are illustrated by diagrams in Figures 1, A.1, and A.2, respectively (Figures A.1 and A.2 are in the appendix.)

(A) From Figure 1, which demonstrates Proposition 1 (A), it is trivial that a separating equilibrium exists (C_A^*, C_B^*). Suppose that an insurer offers a contract C_B' below P_e to attract type *s* only. Then C_B' makes a negative profit, and no insurers will provide such contract. Note that curve Z^s is downward sloping when $U_{CA} > 0$, as depicted in Figure 1. In Figure A.1, curve Z^s is vertical when $U_{CA} = 0$. Lastly, curve Z^s is upward sloping when $U_{CA} < 0$, as illustrated in Figure A.2. In all cases, however, the results are similar and the equilibrium occurs at (C_A^*, C_B^*).

(B) In Figure 2, which describes Proposition 1 (B), the only way to attract type *s* who invests in self-protection is to provide a contract located below J^s where $\Delta^s(Q, I) \geq 0$ and above P_e. However, since this contract lies below $\bar{P}$, it cannot be an equilibrium contract that makes nonnegative profits for insurers.

(C) Figure 3 illustrates Proposition 1 (C). To attract type *s*, an insurer should offer a contract that is located below J^s and above P_e. A contract located below J^s and above Z^s cannot induce type *s* to invest in self-protection. Therefore, the insurers will not deviate to offer a new contract, because P_0 lies above this area. By contrast, a contract located below J^s where $\Delta^s(Q, I) \geq 0$ and above P_e will attract both types of individuals and generate a negative profit. This is because the pooling price line $\bar{P}$ is located above this contract.

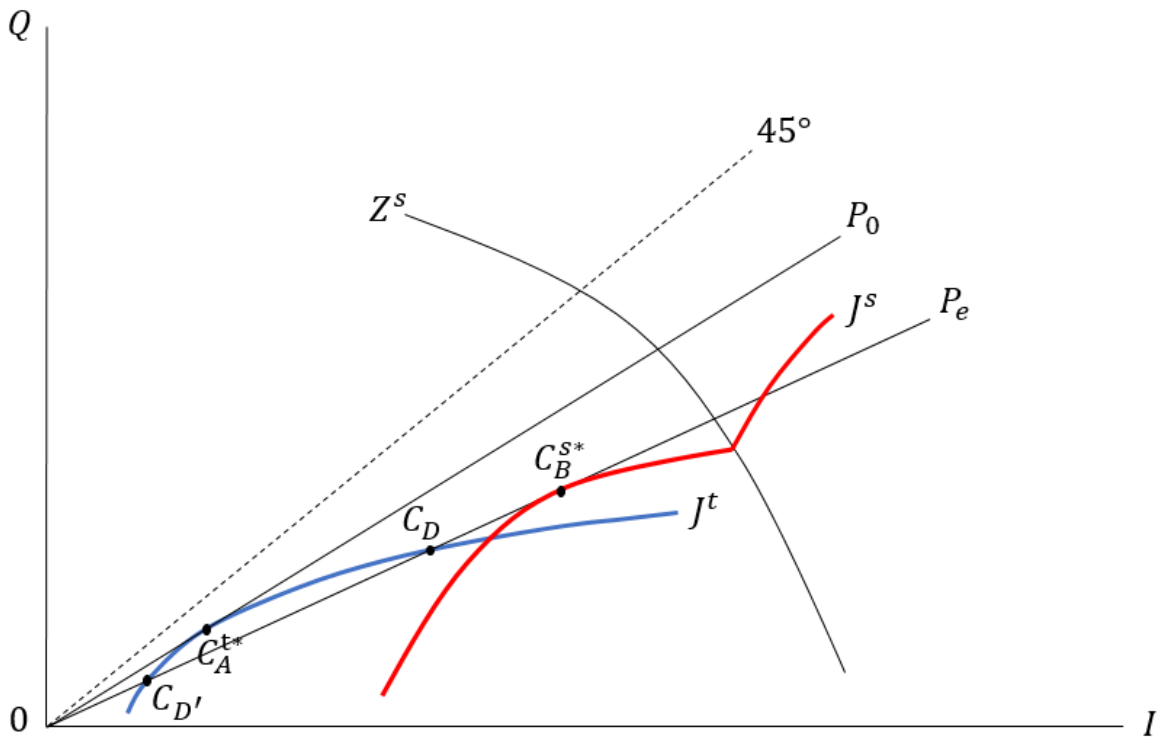

Figure 1. Separating equilibrium with advantageous selection (type *t* prefers contract C_D to C_B^{s*})**,** when $U_{CA} > 0$.

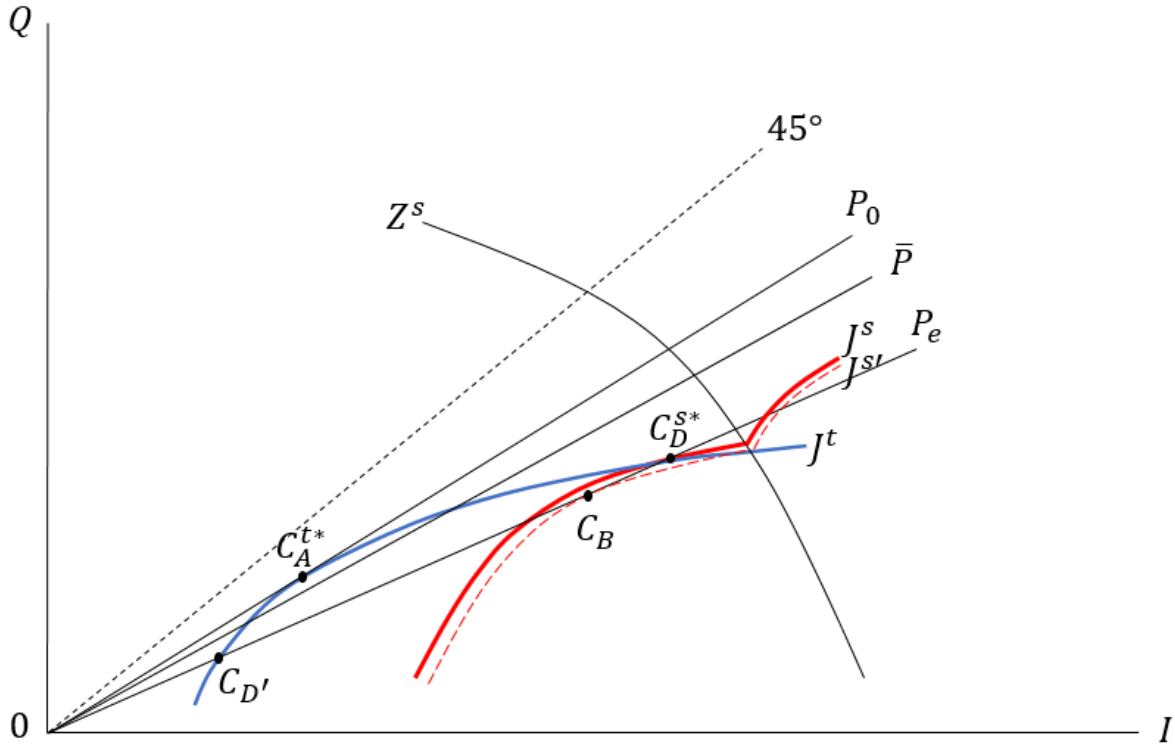

Figure 2. Separating equilibrium with advantageous selection (type t prefers contract C_B to C_D^{s*}), when $U_{CA} > 0$.

