The CAPM and Capital Budgeting as a Mismatched Couple

Jin Yoo*

Abstract

For decades, most business schools and companies have taught and behaved as if the capital asset pricing model (CAPM) can be used to estimate firms' cost of equity capital for capital budgeting purposes. I examine whether the CAPM can be used for such purposes and prove that it cannot and must not. The most obvious reason for this rejection is the CAPM itself, which never allows for use in evaluating firms' investment projects, if understood correctly. Using the CAPM for capital budgeting is essentially applying a price-based theory to a cash-flow-based world and a perfect diversification model to undiversified firms, both of which are illogical jumps and never automatically guaranteed. I prove this in five independent rationales based on differences between securities and real investment, between stock prices and cash flows, between beta and standard deviation of returns, and based on problems in stocks with very low or even negative betas. In addition, a case illustration, two explanations for the ongoing inappropriate use of the CAPM for capital budgeting applications, a testable hypothesis, and a summary of previous surveys on the topic are also provided. This paper will provide theoretical rationales and practical interpretations of the significant differences between firms' hurdle rates and the costs of capital reported in survey studies, in order to help practitioners or researchers establish more scientific and practical methods of determining appropriate discount rates for firms in the future.

Keywords: Capital Asset Pricing Model; Capital Budgeting; Beta; Cost of Capital; Hurdle Rate

JEL classification codes: G31

* College of Economics and Finance, Hanyang University

222 Wangsimniro, Seongdong-gu,

Seoul, 04763, Korea

Email: jyoo@hanyang.ac.kr,

Tel: 82-2-2220-1026, 82-10-8121-3902

I. Introduction

If something is truly right, it is right from multiple different viewpoints. A good example is the option pricing formulas by Black and Scholes (1973). They are not just right as solutions to the partial differential equation and the boundary conditions derived by the authors, but are also right according to the martingale pricing methodology suggested by Harrison and Kreps (1979) and Harrison and Pliska (1981). Furthermore, these formulas can also be proven right in many other ways, as suggested by Wilmott (2009).

Similarly, if something is truly wrong, it can be proved wrong in different ways. A good example is using the capital asset pricing model (CAPM) to estimate the cost of equity, a component of the weighted average cost of capital (WACC), of a given investment project of a firm. In recent decades, most business schools and major corporate finance or financial management textbooks have emphasized that CAPM-based expected returns can be used to estimate the WACC of investment projects. For instance, *Principles of Corporate Finance* by Brealey, Myers and Allen (2017), one of the most popular corporate finance textbooks, says the following:

> The estimated beta of Johnson & Johnson's common stock is 0.53. Suppose that the risk-free interest rate is 2% and the market risk premium is 7%. Then the capital asset pricing model would imply an expected return of 5.7% from J&J's stock:
>
> $$r = r_f + \beta(r_m - r_f) = 2 + .53 \times 7 = 5.7\%$$
>
> If J&J is contemplating an expansion of its existing business, it would make sense to discount the forecasted cash flows at 5.7%.[1]

[1] This is on page 222 in chapter 9, "Risk and Cost of Capital." The authors assume that J&J is all-equity financed.

Similarly, *Fundamentals of Corporate Finance* by Ross, Westerfield and Jordan (2017) says the following:

> We saw that Tesla had an estimated beta of 1.19. We could estimate Tesla's cost of equity as:
>
> $$R_{Tesla} = R_f + \beta_{Tesla}(R_M - R_f) = .0040 + 1.19 \times .07 = .0873, \text{or } 8.73\%$$
>
> Using the SML approach, we calculate that Tesla's cost of equity is about 8.73%.[2]

Financial management textbooks such as Brigham and Ehrhardt (2017) are no exception: they also suggest the CAPM as a method to estimate the cost of capital in real investment projects.

This convention of using the CAPM to estimate cost of capital has not been seriously challenged, although some studies, including Jagannathan et al. (2016), have posed reality-based problems of such usage of the CAPM. It seems that those problems have not been compelling enough to significantly weaken CAPM usage in educational and business applications. In this paper, I contend that, even without such reality-based arguments, there is a very clear and compelling reason why the CAPM should not be used for capital budgeting decisions: the CAPM itself rules out such usage. In other words, using CAPM-derived costs of capital is wrong and even contradictory. In this study, I verify that such usage of the CAPM is wrong even if all the assumptions and predictions of the CAPM are satisfied and even when additional assumptions made in this study in favor of such usage

[2] This is on page 463 in chapter 14, "Risk and Cost of Capital." They used 0.004 as r_f since US Treasury bills were paying about .40 percent when they were writing the book.

of the CAPM are met. This verification is based on five different and independent rationales, where "independent" means that each rationale holds true whether the other rationales are right or wrong. Using the CAPM for capital budgeting is essentially applying a price-based theory to a cash-flow-based world and a model of perfectly diversified investors to undiversified investors (firms), both of which are illogical jumps and never automatically guaranteed.

