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Key words: Options, Volatility Smile, Volatility Surface, Ad Hoc Black Scholes Model, Stochastic Volatility

JEL classification: G13, G14

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1. Introduction

Many theoretical and empirical studies have been conducted since the introduction of the Black and Scholes (1973) option-pricing model (BS henceforth). The fatal error of the BS model is the presence of the "volatility smile" (or volatility term structure) phenomenon, where the volatilities implied in option prices are observed by the exercise price (or time to maturity), a phenomenon called the "volatility surface." The current literature focuses on the usefulness of option-pricing models by extending the BS model's simple assumptions using various models that extend the assumption of the geometric Brownian motion process for the underlying assets assumed in the BS model, Such models are used for the convenience of closed-form solution derivation but are not always useful in practice. For example, academia and practice have developed a stochastic interest rates model that assumes interest rates change over time (e.g., Amin and Jarrow, 1991, 1992; Rindell, 1995; Ho, Stapleton and Subrahmanyam, 1997), a stochastic volatility model that assumes volatility changes over time (e.g., Hull and White, 1987; Johnson and Shanno, 1987; Scott, 1987; Wiggins, 1987; Heston, 1993), a jump diffusion model that assumes large jumps in underlying assets (e.g., Merton, 1976; Naik and Lee, 1990; Kou, 2002; Kou and Wang, 2004), a regime-switching model that assumes jumps in volatility (e.g., Naik, 1993), a variance gamma model (e.g., Madan, Carr and Chang, 1998), and a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model that assumes the process of underlying assets is a discrete model (e.g., Duan, 1995; Heston and Nandi, 2000; Duan and Zhang, 2001; Hsieh and Ritchken, 2005). Recent studies also include the Ad-Hoc Black-Scholes (AHBS) model, which transforms the BS model used by traders in the options market. Which one is the most useful option pricing model for pricing and hedging options?

To address this question, Bakshi, Cao and Chen (1997, 2000), Bates (2003), and Kim and Kim (2004, 2005) compare the empirical performance of various alternative option-pricing models and conclude that the stochastic volatility model improves the BS model most. However, Dumas, Fleming and Whaley (1998), Jackwerth and Rubinstein (2001), Li and Pearson (2004, 2007), Kim (2009, 2014, 2017), Choi and Ok (2011), Choi, Jordan and Ok (2012), and Dixit and Singh (2018) include in their comparison the AHBS model, a simple modification of the BS model, and show in empirical results

that the AHBS models are better than mathematically sophisticated models when stochastic volatility and jumps are considered. In short, the AHBS model is found to be the best alternative to mathematically sophisticated option-pricing models given its simplicity of implementation.

Cross-sectional options data with multiple maturities are used to estimate the parameters of the AHBS models in two ways: Parameters can be estimated using option prices based on maturity or using all option prices at all maturities, regardless of expiration. First, consider the Portfolio of Volatility Smiles method. Each set of parameters is estimated separately for each maturity, which allows a structure of volatility smile whose characteristics differ at each maturity. If the volatility smile structure for each maturity is stable over time, the Portfolio of Volatility Smiles method is useful. The second method is the Volatility Surface method, where the volatility surface is considered as a portfolio consisting of multiple volatility smiles. The Portfolio of Volatility Smiles method assumes that the volatility smile at each maturity is an individual asset and estimates the parameters that describe each asset. In the Volatility Surface method, regardless of the time to maturity, a single parameter set that describes the entire volatility surface as a single asset is estimated, thus describing the entire volatility surface structure at once. If the volatility surface structure for all maturities is stable over time, the Volatility Surface method is useful.

We compare the AHBS models' pricing and hedging performance using two ways to employ cross-sectional options data with multiple maturities. In sample pricing, one-day-ahead or one-week-ahead out-of-sample pricing and one-day or one-week hedging performance are examined. In the Portfolio of Volatility Smiles method, the parameters estimated at each time to maturity apply only to the next-day or next-week options with the corresponding time to maturity. In the Volatility Surface method, a single parameter set that is estimated regardless of expiration is applied to all next-day or next-week options. For the robustness check, we examine the performance of the stochastic volatility (SV) model, a mathematically sophisticated model that is known for its excellent pricing and hedging performance.