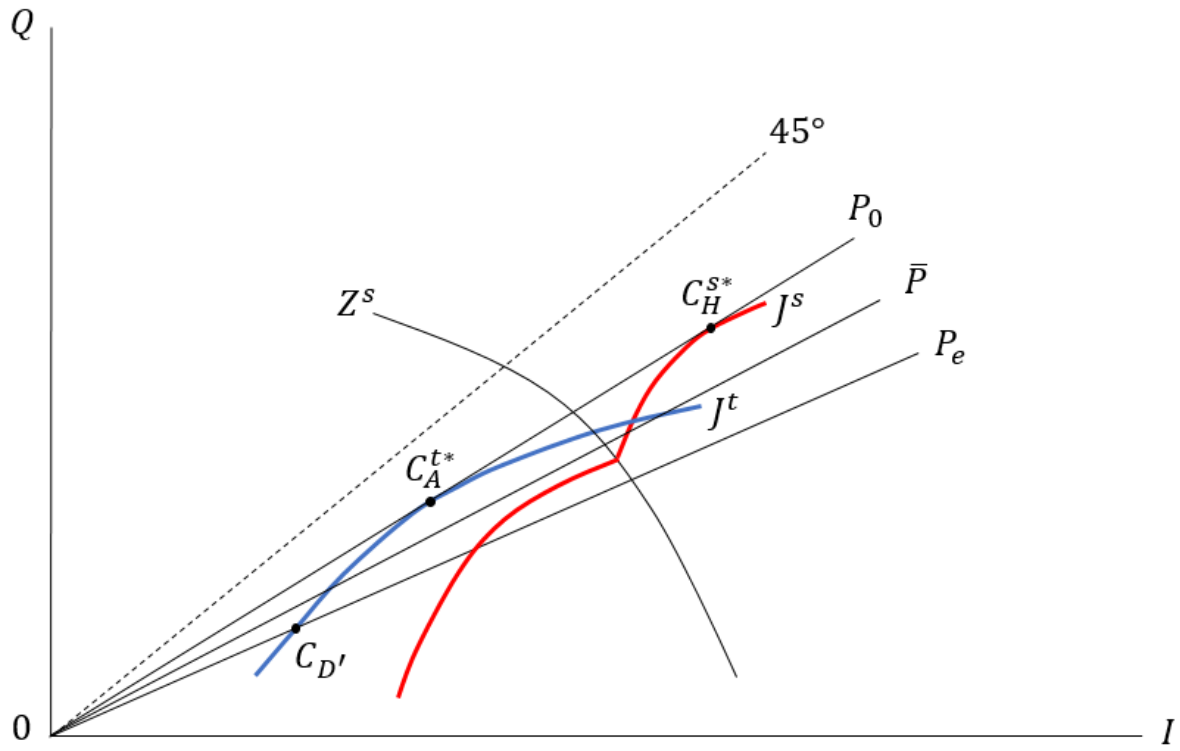

Figure 3. Separating equilibrium for a single premium rate under the single crossing property (SCP), when $U_{CA} > 0$.

Unlike the standard model of Rothschild and Stiglitz (1976), Propositions 1 (A) and (B) indicate that the sensitive type (low-risk) demands more insurance than the insensitive type (high-risk) under advantageous selection. In the case of Proposition 1 (A), both types obtain their first-best contracts that are the optimal contracts under full information. In this case, there is no inefficiency in the market, even though an insurer cannot observe the individual's action. In the case of Proposition 1 (B), by contrast, type t obtains her first-best contract under P_0, whereas type s can only obtain the best fair contract that is not her first-best. This is because insurers should offer higher coverage (C_D) to type s than her optimal level (C_B) to screen the individuals.

The key that causes a difference between Propositions 1 (A) and (B) is $\mathrm{MRS}^s(Q, I; p_e)$ which represents the type s's preference toward income and asset, after investing in self-protection. From Equation (2), type s is asset sensitive enough so that he/she invests in self-protection. In other words, the optimal choice of effort for type i largely depends on U_A^i. However, the optimal choice of insurance contract for type i depends on $\mathrm{MRS}^i(Q, I; p(\varepsilon))$. That is, it depends not only on U_A^i but also on U_C^i (see the appendix). In the case of Proposition 1 (A), type s has high enough $\mathrm{MRS}^s(Q^B, I^B; p_e)$ so that he/she expends effort while demanding much higher coverage than type t. On the other hand, in the case of

Proposition 1 (B), type *s* is asset sensitive enough to invest in self-protection, but he/she does not have sufficiently high $\mathrm{MRS}^s(Q^B, I^B; p_e)$ to be separated from type *t*.

In the case of Proposition 1 (C), as type *s* cannot be separated from type *t* while expending effort, he/she refuses to invest in self-protection and maximizes his/her expected utility by choosing C_H^{s*}. In this case, both types do not invest in self-protection in equilibrium, but type *s* purchases higher coverage than type *t*. It is clear because we assume that $\mathrm{MRS}^s(Q, I; p_0) > \mathrm{MRS}^t(Q, I; p_0)$. Therefore, the equilibrium contracts consist of two insurance contracts with the same premium rate, p_0, but different insurance coverage. Notice that the separating equilibrium for a single premium rate indicates the separation of the type of asset-sensitivity in equilibrium, not the separation of risk type. This separating equilibrium is consistent with Proposition 2 of Huang et al. (2010).

Now, we consider the case of pooling equilibrium. The results are summarized in Proposition 2.

Proposition 2: *[Pooling equilibrium under SCP].*

(A) Suppose that the slope of J^s at C_G is steeper than the unit price of pooling contract, $\bar{p}$. If $EU^s(Q^G, I^G; e) > EU^s(Q^H, I^H; 0)$, $EU^s(Q^G, I^G; e) > EU^s(Q^{D'}, I^{D'}; e)$ and $EU^t(Q^G, I^G; 0) > EU^t(Q^A, I^A; 0)$, there exists pooling equilibrium in which both types choose the pooling contract C_G^{p*}.

(B) Suppose that $EU^s(Q^F, I^F; e) > EU^s(Q, I; e)$, where the contract (Q, I) lies on the pooling price line $\bar{P}$. If $EU^s(Q^F, I^F; e) > EU^s(Q^{D'}, I^{D'}; e)$ and $EU^s(Q^F, I^F; e) > EU^s(Q^H, I^H; 0)$, then there exists partial pooling equilibrium in which type *t* chooses C_A^{t*} and type *s* chooses C_F^{s*}.