A distinction between this study and others related to the CAPM and capital budgeting is that this study does not rely on any reality-based issues but only on the CAPM itself. For example, Jagannathan et al. (2016) point out that firms use discount rates higher than their WACCs computed using the CAPM largely because of operational constraints such as organizational and managerial constraints. However, even without relying on those reality-related problems, the CAPM alone is sufficient to prove that its use for capital budgeting is wrong. Another distinction is that this study is inherently theoretical rather than survey-oriented. In general, the findings of survey-oriented studies may be sensitive to sample firms, periods, and methodologies, among other factors, in ways this study will not be. Third, this study focuses on whether to use CAPM betas for capital budgeting, whereas some other papers have primarily focused on which beta, firm beta or project-specific beta to use for capital budgeting, including Kruger et al. (2015).

In Section II, I verify that the CAPM should not be used for capital budgeting, employing five different rationales. They are i) the difference between securities and real investment, ii) the difference between beta and standard deviation of returns, iii) problems in zero-beta stocks, iv) problems in negative-beta stocks, and v) the difference between (stock) prices and (project) cash flows. In Section III, I propose testable hypotheses and possible reasons for the long-term use of this educational convention, and summarize previous survey papers on the topic. The summary of related survey papers is

presented later because this study has little to do with most survey papers and, for this reason, putting the summary of the papers behind the main body of this study is more effective. Section IV concludes.

II. The Mismatch of the CAPM and Capital Budgeting

The CAPM and capital budgeting represent two different categories: the former represents capital asset pricing, in particular stock valuation, and the latter represents decision making for real projects. Nonetheless, they appear related to each other for multiple reasons. First, both are investment-related. Second, both use the term "rate of return" such as expected rate of return for the CAPM and internal rate of return or IRR for capital budgeting. Third, both are risk-related. Despite these similarities, the CAPM and capital budgeting have many differences. If the similarities appear more compelling than the differences, the CAPM and capital budgeting may be viewed as more closely related than in actuality. Therefore, we need to understand exactly whether and how they are related, which will be the starting point of the first subsection.

In this section, I prove PROPOSITION 1 in five different ways.

PROPOSITION 1: *CAPM-based expected returns should not be used as the costs of equity in the WACC for capital budgeting decisions of firms.*

A. Securities vs. Real Investments

I prove PROPOSITION 1 by proving LEMMA 1 in two steps. First, we examine the CAPM in relation to investment in individual stocks, and then in relation to investment in individual projects.

LEMMA 1: *Stock investors cannot use the CAPM for valuing individual stocks unless they have a well-diversified stock portfolio. Similarly, project investors or firms cannot extend use of the CAPM to evaluating individual projects unless they have a well-diversified real investment portfolio.*

1. *Investment in Individual Stocks*

We re-examine what the CAPM really tells us about securities investment. Suppose we want to buy only one stock, firm "A," out of more than some 2,400 stocks in the New York Stock Exchange. Is it appropriate for us to use its CAPM beta, β_A, to assess whether stock A is currently under- or overpriced? The answer is no because we are investing in one stock only.[3] Since our entire asset is just stock A, the appropriate measure of investment risk for us is the standard deviation, or variance, of the returns on stock A, σ_A, just as the standard deviation of the returns on the market portfolio, σ_M, is the appropriate measure of investment risk when the portfolio is our entire assets. Therefore, the conclusion is clear: even stock investors cannot use the CAPM for individual stock pricing unless they have a well-diversified portfolio of stocks. Then, could investors in real projects, or firms, use the CAPM for individual real projects even though this securities pricing model was not originally developed for capital budgeting of real projects and sometimes cannot even be used for stock pricing as mentioned above?

[3] In contrast, if we have or want to have a well-diversified portfolio including stock A, which is the most rational investment according to the CAPM, then we can use the CAPM to assess whether individual stocks (including A) are under or overpriced. This is because now A is not our entire asset but only a tiny component of it. Since most of our money is or will be invested in the portfolio, (say, P) what matters in the investment in A is not the risk of A but the impact of adding A into P on the risk of P, which is β_A.

2. *Investment in Individual Projects*

Although the CAPM was not originally developed for evaluating real projects, we extend the rationale of the CAPM to real projects to examine whether it can ever be applicable for capital budgeting. The CAPM predicts that stock investors diversify their stock investments perfectly, but it neither predicts nor assumes that firms should diversify their *real* investments in various industries or sectors, let alone diversify them in *all* industries. If the CAPM were to predict that all firms diversify their real investments in almost all industries, the betas of all firms would be nearly 1, and stock investors could achieve the perfect stock diversification effect just by buying any stock. Furthermore, in reality, even the largest firms invest in one or at most a few industries.[4] With regard to this, Kruger et al. (2015) document that, from 1987 to 2007, some 90% of the investment activities of firms in North America were made by *stand-alone* firms and that, even for *conglomerates*, approximately 73% of total sales revenues were made in one division.[5] Since few firms have a well-diversified real project portfolio and CAPM betas can only be used (in an extended way) for investors with a well-diversified portfolio of stocks (of real projects), firms cannot extend the use of their CAPM beta for capital budgeting of their projects, and hence LEMMA 1 is proved.