This study is the first to compare the performance of the Portfolio of Volatility Smiles method with the Volatility Surface method to price and hedge options with multiple times to maturity. The parameter-estimation method is as important as the selection of the option-pricing model in finding

the best option-pricing model, as no matter how good the option-pricing model, incorrectly estimated parameters result in "garbage in, garbage out." This study proposes the option-pricing model's best parameter-estimation method when one is pricing and hedging options with multiple times to maturity. Empirical studies by Kim (2009), Choi and Ok (2011), and Choi, Jordan and Ok (2012) on the AHBS models focus on pricing and hedging short-term options with a single maturity, so they do not include variables related to time to maturity as independent variables in the AHBS model. In contrast, we investigate all options with multiple maturities at any given time.

We find that, in pricing and hedging options with various times to maturity, the Portfolio of Volatility Smiles method generally outperforms the Volatility Surface method. The SV model shows the best in in-sample pricing and one-day-ahead out-of-sample pricing performance but the result is reversed by the AHBS model in one-week-ahead out-of-sample pricing. In hedging performance, the AHBS model outperforms the BS and SV models, regardless of time to maturity or moneyness. As a result, considering the volatility smile individually at each maturity is more effective in pricing and hedging options than is using the volatility surface simultaneously.

The paper is organized as follows. Section 2 introduces the AHBS models and the SV model, presenting two ways to use cross-sectional options data with multiple maturities, the Portfolio of Volatility Smiles and the Volatility Surface methods. Section 3 describes the sample data. In Section 4, we analyze the parameters of the option pricing models and examine the in-sample pricing, out-of-sample pricing, and hedging performance of several option-pricing models. Finally, Section 5 draws conclusions.

2. Option Pricing Models and Parameter Estimation Methods

2.1 The Ad-Hoc Black-Scholes Model

Despite its problems, the BS model is still widely used by market participants. When market traders use the BS model, they assume that volatility changes with exercise price and time to maturity to explain the "volatility smile" or "volatility surface" phenomenon observed in the options market. As

Dumas, Fleming and Whaley (1998) point out, this approach reduces the issues associated with the BS model's constant volatility assumptions.

The AHBS model is implemented assuming that the implied volatility for each option is related to the exercise price and time to maturity. In other words, users assume that the volatility of the underlying assets entered into the BS model is a function of the exercise price, time to maturity, or a combination thereof. However, previous studies on the AHBS model implement the models with only the exercise prices of the short-term options, taking into account that these options are the most liquid. In this study, we compare the performance of the AHBS models with the time-to-maturity factors.

There are two types of AHBS models: The "relative smile" approach, also called the "sticky volatility" model, regresses an option's implied volatility as a dependent variable on the option's relative moneyness as an independent variable. The "absolute smile" approach, also known as the "sticky delta" model, regresses the implied volatility as the dependent variable on the option's absolute strike price as the independent variable. Jackwerth and Rubinstein (2001), Li and Pearson (2007), Kim (2009), and Choi and Ok (2011) compare these two approaches and show that the "absolute smile" approach performs better than the "relative smile" approach. Studies in which the AHBS models outperform other models use the "absolute smile" method (Dumas, Fleming and Whaley, 1998; Jackwerth and Rubinstein, 2001; Li and Pearson, 2007), while studies in which the AHBS models perform poorly compared to other models use the "relative smile" method (Kirgiz, 2001; Kim and Kim, 2004). Kim (2009) also shows that the simplest AHBS model—that with fewer independent variables—outperforms more sophisticated models that assume a large number of independent variables, so the AHBS model can have an over-fitting problem. Therefore, the simple "absolute smile" approach is expected to show the best performance. The AHBS models for the Volatility Surface model are assumed in sixteen forms (Equations (1)-(16)). In the Portfolio of Volatility Smiles method, A1, A2, R1, and R2 models without the time-to-maturity variables are used.