Proof:

(A) In Figure 4, which demonstrates Proposition 2 (A), the pooling equilibrium lies at the intersection of the curve Z^s and the pooling price line $\bar{P}$. As we suppose that $\frac{p_e U_A^s(y-Q^G, w-D+I^G)}{p_e U_C^s(y-Q^G, w-D+I^G)+(1-p_e)U_C^s(y-Q^G, w)} > \bar{p}$, type *s* does not prefer any other pooling contract to C_G while expending effort. To attract type *s*, an insurer should offer a contract located below J^s and above P_e. However, any deviating contract located below J^s where $\Delta^s(Q, I) \geq 0$ and above P_e will also attract type *t* and make a negative profit. This is because $\bar{P}$ is located above this contract.

(B) Figure 5 illustrates Proposition 2 (B). The only way to attract type *s* is to provide contracts located below J^s and above P_e. However, such a contract located in the area where $\Delta^s(Q, I) \geq 0$ attracts not only type *s* but also type *t*. Since this offer lies below the pooling price line $\bar{P}$, it is unprofitable, and thus cannot be an equilibrium contract.

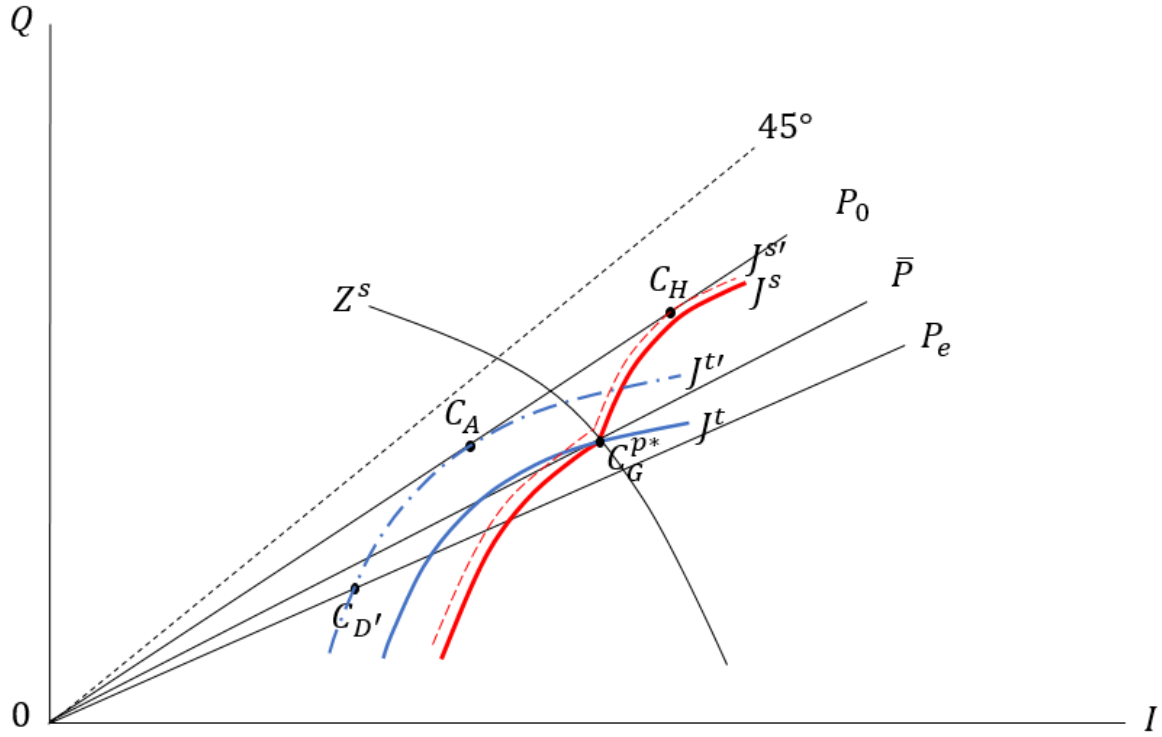

Figure 4. Pooling equilibrium under SCP, when $U_{CA} > 0$

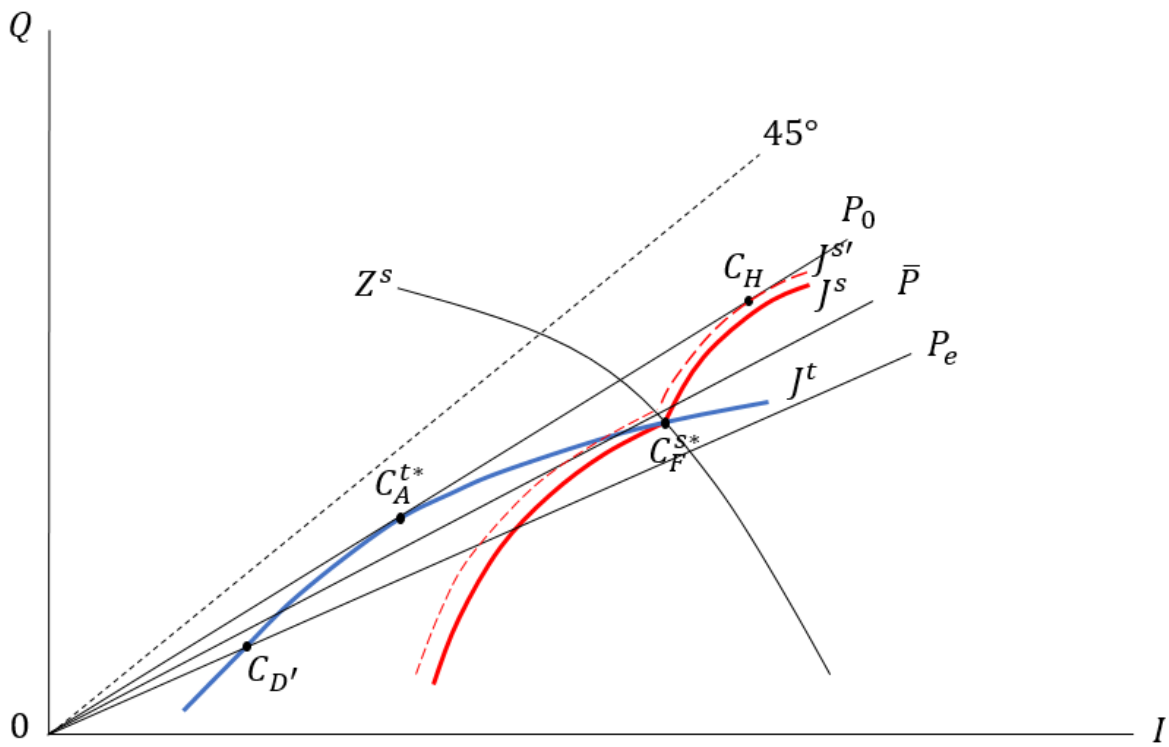

Figure 5. Partial pooling equilibrium under SCP, when $U_{CA} > 0$.