Now let us answer an intriguing question: whether a firm, say "X," could employ its CAPM-based expected return, $r_f + (\overline{r_M} - r_f)\beta_X$, as the hurdle rate for its project(s) if it were currently operating in numerous industries. First, its beta would be close to 1 or $\beta_X \approx 1$. Second, X might be tempted to consider using $r_f + (\overline{r_M} - r_f)\beta_X$ as the cost of

[4] Such large firms are numerous and include Microsoft, Exxon Mobile, Coca-Cola, General Motors, Intel, Boeing and Walt Disney, among others.

[5] Kruger et al. (2015) use the two-digit Fama-French industry classification codes (Fama and French (1997)). Stand-alone firms and conglomerates refer to firms operating in one industry and more than one industry, respectively.

equity in the WACC for one of its investment projects, say "IP." However, even in this case, X should use the beta of the project, not its firm beta (= β_X), since the project beta is the appropriate (systematic) risk of project IP for X with the well-diversified portfolio of real investments. Remember that, when pricing stock A, stock investors with a well-diversified or the market portfolio do not use the beta of the market portfolio but the beta of stock A according to the CAPM. Similarly, when evaluating project IP, firm X with a well-diversified portfolio of real projects should not use the beta of the portfolio, which is the firm beta itself in this case, but the beta of project IP.

In summary, it does not make sense that X with the well-diversified real project portfolio always uses a fixed constant, β_X, when evaluating different projects, including IP, of different industries with different (systematic) risks. Moreover, no firm, even the largest ones, has such a well-diversified real investment portfolio, which violates the crucial assumption of the (extended, real investment version of) CAPM. These are a few reasons why CAPM betas should not be used for capital budgeting of firms' projects.

B. Beta vs. Standard Deviation

Hereafter, for convenience, I assume that firms under consideration are 100% equity-financed. This is because the CAPM will only determine the component of cost of equity in the WACC. This assumption does not directly affect judgments regarding whether the CAPM-based returns can be used for the determination of WACC for capital budgeting, but makes the judgment simpler.

ASSUMPTION 1: Firms under consideration are all-equity financed.

Here I prove PROPOSITION 1 by proving LEMMA 2.

LEMMA 2: *The CAPM-based costs of capital and firms' own hurdle rates are not necessarily equal values since their sensitivity to project risks differs.*

I prove PROPOSITION 1 by directly comparing costs of capital calculated with and without the CAPM. Suppose that firm X is all-equity financed and that it operates in industry I, whose beta is β_I. X has invested in projects that are typical of industry I, and is currently considering investment project IP, which is also typical of that industry, so the industry beta, the firm beta, and the project beta are the same (or very similar); thus, $\beta_I = \beta_X = \beta_{IP}$. Let us denote the standard deviation of the returns on IP by σ_{IP}. Since X is not diversifying its businesses in various industries but is considering project IP only, the appropriate risk measure in investing in IP for X is not β_X but the risk of IP itself, σ_{IP}. Specifically, the firm's hurdle rate is likely to be proportional to σ_{IP}. Let us assume that the hurdle rate of IP for X is

$$r_{IP,HR} = r_f + k_{HR} \cdot \sigma_{IP}, \tag{1}$$

subject to

$$k_{HR} > 0, \tag{2}$$

where k_{HR} refers to the risk-averseness coefficient for the firm.

In contrast, the CAPM-based required return on stock X, $r_{X,CAPM}$, is

$$r_{X,CAPM} = r_f + (\overline{r_M} - r_f)\beta_X, \tag{3}$$

where $\overline{r_M}$ is the expected value of the return on the whole market ($= r_M$). If (3) is used as the cost of capital for IP, its notation can be changed to $r_{IP,CAPM}$, where

$$r_{IP,CAPM} = r_f + (\overline{r_M} - r_f)\beta_X. \qquad (3-1)$$

Since $\beta_I = \beta_X = \beta_{IP}$, $(3-1)$ can also be re-written as

$$r_{IP,CAPM} = r_f + (\overline{r_M} - r_f)\beta_{IP}. \qquad (3-2)$$

To compare formulas $r_{IP,HR}$ and $r_{IP,CAPM}$, $(3-2)$ can be re-written such that both have σ_{IP} in common.