A1T1 $$\sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot \tau_i \quad (1)$$

A1T1C $$\sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot \tau_i + \beta_4 \cdot K_i \cdot \tau_i \quad (2)$$

A1T2 $\sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot \tau_i + \beta_4 \cdot (\tau_i)^2$ (3)

A1T2C $\sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot \tau_i + \beta_4 \cdot (\tau_i)^2 + \beta_5 \cdot K_i \cdot \tau_i$ (4)

A2T1 $\sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot (K_i)^2 + \beta_4 \cdot \tau_i$ (5)

A2T1C $\sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot (K_i)^2 + \beta_4 \cdot \tau_i + \beta_5 \cdot K_i \cdot \tau_i$ (6)

A2T2 $\sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot (K_i)^2 + \beta_4 \cdot \tau_i + \beta_5 \cdot (\tau_i)^2$ (7)

A2T2C $\sigma_i = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot (K_i)^2 + \beta_4 \cdot \tau_i + \beta_5 \cdot (\tau_i)^2 + \beta_6 \cdot K_i \cdot \tau_i$ (8)

R1T1 $\sigma_i = \beta_1 + \beta_2 \cdot (S / K_i) + \beta_3 \cdot \tau_i$ (9)

R1T1C $\sigma_i = \beta_1 + \beta_2 \cdot (S / K_i) + \beta_3 \cdot \tau_i + \beta_4 \cdot (S / K_i) \cdot \tau_i$ (10)

R1T2 $\sigma_i = \beta_1 + \beta_2 \cdot (S / K_i) + \beta_3 \cdot \tau_i + \beta_4 \cdot (\tau_i)^2$ (11)

R1T2C $\sigma_i = \beta_1 + \beta_2 \cdot (S / K_i) + \beta_3 \cdot \tau_i + \beta_4 \cdot (\tau_i)^2 + \beta_5 \cdot (S / K_i) \cdot \tau_i$ (12)

R2T1 $\sigma_i = \beta_1 + \beta_2 \cdot (S / K_i) + \beta_3 \cdot (S / K_i)^2 + \beta_4 \cdot \tau_i$ (13)

R2T1C $\sigma_i = \beta_1 + \beta_2 \cdot (S / K_i) + \beta_3 \cdot (S / K_i)^2 + \beta_4 \cdot \tau_i + \beta_5 \cdot (S / K_i) \cdot \tau_i$ (14)

R2T2 $\sigma_i = \beta_1 + \beta_2 \cdot (S / K_i) + \beta_3 \cdot (S / K_i)^2 + \beta_4 \cdot \tau_i + \beta_5 \cdot (\tau_i)^2$ (15)

R2T2C $\sigma_i = \beta_1 + \beta_2 \cdot (S / K_i) + \beta_3 \cdot (S / K_i)^2 + \beta_4 \cdot \tau_i + \beta_5 \cdot (\tau_i)^2 + \beta_6 \cdot (S / K_i) \cdot \tau_i$ (16)

where σ_i is the implied volatility of option i with the strike price K_i, the time to maturity τ_i and the underlying asset S.

The first eight models, which use the exercise price directly as an independent variable are "absolute smile" models, and the remaining eight models, which use moneyness as the independent variable are "relative smile" models. For example, the A2T2C model runs with the intercept, exercise price, exercise price squared, time to maturity, time to maturity squared, and the product of exercise price and time to maturity, while the R2T2C model runs with the intercept, moneyness, moneyness squared, time to maturity, time to maturity squared, and the product of moneyness and time to maturity. More independent variables can be used, such as the cube of the time to maturity or the strike price (moneyness), but since the additional independent variables' marginal effects are not significant, we consider only linear and squared terms as independent variables.

The AHBS models are implemented using four steps. First, the implied volatility is estimated from each option. Then the regression coefficients $\beta_i (i = 1, \cdots, 6)$ are estimated with implied volatility as

a dependent variable and exercise price (or moneyness) and time to maturity as independent variables. Next, theoretical implied volatility is obtained by substituting the regression coefficients estimated in the second step—the exercise price (or moneyness), time to maturity of each option, and the product of the exercise price (or moneyness) and time to maturity—into the AHBS model. Finally, the theoretical option price is derived by substituting the theoretical implied volatility calculated in the third step into the BS model (Equations (17)-(19)).

$$C(t,\tau;K) = S_t N(d_1) - Ke^{-r\tau} N(d_2) \tag{17}$$

$$P(t,\tau;K) = Ke^{-r\tau} N(-d_2) - S_t N(-d_1) \tag{18}$$

$$d_1 = \frac{\ln[S_t / K] + (r + \sigma^2 / 2)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau} \tag{19}$$

where $C(t,\tau;K)$ and $P(t,\tau;K)$ are the call and put option prices with strike price K and time to maturity τ at time t, respectively; S_t is the underlying asset value at time t; r is the risk-free interest rate; σ is the volatility; and $N(\cdot)$ is the cumulative standard normal distribution function.