Contrary to the seminal work of Rothschild and Stiglitz (1976), Propositions 2 (A) and (B) indicate that pooling equilibrium can exist. In these cases, an actuarially unfair insurance contract is provided to type *s*. In a pooling equilibrium demonstrated in Proposition 2 (A), all risk types choose the same insurance contract C_G^{p*}. Since risk type is endogenously determined by individuals, the area of interest for existence of a pooling contract is where $\Delta^s(Q,I) \geq 0$. As illustrated in Figure 4, the only possible pooling contract is a particular point (C_G^{p*}), i.e., the point of intersection between the pooling price line $\bar{P}$ and Z^s. Therefore, unlike the results of the conventional model, a pooling contract can remain stable under some conditions described in Proposition 2 (A). In a pooling equilibrium, the sensitive type of the insured (low-risk) subsidizes the insensitive type of the insured (high risk).

In a partial pooling equilibrium described in Proposition 2 (B), there is a potential for a positive profit in the competitive market. This is because type *t* is indifferent between contracts C_A^{t*} and C_F^{s*}. As De Meza and Webb (2001) pointed out, a chance of a positive profit is an artifact of the discontinuous choice of effort in the model. Suppose that only some of type *t* choose to purchase C_F^{s*} and the others purchase C_A^{t*}, so that C_F^{s*} generates zero profit. Then, a partial pooling equilibrium can be maintained in the competitive market in which no firms can earn a positive profit. In this case, the sensitive type of the insured (low-risk) provides a subsidy only to the insensitive type of the insured (high-risk) who chooses C_F^{s*}. This partial pooling equilibrium is consistent with the results of a partial pooling equilibrium of De Meza and Webb (2001) and Huang et al. (2010).

In the second place, we suppose that double crossing property (DCP) holds, i.e., $\frac{p_e U_A^s(y-Q,w-D+I)}{p_e U_C^s(y-Q,w-D+I)+(1-p_e)U_C^s(y-Q,w)} < \frac{p_0 U_A^t(y-Q,w-D+I)}{p_0 U_C^t(y-Q,w-D+I)+(1-p_0)U_C^t(y-Q,w)}$ in the relevant region of the contract space. The following proposition indicates the possible cases in which a separating equilibrium occurs in the market.

Proposition 3: *[Separating equilibrium under DCP].*

(A) Suppose that $EU^s\left(Q^{D'},I^{D'};e\right) > EU^s(Q^H,I^H;0)$. Further, suppose that $EU^s(Q^{D'},I^{D'};e) > EU^s(Q,I;e)$, where the contract (Q,I) lies on the pooling price line $\bar{P}$. Then, there exists separating equilibrium with adverse selection in which type *t* chooses C_A^{t*} and type *s* chooses $C_{D'}^{s*}$.

(B) Suppose that $EU^s(Q^H,I^H;0) > EU^s(Q^{D'},I^{D'};e)$. Further, suppose that $EU^s(Q^H,I^H;0) > EU^s(Q,I;e)$, where the contract (Q,I) lies on the pooling price line $\bar{P}$. Then, there exists separating equilibrium for a single premium rate in which type *t* chooses contract C_A^{t*} and type *s* chooses C_H^{s*}. In this case, both types do not invest in self-protection in equilibrium.

Proof:

(A) Figure 6 illustrates Proposition 3 (A). The only way to attract type *s* is to provide a contract

located below J^s and above P_e. However, such a contract located in the area where $\Delta^s(Q, I) \geq 0$ is preferred by both type *s* and type *t*. Then, the insurer will earn a negative profit, because $\bar{P}$ is located above this contract.

(B) Figure 7 illustrates Proposition 3 (B). The proof is similar to the proof of Proposition 1 (C).

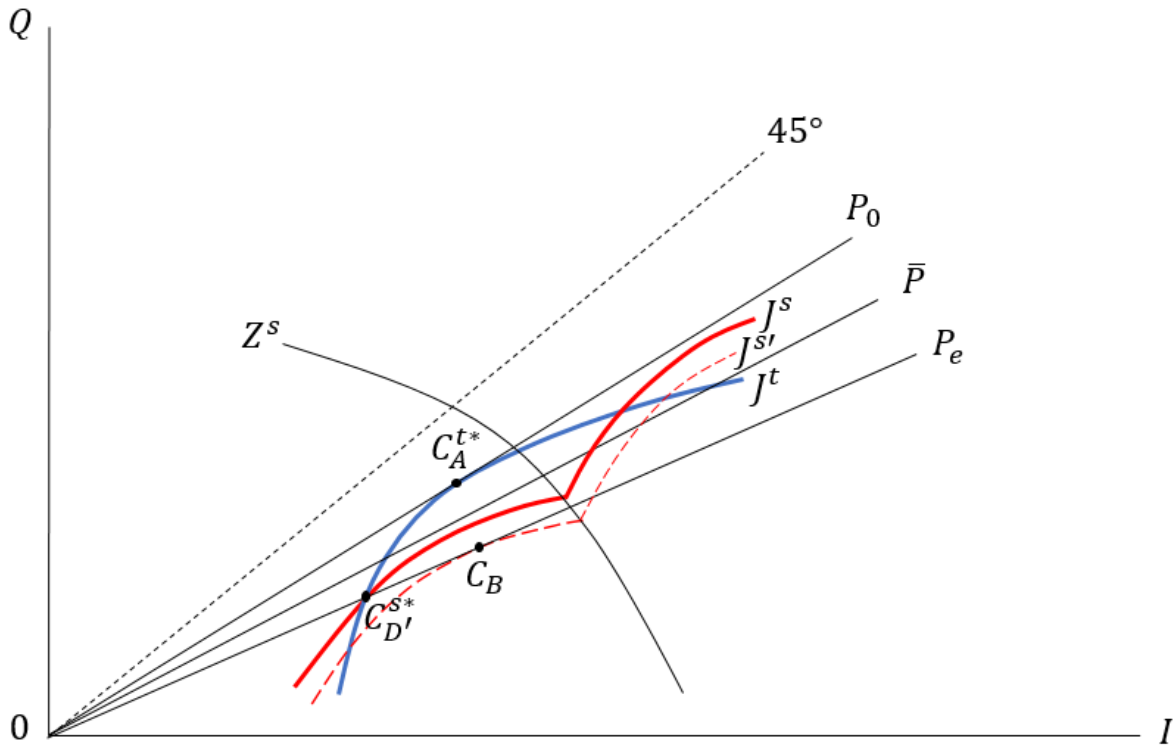

Figure 6. Adverse selection under the double crossing property (DCP), when $U_{CA} > 0$.