$$\begin{aligned} r_{IP,CAPM} &= r_f + (\overline{r_M} - r_f)\beta_{IP} \\ &= r_f + (\overline{r_M} - r_f)\frac{Cov(r_M, r_{IP})}{Var(r_M)} \\ &= r_f + (\overline{r_M} - r_f)\frac{\sigma_M \cdot \sigma_{IP} \cdot \rho_{M,IP}}{{\sigma_M}^2} \\ &= r_f + \frac{(\overline{r_M} - r_f)\rho_{M,IP}}{\sigma_M} \cdot \sigma_{IP} \\ &= r_f + K_{CAPM} \cdot \sigma_{IP}, \qquad (4) \end{aligned}$$

where $\rho_{M,IP} = \frac{Cov(r_M, r_{IP})}{\sigma_M \cdot \sigma_{IP}}$ and $K_{CAPM} = \frac{(\overline{r_M} - r_f)\rho_{M,IP}}{\sigma_M}$. Now we compare the right hand sides of (1) and (4), which are identical except for k_{HR} and K_{CAPM}. Since $k_{HR} = K_{CAPM}$ is not automatically guaranteed, $r_{IP,HR} \neq r_{IP,CAPM}$ in general. That is, using CAPM-based required returns such as $r_{IP,CAPM}$ as project hurdle rates such as $r_{IP,HR}$ cannot be justified (unless their values are always equal or close). That is, both $r_{IP,HR} > r_{IP,CAPM}$

and $r_{IP,HR} < r_{IP,CAPM}$ are possible according to (1) and (4). This sheds light on the discrepancies between hurdle rates and costs of capital for most firms that have been reported by survey studies on using the CAPM for capital budgeting. This includes Jagannathan et. al. (2016), who find that firms' hurdle rates are higher than their weighted average costs of capital computed using the CAPM.

C. Zero Beta

Here I prove PROPOSITION 1 by proving LEMMA 3.

LEMMA 3: *A firm with a beta of 0 or close to 0 should not use its CAPM-based expected return to evaluate investment projects. If it does, the hurdle rates for its risky projects will be the risk-free rate.*

Can the beta of a stock be 0 or close to 0? A crucial element of this question is that the weighted average of the betas of all N firms in a stock market is 1 or

$$\sum_{i=1}^{N} (\beta_i \cdot w_i) = 1, \tag{5}$$

subject to

$$\beta_i = \frac{Cov(r_M, r_i)}{Var(r_M)}, \tag{6}$$

$$\sum_{i=1}^{N} (w_i) = 1, \tag{7}$$

$$w_i > 0, \forall i \in \{1, \dots, N\}, \tag{8}$$

where r_i and w_i are the return on stock i and firm i's capitalization weight as a fraction of the whole market capitalization, respectively. Since the weighted average beta of the market is 1, some firms have betas greater than 1 and others less than or close to 1. Now, is it theoretically impossible that $\beta_i \approx 2 \; or \; \beta_i \approx 0$ for some firm i? There are no such restrictions in the theory of the CAPM and both are theoretically possible.

Next, is it empirically impossible that $\beta_i \approx 0$? No, it is not. Note that the beta of a firm is not determined (a priori) theoretically but estimated from its price data by (6). If the value of (6) for a firm in a given sample period is 0, the beta of the firm is (estimated to be) 0. For instance, Brealey, Myers and Allen (2017) show that the beta of Newmont (Ticker NEM) is 0 based on its price data from November 2009 to October 2014.[6] This is also intuitively appealing since the performance of this mining company is not expected to be (significantly) correlated to that of the stock market or other firms.

Zero-beta stocks are those whose returns are not correlated to those of the whole market or economy. For instance, firms producing daily necessities can have a beta close to zero. Nevertheless, the stocks of these firms are not necessarily risk free.[7] In general, given that x is a random variable, $Cov(x, y) = 0$ does not mean that y must be a constant since the covariance of *two* random variables can be 0, too. That is, $Cov(r_M, r_i) = 0$ or $\beta_i = 0$ does not mean that r_i must be a constant or that stock i must be a risk-free asset.

Suppose firm X is all-equity financed and has a beta equal or close to zero and its CAPM-based expected return is r_f, or $r_{X,CAPM} = r_f$. X is contemplating an investment

[6] Refer to pages 196 to 200. In addition, the beta of Newmont (Goldcorp Corporation) has been reported to be very close to 0 or even negative by different sources including Infront Analytics (https://talkmarkets.com/content/stocks--equities/newmont-goldcorp-why-this-negative-beta-stock-could-outperform-in-a-market-downturn?post=238940). Note that estimating beta is relatively easy and can be done by not only finance professors but also practitioners.

[7] On the contrary, the betas of risk-free assets such as US Treasury bills are (close to) zero.

project, IP, the cash flow of which is typical of a zero-beta business. In other words, the correlation between the return on the market portfolio and the return on IP is (close to) 0.[8] This, however, does not mean that IP must be a risk-free project as explained above. Given that IP is not always risk free, could X use $r_{X,CAPM} = r_f$ as the cost of capital for IP? The answer is no, and this is another proof of why CAPM-based capital budgeting criteria should not be used.

D. Negative Beta

Here I prove PROPOSITION 1 by proving LEMMA 4.