2.2 The Stochastic Volatility Option Pricing Model

We consider Heston's (1993) model, which assumes continuous-time stochastic volatility and that the volatility correlates with the underlying asset prices and shows a mean-reverting tendency. It is well suited for a benchmark because it offers a closed-form solution and has excellent pricing and hedging performance in several option markets. The diffusion processes for the underlying asset and the volatility are defined as in Equations (20) and (21).

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_{S,t} \tag{20}$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_{v,t} \tag{21}$$

where $dW_{S,t}$ and $dW_{v,t}$ are wiener processes with a correlation of ρ, v_t is the instantaneous

variance, κ is the speed of adjustment to long-run variance θ, and σ_v is the volatility of the variance. Assuming the diffusion process of underlying assets and volatility as shown in equations (20)

$$C(t,\tau;K) = S_t P_1 - Ke^{-r\tau} P_2 \tag{22}$$

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left[\frac{e^{-i\phi \ln[K]} f_j(x, v_t, \tau;\phi)}{i\phi}\right] d\phi \quad (j = 1,2) \tag{23}$$

where $\mathrm{Re}[\cdot]$ is the real part of the complex number, i is an imaginary number, $f_j(x, v_t, \tau;\phi) = \exp[C(\tau;\phi) + D(\tau;\phi)v_t + i\phi x]$, and $C(\tau;\phi)$ and $D(\tau;\phi)$ are the functions of θ, κ, ρ, σ and v_t. The put option price can be obtained from the put-call parity.

In the SV model, volatility and structural parameters are not observable, so they are estimated daily. Since the SV model exists in the closed form solution, the most commonly used way of estimating parameters is a non-linear least squares method that minimizes the sum of squared errors between model and market prices. While parameters can also be estimated from a historical time series of underlying asset prices, historical data reflect only what has happened in the past, do not provide good estimates of future volatility, and cannot estimate risk premiums contained in the volatility. Cross-sectional option prices can be used to estimate the risk premium of risk factors, with the advantage of using the forward-looking information contained in the option prices.

To estimate the parameters, we minimize the sum of squared errors between the model and market prices, as shown in equation (24).

$$\min_{\varphi_t} \sum_{i=1}^{N_t} \left[O_{i,t}^* - O_{i,t}\right]^2 \quad (t = 1,\ldots,T) \tag{24}$$

where $O_{i,t}^*$ is the model price of option i at time t, $O_{i,t}$ is the market price of option i at time t, N_t is the number of options traded at time t, and T is the number of trading days included in our sample.

2.3 The Portfolio of Volatility Smiles Method and the Volatility Surface Method

The central purpose of this study is to compare two ways of using cross-sectional options data for estimating parameters. The Portfolio of Volatility Smiles method allows each set of parameters to be estimated separately for each maturity. Since we consider six expirations, from T1 to T6, six parameter sets are estimated per day. If the number of parameters in an option-pricing model is five, thirty (5 parameters × 6 maturities) parameters are estimated per day. This method has the advantage of applying the structure of volatility smile whose characteristics differ at each maturity. The Portfolio of Volatility Smiles method is useful if the volatility smile structure for each maturity is stable over time. The Volatility Surface method, on the other hand, allows a single parameter set that describes the entire volatility surface to be estimated. The volatility surface can be thought of as a portfolio of multiple volatility smiles. The Portfolio of Volatility Smiles method assumes that the volatility smile at each maturity is an individual asset and estimates the parameters that describe each volatility smile. Regardless of the time to maturity, the Volatility Surface method estimates a single parameter set that describes the entire volatility surface, which is assumed to be a single asset. This method has the advantage of considering the entire volatility surface structure at once. Therefore, if the volatility surface structure for all maturities is stable over time, the Volatility Surface method is useful.

We examine several options-pricing models' pricing and hedging performance using two ways of employing cross-sectional data. In the Portfolio of Volatility Smiles method, the parameters estimated at each time to maturity apply only to the next-day or next-week options with the corresponding time to maturity. In the Volatility Surface method, a single-parameter set, estimated regardless of expiration, is applied to all next-day or next-week options.