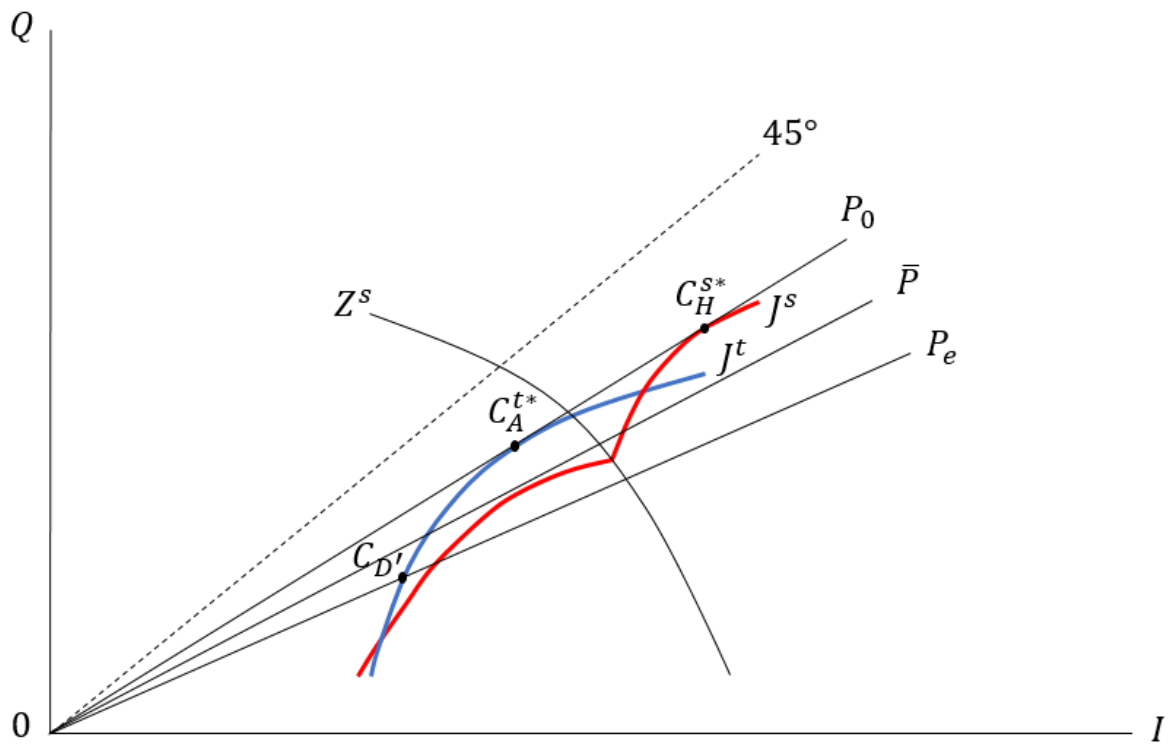

Figure 7. Separating equilibrium for a single premium rate under DCP, when $U_{CA} > 0$.

If DCP holds in the relevant region, adverse selection may occur in the market, as depicted in Figure 6. In this case, risk and coverage are positively correlated. Another possible equilibrium is the separating equilibrium for a single premium rate, as demonstrated in Proposition 3 (B). The equilibrium result is similar to Proposition 1 (C). That is, both types do not invest in self-protection, but type *s* purchases higher coverage than type *t* in the equilibrium. The only difference between Proposition 1 (C) and Proposition 3 (B) is whether the SCP holds or not. In Proposition 3 (B), the degree of asset sensitivity for type *s* is not that high, but he/she has large enough $\mathrm{MRS}^s(Q, I; p_0)$, so that he/she obtains higher expected utility by choosing C_H^{s*}, rather than any other contracts located in the region where $\Delta^s(Q, I) \geq 0$. Again, the type of asset sensitivity, not the risk type, is revealed in the separating equilibrium for a single premium rate.

Now, we will demonstrate that a pooling equilibrium is ruled out when DCP holds, as described in Proposition 4.

Proposition 4: *[No pooling equilibrium under DCP].*

If double crossing property holds, pooling equilibrium cannot exist.

Proof: Figure 8 illustrates Proposition 4. Suppose on the contrary that there exists a pooling contract $C_{G'}$ when DCP holds. To induce type *s* to expend effort, the pooling contract $C_{G'}$ should lie on the line $\bar{P}$ where $\Delta^s(Q, I) \geq 0$. That is, $EU^s(Q^{G'}, I^{G'}; e) \geq EU^s(Q^{G'}, I^{G'}; 0)$. In this region, the slope of the indifference curve of type *t* is steeper than that of type *s* by

assumption. Then an insurer will deviate to offer a new contract located in the region below J^s where $\Delta^s(Q,I) \geq 0$, and surrounded by P_e and J^t. Since this contract lies above the line P_e and attracts type *s* only, it generates a positive profit. Therefore, a pooling equilibrium cannot occur when the DCP holds.

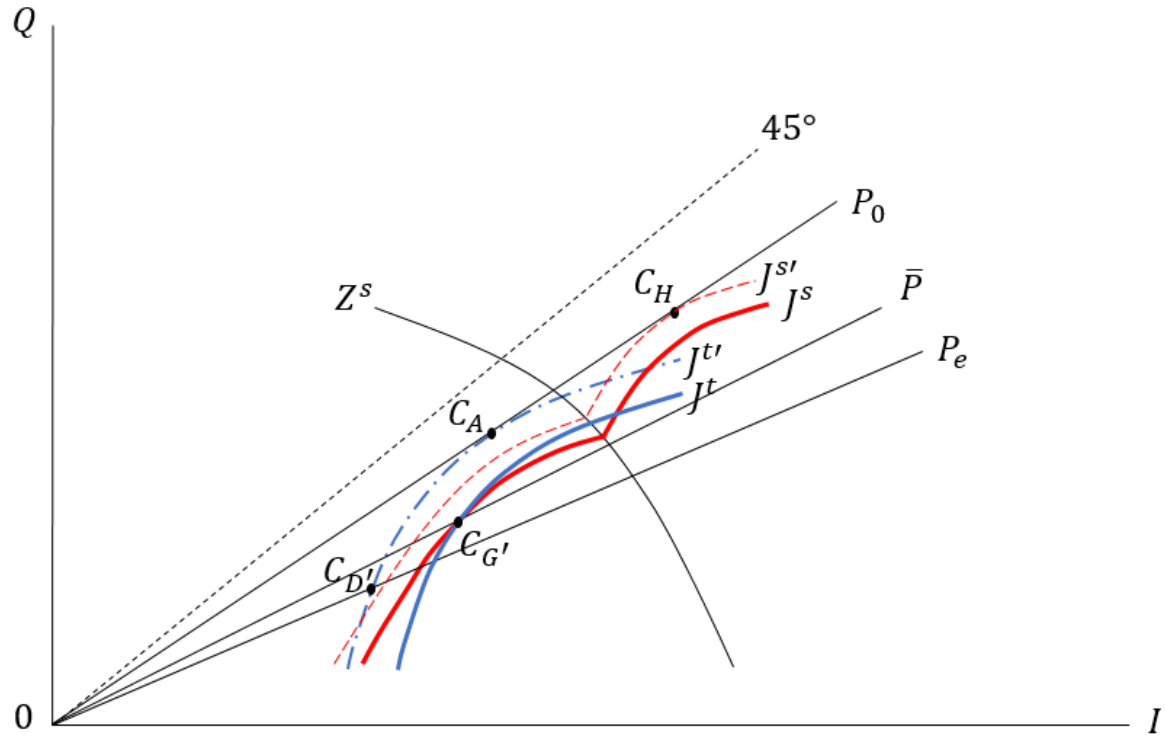

Figure 8. No pooling equilibrium under DCP, when $U_{CA} > 0$.