LEMMA 4: *A firm with a negative beta should not use its CAPM-based return to evaluate its investment project. If it does, the hurdle rates for risky projects are lower than the risk-free rate.*

Can the beta of a stock be negative? The CAPM does not rule out such a possibility. For instance, is it impossible that $\beta_i > 2$ *or* $\beta_i < 0$ for some firm i? There are no such restrictions in the CAPM and both are theoretically possible. Next, is it empirically impossible that $\beta_i < 0$? No, it is not. Recall that the beta of a firm is estimated from its price data by (6). Consequently, if the value of (6) for a firm in a given time period is negative, the beta of the firm is (estimated to be) negative. In fact, Aktas and McDaniel (2009) find, using a positive risk-free rate and two positive market risk premiums, that the CAPM-generated costs of equity of 6.7% (7.2%) of the sample of 1,229 US companies

[8] Since project IP is not a traded asset, the return on IP will be in terms of cash flows, not in terms of (stock) prices. For example, if IP is a one-year project, the return on IP in terms of cash flows would be the cash inflow into the project at the end of the year divided by the initial outlay of IP minus 1.

are less than the risk-free rate because of their negative betas.[9]

Stocks with a negative beta are not limited to the ones whose returns are likely to be negative when market returns are positive and vice versa. A negative beta can occur even when both the market and the stock under consideration have positive returns. For example, lower (higher) positive returns on the market might coincide with higher (lower) positive returns on the stock. As a matter of fact, there can be multiple cases where a firm's beta is negative. A "dedicated short bias" hedge fund can have a negative beta: its return will be positive (negative) when that of the market is negative (positive). If a firm mainly buys index put options, it can have a negative beta. If a firm mostly invests in gold, a standard example of a hedge against some macroeconomic risk, its beta can be negative, too.

If the CAPM-based required return of a negative-beta firm is used for the firm's capital budgeting decisions, the discount rate will be less than the risk-free rate. However, a negative-beta firm does not necessarily mean that its business or project is risk-free *per se*, let alone *risk freer*. For instance, a gold investment firm or a hedge fund using the dedicated short bias strategy is more likely to have a negative beta, but both are risky by nature since gold price or the short strategy does not guarantee a risk-free return. Suppose firm X is all-equity financed and has a negative beta. X is investing in an investment project IP, the cash flow of which is typical for a negative-beta business. Given that IP is a risky investment, could X use its CAPM-based discount rate, which is lower than r_f, for project IP? If the CAPM-based discount rate and r_f were 0.011 and 0.016, respectively, would X accept IP whose IRR is 0.013? The answer is no. This is another reason why CAPM-based required returns should not be used as discount rates for capital budgeting. Moreover, if a firm's beta is negative and the market premium, $\overline{r_M} - r_f$, is

[9] The fraction of companies with a cost of equity less than the risk-free rate, 6.7% or 7.2%, depends on the two market risk premiums used.

positive and high enough, its CAPM-based cost of equity can even be negative, i.e., $r_f + \beta(\overline{r_M} - r_f) < 0$.

Meanwhile, how can we interpret a negative-beta stock, say, NG? According to the CAPM, a rational investor, who already has PP, a well-diversified portfolio without NG, can invest in NG even if he or she gets a mean return on NG of less than r_f. The reason is as follows. When NG is included as a new component stock in his or her (expanded) portfolio, say PP+NG, the risk of the portfolio will be lower than before since NG's beta is negative: PP and NG tend to move in the opposite directions so the volatility of "PP+NG" will be less than that of PP. Therefore, as long as the reduced risk of the portfolio compensates for the loss in its mean return, the investor can buy NG even though its mean return is lower than the risk-free rate.

Suppose $r_f = 0.016$ and firm X is 100% equity financed and its number of shares outstanding is 5,000. Currently, X has only \$1,000,000 cash, so its stock price is \$200. X is contemplating a one-year investment project, XP, whose initial outlay is \$1,000,000. The mean return and standard deviation of the returns on XP are 0.05 and 0.02, respectively. Specifically, there are two equiprobable states in one year, G and B. The return on XP is expected to be 0.07 in state B and 0.03 in state G, whereas the market return in state B is expected to be lower than in G. Therefore, the beta of X is negative and let us say that its CAPM-derived expected return, $r_{X,CAPM}$, is 0.010, smaller than r_f. If X decides to invest in XP and this information is known to market participants, most of whom have a well-diversified stock portfolio, the stock price of X will change to

$$P_{x,0} = \frac{E(P_{X,1})}{1 + r_{X,CAPM}} = \frac{\frac{E(V_{X,1})}{5{,}000}}{1 + 0.010} = \frac{\frac{1{,}000{,}000 \times (1+0.05)}{5{,}000}}{1.01} = \frac{210}{1.01} = 207.92$$

where $P_{X,t}$ and $V_{X,t}$ denote the stock price and the firm value of X at time t (years from now), respectively, and $E(\cdot)$ is the expectation operator. After all, investors in stock X, who pay \$207.92 per share today, are expected to achieve the average return of 1% in a year, which is less than r_f. This does not mean that firm X will earn the expected return on XP less than r_f: X will earn the expected return of 5% from project XP. Second, according to the CAPM, investors with a well-diversified stock portfolio are willing to buy stock X and earn the expected return on X lower than r_f because it is worth it: the risk of their portfolios, now including stock X, will be reduced.