3. Data

The SPX (S&P 500 index) option data used in this paper are from the IVY DB database of OptionMetrics LLC. The data include end-of-day bid and ask quotes, implied volatilities, open interest, and daily trading volume for the SPX options traded on the Chicago Board Options Exchange from

January 4, 1996, through August 31, 2015. SPX option contracts have three advantages. First, it is easy to compare our results with those of extant research, as several extant studies analyze the S&P 500 options markets, so implications can be easily derived by comparing our results with those of previous studies. Second, SPX option contracts trade options with various times to maturity. Although the trading volumes in emerging options markets are plentiful, they are concentrated in the short-term options, but a fully traded options market with a variety of maturities is required to conduct our study on the volatility surface. Third, the S&P 500 option is a European option, and a number of option pricing models have been derived to price European options. Evaluating the American options requires additional tweaking.

The price of the option is calculated as the average of the best bid and ask quotes at the end of each trading day. We use all out-of-the-money (OTM)[1] call and put options but exclude duplicates and missing values in bid or ask quotes, strike prices or times to maturity; options with less than seven days (to avoid liquidity bias) prices lower than 3/8 (to control for price discreteness) and prices that do not satisfy the no-arbitrage conditions (Bakshi, Cao and Chen, 1997, 2000; Kim and Kim, 2004, 2005; Choi and Ok, 2011; Choi, Jordan and Ok, 2012).

Table 1 summarizes the option prices and numbers by dividing them into time to maturity and moneyness. We use six expressions of moneyness as S (S&P 500 index)/K (strike price) and six maturities, where T1 is the shortest expiration on each trading day and T6 is the sixth-shortest maturity on each trading day. The average option price for the entire sample is 15.0997, and our sample consists of 1,366,012 option prices. The options with the second-shortest maturity, T2, make up 23 percent of the sample, and the options with relatively short maturities, T1, T2, and T3, account for about 60 percent of the sample. These percentages are consistent the fact high trading volume of short-term options. The options whose S/K is less than 0.94 or greater than 1.06, which are deep OTM options,

[1] In-the-money (ITM) options are not considered because their trading volumes are low, so information about them can be doubtful. Moreover, including them may cause duplicates from double counting since ITM calls and OTM puts are equivalent for a specified strike price according to put-call parity.

account for the largest percentage of the sample (about 67%). Among these, deep OTM put options with S/K greater than 1.06 account for about half of the sample, which is consistent with the high trading volume of OTM put options.

Table 2 shows the average implied volatilities of options by time to maturity and moneyness. The table shows a humped pattern by the time to maturity. The average implied volatilities of T1 (with the shortest expiration) and T6 (with the longest expiration) are 0.2486 and 0.2465, respectively; the implied volatility increases from T1 to T2 and then decreases from T3 to T6. Implied volatility by the moneyness reveals an obvious pattern, as regardless of expiration, the implied volatility decreases as from deep OTM call options to near-the-money options and then increases from near-the-money options to deep OTM put options, revealing the so-called "volatility smile" or "volatility sneer." Deep OTM put options with S/K greater than 1.06 have the highest implied volatility, which is in line with Black's (1976) leverage effect hypothesis and the fact that the risk-neutral distribution estimated from option prices is negatively skewed. This "volatility smile" or "volatility sneer" phenomenon confirms the need for options-pricing models other than the BS model.

4. Pricing and Hedging Performance

We examine the in-sample pricing, out-of-sample pricing and hedging performance of several option pricing models. To compare pricing and hedging performance, we use mean absolute errors (MAE) and root mean squared errors (RMSE):

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{i=1}^{N_t}\left|O_i(t,\tau;K) - O_i^*(t,\tau;K)\right| \tag{25}$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{i=1}^{N_t}\left[O_i(t,\tau;K) - O_i^*(t,\tau;K)\right]^2} \tag{26}$$

where $O_{i,t}$ is the market price of option i at time t, $O_{i,t}^*$ is the model price of option i at time t, N_t is the number of options traded at time t, and T is the number of trading days included in

our sample. In the Portfolio of Volatility Smile method, N_t is the number of options with a specific time to maturity at time t, and in the Volatility Sneer method, N_t is the number of options at time t.