In this paper, the individuals' preferences toward income and asset are the main key determining the type of equilibrium. Additionally, the proportion of the sensitive type in the market (θ) is another factor that affects the existence of equilibrium. If θ is not sufficiently low, the separating equilibrium may not exist, as we have discussed in Propositions 1 and 3. According to this proportion, there may also exist a pooling equilibrium. In particular, we have the interval of value θ satisfying the following inequality for a complete risk pooling to exist: $\frac{p_e U_A^s(y-Q^G, w-D+I^G)}{p_e U_C^s(y-Q^G, w-D+I^G)+(1-p_e)U_C^s(y-Q^G, w)} > \bar{p} = \theta p_e + (1-\theta)p_0$. In conclusion, when we consider some appropriate utility functions of type *i* and the proper risk reduction technology, the proportion of the sensitive type in the market plays an important role in determining the existence of equilibrium.

Ⅳ. Discussion

In a single-argument utility framework, full insurance is optimal if it is actuarially fair (Mossin, 1968; Smith, 1968). However, in a two-argument utility framework, a fair premium

is neither necessary nor sufficient for optimality of full insurance (Lee, 2007). In this paper, optimal level of insurance coverage depends on the individual's preference toward income and asset, which is represented by MRS between insurance premium and indemnity. Therefore, the degree of asset sensitivity also affects the optimality of insurance contracts. Note that the equilibrium contract for type *s* obtained in Proposition 1 (A) is partial coverage insurance such that the marginal expected benefit of an additional coverage equals its marginal expected cost.

Unlike the standard model of Rothschild and Stiglitz (1976), when individuals differ with respect to both loss probability and degree of risk aversion, it is well recognized that more risk-averse individuals may purchase more insurance in the market with asymmetric information.[15] For example, Smart (2000) and Wambach (2000) demonstrated that at least for one risk group, more risk-averse individuals purchase more insurance than less risk-averse individuals in the same risk group. The key to these results is that high-risk types with higher risk aversion are less distracted by a partial insurance contract that is offered to the low-risk types than high-risk types with lower risk aversion. In the model of de Meza and Webb (2001), by contrast, the risk type is not exogenously given but is determined by the degree of risk aversion. They demonstrated that those with lower risk aversion buy less insurance and take fewer precautions than those with higher risk aversion. The mechanism of this equilibrium is that less risk-averse individuals can tolerate higher uncertainty than more risk-averse individuals.

From a different perspective, as we utilize a two-argument utility function, we can define risk aversion with respect to asset. As mentioned in Section 2, type *s* can be less risk averse in terms of asset. Lower risk aversion does not necessarily imply that this type of individual can tolerate higher uncertainty. Rather, type *s* with lower risk aversion with respect to asset is more likely to invest in self-protection. In addition, we can easily demonstrate that in separating equilibrium with advantageous selection, type *s* with lower risk aversion invests in self-protection and purchases more insurance than type *t* with higher risk aversion who never expends effort (see, for example, Kim and Seog, 2019). This result is perhaps surprising given that it is opposite to that of the standard model. Note that the key to this result is not the risk aversion but the degree of asset sensitivity. However, the degree of asset sensitivity and risk aversion with respect to asset are both relevant to the shape of a two-argument utility function associated with an asset. Moreover, we can generally demonstrate the cases, in which those who are more asset sensitive are less risk averse with respect to asset. Our finding emphasizes the fact that less risk averse individuals can expend more efforts and purchase more insurance than those with higher risk aversion; this implies that the present model considers the characteristics of utility, which cannot be captured by the standard model.

Other explanations about advantageous selection explored in the literature are cognitive biases. Huang et al. (2010) and Spinnewijn (2013) modeled the individual's decision making as if the individual were trying to maximize a perceived expected utility function that incorporates risk perception bias, when the true expected utility function is that of an unbiased individual. By contrast, the first-best decision is the optimal level of effort and insurance

[15] In a single-argument utility framework, the utility function depends only on wealth. Then, the coefficient of absolute risk aversion is defined by $A(w) = -\frac{u''(w)}{u'(w)}$, where $u'(w)$ and $u''(w)$ denote the first and second derivatives with respect to w of $u(w)$.

coverage that maximize the true expected utility. Therefore, in equilibrium, welfare loss arises not only because the insurer cannot observe the individual's action but also because the optimistic individual believes that his subjective loss probability is lower than his objective loss probability. In other words, a full information contract does not maximize the true expected utility of an optimistic individual. Huang et al. (2010) argued that in the first case of Proposition 1, an optimistic individual does not impose any negative externality on an unbiased individual. However, this outcome is clearly not first-best. By contrast, in Proposition 1 (A), we show that both types obtain their first-best contracts and that there is no welfare loss even though an insurer cannot observe the individual's action. The present model is thus differentiated from those of Huang et al. (2010) and Spinnewijn (2013) in that we provide our results under full rationality.

In the seminal work of Rothschild and Stiglitz (1976), pooling equilibrium cannot exist. In practice, however, pooling contracts are prevalent, notably in health insurance. Moreover, in group life insurance, a single contract covers an entire group of people who differ in their risks. Under the alternative equilibrium concept of Wilson (1977), however, pooling equilibrium can occur in a competitive insurance market.

Even under the Rothschild-Stiglitz conjecture, however, we proved that pooling equilibrium can exist, as demonstrated in Proposition 2 (A). Wambach (2000) also demonstrated that as a very special case, a complete risk pooling can occur. However, he introduced four types of individuals and defined a complete risk pooling as a situation in which more than one risk type chooses one specific contract. That is, not all of the individuals in the economy, but three types of individuals out of four types, purchase a particular contract. In this study, by contrast, there are only two types of individuals, and in a pooling equilibrium, all individuals in the economy choose the same contract. Even under Rothschild-Stiglitz conjecture, this is a new finding that demonstrates the existence of a complete risk pooling in the competitive insurance market, in which two different types of the insured exist.

V. Conclusion

According to the asset sensitivity, people have different preferences towards income and asset. This paper develops an endogenous selection model under asymmetric information and investigates how insurers screen individuals who differ in asset sensitivity. The analysis indicated that in equilibrium, the asset sensitive type may invest in self-protection and become a low-risk, whereas the insensitive type never chooses to expend effort. Unlike the standard model of Rothschild and Stiglitz (1976), we demonstrated that sensitive type (low-risk) purchases higher insurance coverage than insensitive type (high-risk) under advantageous selection. We also find other types of equilibrium including adverse selection, single premium rate, and pooling equilibrium.