E. Stock Prices vs. Cash Flows

Price and cash flow are two different entities or concepts. Thus far I have not pointed out this difference since I can prove the previous four lemmas without relying on it. However, price and cash flow are, or at least can be, different in terms of meaning, magnitude, signs, etc. For example, although firms are generating less (more) cash flow from their businesses than before, their stock prices can be rising (falling). Even if firms are generating negative (positive) cash flow from their projects, their stock prices can be rising (falling). Furthermore, it is not unusual for the stock prices of many young firms or startups not to be successfully explained by their cash flow alone. Moreover, even the CAPM does not assume or predict that price and cash flow should coincide with each other.[10] In fact, the CAPM assumes all investors' homogeneous expectations of the expected return and volatility of every stock and the correlation with each stock's return and the market return, but addresses neither firms' cash flow nor the relationship between firms' cash flow and prices. Given this, using the CAPM for firms' project

[10] In finance or accounting, there is virtually no theory that assumes or predicts that price and cash flow coincide with each other (except for convenience).

evaluation means applying a price-based valuation model to cash flow-based world. This kind of illogical jump is wrong and flawed. Hence LEMMA 5 is proved.

LEMMA 5: *A firm cannot use the CAPM, which is based on stock prices, for evaluating its investment project because price and cash flow are generally not the same.*

F. Illustration

A case illustration can help clarify the various problems in employing the CAPM for capital budgeting. Suppose that there are 19 distinct industries in the economy, $I_1, I_2, I_3, \ldots, I_{19}$, whose industry betas are 0.1, 0.2, 0.3, ..., 1.9, respectively. In this economy, there is a stock exchange on which 19 all-equity firms are listed, $F_1, F_2, F_3, \ldots, F_{19}$, whose share prices are $P_1, P_2, P_3, \ldots, P_{19}$, respectively. F_k is investing in project IP_k, which is typical of industry k, I_k, and the beta of F_k, or I_k, is $\beta_k = \frac{k}{10}$, where k = 1, 2, ..., 19. The capitalization weight of each firm as a fraction of the whole market capitalization is $w_k = \frac{1}{19}$, and the weighted average of the 19 firms' betas is

$$\sum_{k=1}^{19} (\beta_k \cdot w_k) = \frac{\sum_{k=1}^{19} \beta_k}{19} = \frac{(0.1 + 0.2 + \cdots + 1.9)}{19} = 1. \qquad (9)$$

The standard deviation of the returns on IP_k is $\sigma_{k,IP}$.

If the CAPM is used for capital budgeting of these firms, the first firm, F_1, will discount its cash flow from IP_1 in a year at $d_1 = r_f + (\overline{r_M} - r_f)\beta_1 = r_f + 0.1(\overline{r_M} - r_f)$. However, d_1 would be the right discount rate for IP_1 for a firm which would invest $\frac{1}{19}$ of its money in each of the 19 different projects, not for F_1 that is investing in IP_1 only. Consequently, F_1

should not use d_1 as the appropriate hurdle rate for project IP_1; instead, F_1 should use a discount rate based on $\sigma_{1,IP}$. Similarly, F_k should not use d_k as the appropriate hurdle rate for project IP_k, where $d_k = r_f + (\overline{r_M} - r_f)\beta_k$ is the CAPM-based discount rate for F_k and k = 2, 3, …, 19. This proves that no firm can use its CAPM beta for the capital budgeting of its project.

Additionally, strictly speaking, even if a firm (say, F_{all}) were investing $\frac{1}{19}$ of its money in each of the 19 projects, it could not use the CAPM-based expected return, $d_{All} = r_f + (\overline{r_M} - r_f)\beta_{All}$ as the hurdle rate for IP_k unless $\beta_{All} = \beta_k$, where k = 1, 2, …, 19. The reasons are as follows. When evaluating project IP_k, F_{all} should use the project beta of IP_k, β_k, not its firm beta, β_{All}, just as a stock investor with a market portfolio, when pricing an arbitrary stock A, uses the beta of the stock, not the beta of its market portfolio. Should F_{all} use d_{All} for evaluating its projects, it would be using a single, common discount rate for all 19 projects with different contributions to the risk of the firm's portfolio of the 19 projects. Moreover, since β_{All} is 1, $\beta_{All} \neq \beta_k = \frac{k}{10}$ for all k except k = 10.

III. Discussion

Here I suggest two possible reasons for the use of the CAPM for capital budgeting, present a testable hypothesis, and summarize previous survey papers.

A. Possible Reasons for CAPM's use as an educational convention

Why have the most popular finance textbooks such as Brealey, Myers, and Allen (2017) and Ross, Westerfield and Jordan (2017) stated that firms' hurdle rates for capital budgeting can be estimated using CAPM betas, as illustrated in Section I? In addition, why

have researchers not posed serious theoretical, not just practical, contradictions to this contention? A possible reason is the "argument from authority," or "appeal to authority."

i) For most readers, practitioners and researchers, the idea that prominent authors in corporate finance may have made mistakes in their books is inconceivable. Therefore, there have been few negative responses from readers against using the CAPM for capital budgeting.

ii) For most authors, it is unfathomable that the decades-long notion and educational convention of employing the CAPM for capital budgeting could be seriously flawed and contradictory.