These results are partially analogous to those of De Meza and Webb (2001), Huang et al. (2010), and Spinnewijn (2013). However, in contrast to all previous papers, we propose a model reflecting the reality, which captures the trade-offs that individuals make between income and asset. Unlike de Meza and Webb (2001), we consider the heterogeneity in preferences by utilizing a two-argument utility function that depends on income and asset. This

realistic modification of the utility function is the foundation of a rationale for the existence of advantageous selection that the sensitive type may demand more insurance than the insensitive type while expending effort. In addition, we provide equilibrium results under full rationality, while Huang et al. (2010) and Spinnewijn (2013) introduce irrational (optimistic) individuals into their models. Furthermore, our model even shows that a complete risk pooling can occur in the market.

Moreover, this paper can provide a different theoretical foundation for future empirical studies investigating the relationship between risk and insurance demand. Heterogeneity in asset sensitivity can also explain the mixed results in empirical papers that find positive, negative, or even no correlation between risk and insurance coverage in some insurance markets. Furthermore, understanding the heterogeneity in preferences is important in that an insurer can design the insurance contract accordingly, and our analysis can therefore contribute to better underwriting performance.

Appendix 1. (Proofs)

Footnote 4. Insurers provide an insurance contract (Q, I), where $Q = p'\alpha D$, $I = \alpha D$, $0 \leq \alpha \leq 1$, and p' is the unit price of insurance. An individual will choose the efficient level of effort and insurance coverage to maximize his/her expected utility. Since the individual chooses whether to make an effort or not, she will compare the maximum expected utilities given $\varepsilon \in \{0, e\}$. Let us first consider the following maximization problem given an effort level of ε:

$$\begin{aligned} EU\big(\alpha^*(\varepsilon)\big) &\equiv Max_{\alpha} EU(\varepsilon) \\ &= p(\varepsilon)U(y - Q, w - D + I) + \big(1 - p(\varepsilon)\big)U(y - Q, w) - c(\varepsilon), \end{aligned} \tag{A.1}$$

$$\text{subject to } Q = p'\alpha D \text{ and } I = \alpha D.$$

Solving (A.1), the first-order condition is

$$\begin{aligned} p(\varepsilon)[U_C(y_1, w_1)(-p'D) + U_A(y_1, w_1)D] + \big(1 - p(\varepsilon)\big)U_C(y_1, w_0)(-p'D) \\ = 0. \end{aligned} \tag{A.2}$$

Then, the final solution would be $\{\alpha^*(\varepsilon), \varepsilon\}$ such that $EU(\alpha^*(\varepsilon)) = Max\{EU(\alpha^*(0)), EU(\alpha^*(e))\}$. Equation (A.2) implies that the optimal insurance coverage is determined such that $\mathrm{MRS}(Q, I; p(\varepsilon))$ equals the unit price of insurance, p'. Therefore, if $\mathrm{MRS}\big(0, 0; p(\varepsilon)\big) = \frac{p(\varepsilon)U_A(y, w-D)}{p(\varepsilon)U_C(y, w-D) + \big(1-p(\varepsilon)\big)U_C(y, w)} < p'$, an individual will exit the insurance market. As we are interested in the equilibrium contracts of each type, we want to exclude the obvious cases where one or all types of the insured decide to exit the market. Thus, we assume that the individuals have small enough $U_C(C, A)$ relative to $U_A(C, A)$ to guarantee the existence of sufficient contract space for the voluntary participation of the insured.

Footnote 8. Consider the curvature of the indifference curve of type *i*:

$$\frac{d^2Q}{dI^2} = \frac{1}{\{E[U_C^i]\}^2}\Big\{p(\varepsilon)\left[U_{CA}^i(y_1,w_1)\left(-\frac{dQ}{dI}\right) + U_{AA}^i(y_1,w_1)\right]\cdot E\left[U_C^i\right] - p(\varepsilon)U_A^i(y_1,w_1)\left[p(\varepsilon)\left[U_{CC}^i(y_1,w_1)\left(-\frac{dQ}{dI}\right) + U_{CA}^i(y_1,w_1)\right] + \left(1 - p(\varepsilon)\right)U_{CC}^i(y_1,w_0)\left(-\frac{dQ}{dI}\right)\right]\Big\}. \tag{A.3}$$

The sign of (A.3) coincides with that of the numerator. It is easy to prove that (A.3) is negative if $U_{CA} \geq 0$; thus J^i is increasing and concave. Now consider the case of $U_{CA} < 0$. The sign of (A.3) can be positive, negative or zero, and can be changed according to the given insurance contracts. We rewrite the numerator of $\frac{d^2Q}{dI^2}$ as

$$p(\varepsilon)\left[p(\varepsilon)U_A^i(y_1,w_1)\left\{-2U_{CA}^i + E\left[U_{CC}^i\right]\cdot\frac{U_A^i(y_1,w_1)}{E[U_C^i]}\right\} + U_{AA}^i(y_1,w_1)E\left[U_C^i\right]\right]. \tag{A.4}$$

The indifference curve in the (Q, I) plane is concave if (A.4) is negative, and its sign depends on the shape of the utility function. Note that the only positive term inside the bracket in (A.4) is $-2U_{CA}^i$. If the utility function is concave enough, i.e., $\left|U_{CC}^i\right|$ or $\left|U_{AA}^i\right|$ are large enough, (A.4) is negative. In addition, if income and insurable asset are almost independent of each other, that is, the absolute value of U_{CA} is small enough, indifference curves are concave. For simplicity, we assume that the indifference curves of both types are increasing and concave, regardless of the interaction between income and insurable asset.

Existence of advantageous selection in Proposition 1 (A). Consider the optimal expected utilities of type *t* and type *s* in Proposition 1 (A), as follows:

$$J^t(\mathrm{C_A}) = p_0U(y - p_0I^A, w - D + I^A) + (1 - p_0)U(y - p_0I^A, w),$$

$$J^s(\mathrm{C_B}) = p_eU(y - p_eI^B, w - D + I^B) + (1 - p_e)U(y - p_eI^B, w) - c(e).$$

Note that type *t* is indifferent between choosing C_A and C_D, even though C_D is actuarially favorable for type *t*. In other words, type *t* cannot obtain higher expected utility than $EU^t(Q^A, I^A; 0)$ by choosing C_D, because he gives up too much of expected consumption of composite good in exchange for the higher expected loss recovery. Now, we intend to show that $I^D < I^B$ such that $\mathrm{MRS}^s(Q^B, I^B; p_e) = p_e$ and $EU^t(Q^D, I^D; 0) = p_0U(y - p_eI^D, w - D + I^D) + (1 - p_0)U(y - p_eI^D, w) > EU^t(Q^B, I^B; 0) = p_0U(y - p_eI^B, w - D + I^B) + (1 - p_0)U(y - p_eI^B, w)$. If $I^D < I^B$, it is clear that $EU^t(Q^D, I^D; 0) > EU^t(Q^B, I^B; 0)$, because $\mathrm{MRS}^t(Q^D, I^D; p_0) < p_e$. Thus, it suffices to show that type *s* demands higher insurance coverage (I^B) than I^D, while expending effort. If the degree of asset-sensitivity of type *s* is sufficiently large so that $\Delta^s(Q^B, I^B) \geq 0$, he/she will invest in self-protection, given insurance contract C_B. At the same time, if $\mathrm{MRS}^t(Q^B, I^B; p_0)$ is much smaller than

$\mathrm{MRS}^s(Q^B, I^B; p_e) = p_e$, we can obtain $I^D < I^B$. In summary, if not only U_A^s is large but also the difference between $\mathrm{MRS}^t(Q, I; p_0)$ and $\mathrm{MRS}^s(Q, I; p_e)$ is large enough, advantageous selection can occur in the insurance market in which both types obtain their first-best contracts. Notice that if we consider some appropriate utility functions of type *i*, all the assumptions and conditions in Proposition 1 (A) can be satisfied with a proper proportion of type *s* (θ).