In short, just as the professional dominance of the efficient market hypothesis (EMH) in the 1960s and 1970s delayed and hindered the emergence of behavioral finance, as pointed out by Shleifer (2000), the decades-long dominance of "the CAPM for capital budgeting" in books and classrooms may have prevented authors and readers from questioning the validity of the CAPM as a tool to estimate firms' cost of equity capital.

B. A Hypothesis on Beta and the Discrepancy between the Two Rates

To survive, corporate firms should earn a return on their projects greater than the risk-free rate whether their CAPM betas are positive, zero or negative. Then, we can conjecture that their target returns or hurdle rates may not vary as much across firms as their CAPM beta-based discount rates. For example, most firms' hurdle rates may be of similar values (except for highly profitable firms such as Apple or Microsoft), whereas their betas can be very different such as 1.5, 0.2, or -0.3 depending on industry.

Equivalently, just because firm X has a lower beta than firm Y does not mean that X always sets a lower hurdle rate on its project than Y. For instance, X and Y may require similar hurdle rates whether (X's beta, Y's beta) is (1.7, 0.2) or (1.2, 0.8) or whether the discrepancy between their betas is large or small. If firms' hurdle rates are independent of their betas or the former do not vary in proportion to the latter, the discrepancy between firm's hurdle rates and (CAPM-based) discount rates will differ depending on the size of beta. That is, as firms' betas increase, so do their discount rates, while their hurdle rates remain unchanged or less changed. Therefore, the value of "hurdle rate minus discount rate for a firm" is a negative function of firm beta.

Hypothesis 1.

The discrepancy between firm hurdle rate and discount rate (= hurdle rate minus discount rate) is a negative function of the size of the firm's beta.

C. Previous Studies on the Topic

No particular relationship between $r_{IP,HR}$ and $r_{IP,CAPM}$ in (1) and (4) on an ex ante basis provides the theoretical background for the finding of previous survey papers that firms' hurdle rates usually differ significantly from their CAPM-derived costs of capital. Based on their survey of Chief Financial Officers of US companies listed in the Compustat name file, Jagannathan et. al. (2016) report that firms use discount rates almost twice as high as their costs of capital. Jacobs and Shivdasani (2012) find that, in a survey conducted by the Association for Financial Professionals (AFP), more than 300 respondents use a discount rate different from the WACC, where the former can be either greater or smaller than the latter. Driver and Temple (2010) analyze a sample of business

units from the PIMS (Profit Impact of Marketing Strategy) cross-sectional database of large firms, which consists of more than 3,000 business units of 450 North American companies, and provides data on hurdle rates. They find that well over one-fifth of the total sample reports hurdle rates below discount rates and that approximately one fifth of the sample reports hurdle rates above discount rates. Poterba and Summers (1995) find the lack of correlation between betas and hurdle rates in a survey of CEOs of all firms in the *Fortune* 1,000.[11] Graham and Harvey (2018) also find that the actual hurdle rates used by firms are 400 bp higher than their reported WACCs on average in their 18 year-long quarterly surveys from June 2000 to December 2017of U.S. Chief Financial Officers.

Most of the papers mentioned above provide their own reality-based explanations for the difference between the two rates. Jagannathan et. al. (2016) find that firms' hurdle rates are higher than their costs of capital both for financial reasons such as idiosyncratic risk of projects and for non-financial reasons such as managerial or organizational requirements. Amongst the reasons, "idiosyncratic risk" is in line with my finding from the comparison of $r_{IP,HR}$ and $r_{IP,CAPM}$. They find that firms that add the biggest risk premiums are exposed to high levels of idiosyncratic risk.[12] This implies that, if σ_{IP} is higher, firms add a bigger $k_{HR} \cdot \sigma_{IP}$ in (1) to get a higher hurdle rate, $r_{IP,HR}$, and that this rule or pattern has little to do with their betas. The rule is trivial if k_{HR} is a positive constant, and more trivial if k_{HR} is positive and $\frac{\partial k_{HR}}{\partial \sigma_{IP}} > 0$. Since firms increase their hurdle rates to account for idiosyncratic risk, σ_{IP}, regardless of their betas, $r_{IP,HR}$ in (1) is determined independently of their betas but $r_{IP,CAPM}$ in (4) is dependent on their betas, which causes the two rates to differ. This conclusion is also strengthened by Poterba and Summers (1995), who find the lack of correlation between firms' betas and hurdle

[11] Other papers such as Graham and Harvey (2001) only surveyed whether firms use a single firm-wide discount rate or project-specific ones.
[12] This is on page 460.

rates. Panousi and Papanikolaou (2012) also empirically document that firm investment falls as idiosyncratic risk rises, implying that not (only) firm beta but idiosyncratic risk matters in firms' investment decisions. Jacobs and Shivdasani (2012) find that the two rates differ since managers do not undertake analyses necessary for adjusting project risks, but instead just add one or more percentage points to costs of capital. Again, this implies that managers just set $k_{HR} \cdot \sigma_{IP}$ independent of their firms' betas.[13] Driver and Temple (2010) attribute the difference between the two rates to the existence of principal-agent relationships and real options embedded in investment projects. I contend that, in addition to these sample-based, case-specific, or reality-based reasons, the two rates, $r_{IP,HR}$ and $r_{IP,CAPM}$, are inherently different, as shown in formulas (1) and (4) and in the four other rationales.