Appendix 2. (Figures)

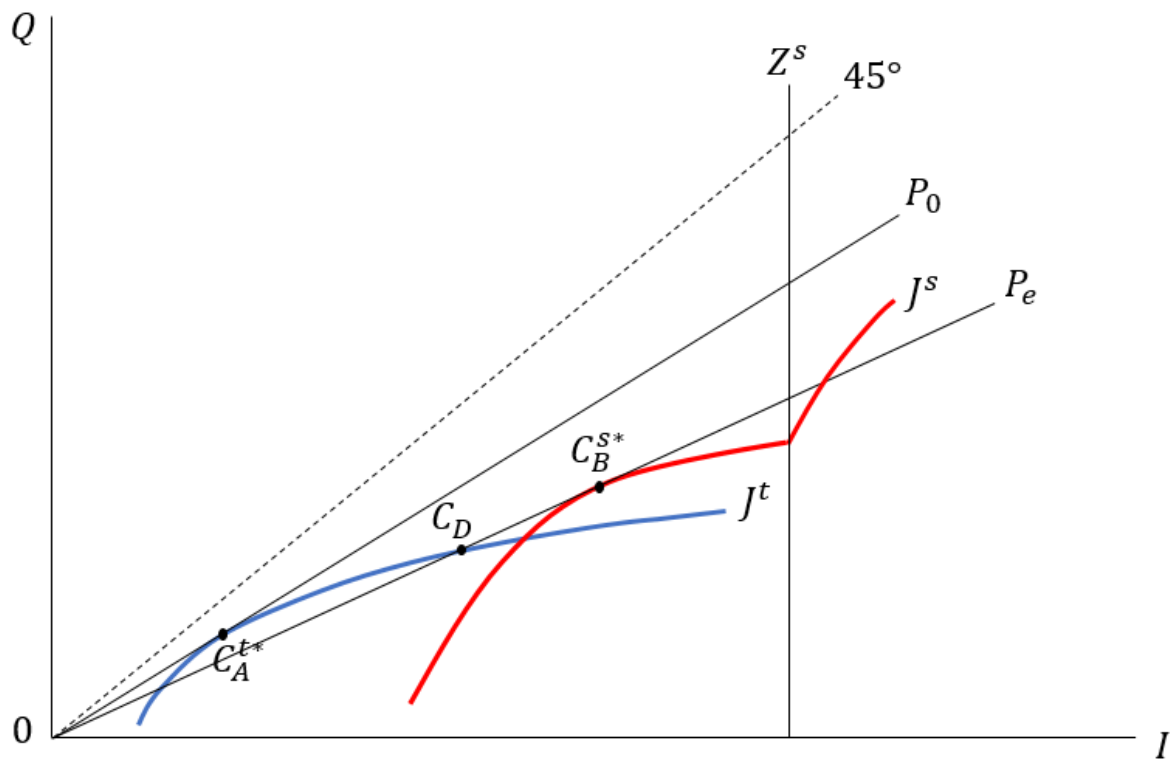

Figure A.1. Separating equilibrium with advantageous selection (type *t* prefers contract C_D to C_B^{s*})**,** when $U_{CA} = 0$.

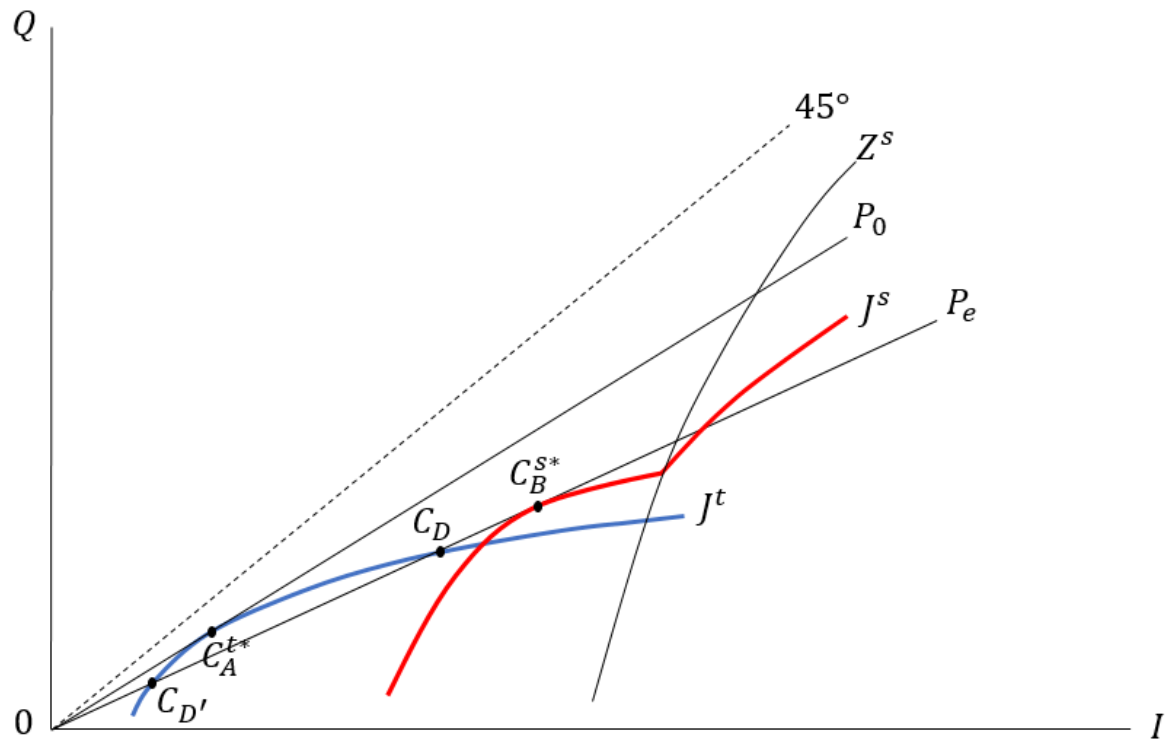

Figure A.2. Separating equilibrium with advantageous selection (type *t* prefers contract C_D to C_B^{s*}), when $U_{CA} < 0$.

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