IV. Concluding Remarks

It is difficult to attempt a task that has rarely or never been attempted by others. It is even harder if the task is to critique a longstanding, well-established convention in theory or practice. However, just because others have not tried it does not mean that the task is not worth attempting. This may have occurred to behavioral finance researchers in the 1970s when the EMH dominated the finance world in theory and practice. Similarly, the convention of using the CAPM for capital budgeting has not been decisively challenged previously. In this study, I propose and prove theoretical problems and contradictions in employing the CAPM for capital budgeting in five different ways, using only the CAPM itself, not reality-based problems or any other theories. Using the CAPM for capital budgeting is applying a price-based theory to a cash-flow-based world and a perfect diversification

[13] It can be conjectured that the "one or more percentage points" may have been determined from the size of their firms' overall business risk, σ_X, or industry risk, σ_I, in the first place.

model to undiversified firms, both of which are illogical jumps and never automatically guaranteed. The CAPM must not be used for capital budgeting because of the essential differences between securities and real investments, between stock prices and cash flows, between beta and standard deviation of returns and stocks with zero or negative betas. After the five proofs, two possible reasons for the long-term inappropriate use of the CAPM and a summary of the previous survey studies on the topic are provided. In this sense, this paper can shed light on the findings of previous studies by providing theoretical rationales for and practical interpretation of them.

The CAPM may need to lose its central characteristics, and even its name, in order to become a usable tool for capital budgeting, because stock pricing and project evaluation do not have much in common. I hope that this study will help establish that the CAPM is not an appropriate tool for capital budgeting, and that practitioners or researchers will find more scientific and practical cost of equity capital theories in the future.

References

Aktas, Elvan, and Wm McDaniel, 2009, Pragmatic problems in using beta for managerial financial applications, *Applied Financial Economics*, 19, 1345–1354.

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy*, 81, 637–659.

Brealey, Richard, Stewart Myers, and Franklin Allen, 2016, *Principles of Corporate Finance, 12th ed.* (McGraw-Hill Education, New York, NY)

Brigham, Eugene, and Michael Ehrhardt, 2017, *Financial Management, 17th ed.* (Cengage Learning, Boston, MA)

Driver, Ciaran, and Paul Temple, 2010, Why do hurdle rates differ from the cost of capital, *Cambridge Journal of Economics*, 34, 501-523.

Fama, Eugene, and Kenneth French, 1997, Industry Costs of Equity, *Journal of Financial*

Economics, 43, 153–193.

Graham, John, and Campbell Harvey, 2001, The theory and practice of corporate finance: evidence from the field, *Journal of Financial Economics*, 60, 187–243.

Graham, John, and Campbell Harvey, 2018, The equity premium in 2018, Working paper.

Harrison, Michael, and David Kreps, 1979, Martingales and arbitrages in multiperiod securities markets, *Journal of Economic Theory*, 20, 381–408.

Harrison, Michael, and Stanley Pliska, 1981, Martingales and stochastic integrals in the theory of continuous trading, , *Stochastic Processes and their Applications*, 11, 215–260.

Jacobs, Michael, and Anil Shivdasani, 2012, Do you know your cost of capital?, *Harvard Business Review*, 7/8, 118–124.

Jagannathan, Ravi, David Masta, Iwan Meier, and Vefa Tahran, 2016, Why do firms use high discount rates?, *Journal of Financial Economics*, 120, 445–463.

McNulty, James, Tony Yeh, William Schulze, and Michael Lubatkin, 2002, What's your real cost of capital?, *Harvard Business Review*, 80(10), 114-121.

Panousi, Vasia, and Dimitris Papanicolaus, 2012, Investment, Idiosyncratic risk, and Ownership, *Journal of Finance*, 67, 1113–1148.

Poterba, James, and Lawrence Summers, 1995, A CEO survey of US companies' time horizons and hurdle rates, *Sloan Management Review*, 37, 43–53.

Ross, Stephen, Randolph Westerfield, and Bradford Jordan, 2019, *Fundamentals of Corporate Finance, 12th ed.* (McGraw-Hill Education, New York, NY)

Shleifer, Andrei, 2000, *Inefficient Markets: An Introduction to Behavioral Finance* (Oxford University Press, Oxford, United Kingdom).

Wilmott, Paul, 2009, *Frequently Asked Questions in Quantitative Finance* (John Wiley and Sons, Sussex, United Kingdom).