The MAE measures the average pricing and hedging error of each option-pricing model, and the RMSE measures the volatility of pricing and hedging errors. If both the MAE and the RMSE of an option-pricing model are small, we can assume that the errors are clustered around zero, indicating that the model performs well.

4.1 In-sample Pricing Performance

Table 3 shows the mean and the standard errors of the parameter estimates for each model. The average and the standard deviation of R^2s for the AHBS models are reported. In "absolute smile" approaches, the implied volatility is treated as a function of strike price (K) and the time to maturity (T) with higher-order terms and/or interaction terms. In "relative smile" approaches, the implied volatility is treated as a function of the moneyness (S/K) and the time to maturity (T) with higher-order terms and/or interaction terms. For the AHBS models, each parameter is estimated using ordinary least squares. For the BS and SV models, each parameter is estimated by minimizing the sum of squared errors between the model and market option prices.

Panel A shows the estimates of the parameters of the AHBS models. The coefficient for K or S/K is negative for the "absolute smile" approaches and positive for the "relative smile" approaches. This result is consistent with the results shown in Table 2, where the implied volatility increases as K decreases (as S/K increases). For the squared terms K^2 and $(S/K)^2$, the coefficient for the "absolute smile" approaches is positive and negative for the "relative smile" approaches. This result supports the concave property of implied volatility for K and its convex property for S/K. Consistent with decreasing implied volatility as the time to maturity increases, the coefficient for T is negative for both the "absolute smile" and the "relative smile" approaches. The coefficient for T^2 is positive independent of the type of the AHBS model and is in line with the property of concave function of implied volatility

with T. The combination term, the product of K (or S/K) and T, is positive in the "absolute smile" approaches and negative in the "relative smile" approaches. Panel B shows the estimates of the parameters of the structural models. The average implied volatility of the BS model is approximated at about 19 percent, and taking the root of the variance estimate of the SV model, the volatility is estimated to be about 20 percent. The average correlation between stock index returns and volatility is negative, which demonstrates the sneer pattern of implied volatility and the shape of the negatively skewed risk-neutral distribution (Dennis and Mayhew, 2002). Similar estimates are shown for both the Portfolio of Volatility Smiles and the Volatility Surface methods.

Table 4 reports in-sample pricing and one-day-ahead and one-week-ahead out-of-sample pricing. Each model is estimated every day during the sample period. In-sample pricing errors are calculated by applying the current estimated parameters to price the current options. In-sample pricing performance measures how well each option-valuation model explains the current market prices. As expected, the models with many parameters are more likely to explain the current options market than the models with few parameters. For example, in the regression analysis, the number of independent variables coincides with the increase in the R^2 value that measures the explanatory power.

In comparing the Portfolio of Volatility Smiles method and Volatility Surface method, we find that, regardless of the option-pricing model used, the Portfolio of Volatility Smiles method provides better in-sample pricing performance than the Volatility Surface method. This result seems obvious, as the Portfolio of Volatility Smiles method, which estimates parameters individually for each time to maturity, uses more parameters than the Volatility Surface method, which estimates a single parameter set irrespective of time to maturity for the same option sample. In this study, the option sample consists of six types of maturities (from T1 to T6). For example, the SV model with five parameters uses thirty parameters (5 parameters × 6 types of maturities) in the Portfolio of Volatility Smiles method and just five parameters in the Volatility Surface method. This result is consistent with our conjecture that the models with a large number of parameters have better in-sample pricing performance.

We also find that, the more complex the model, the better the in-sample pricing performance. The SV model with the most parameters shows the smallest error, and the BS model with the smallest number of parameters shows the largest error, as expected in both the Portfolio of Volatility Smiles and the Volatility Surface methods. As the number of parameters increases, the models' errors decrease. In the Portfolio of Volatility Smiles, A2 and R2 show better performance than A1 and R1. In the Volatility Surface method, in-sample pricing performance improves when the square term of the strike price (or moneyness) or the time to maturity is considered; that is, the error in A2T2 is smaller than the error in A1T1. Combining strike price (or moneyness) and time to maturity is the most obvious way to improve the in-sample pricing performance.

Finally, we find that the "absolute smile" approaches have smaller errors than the "relative smile" approaches, as with previous studies. When considering the time to maturity factor along with the strike price or moneyness, we observe that the "absolute smile" approaches have excellent pricing performance.

In conclusion, our study shows that the best in-sample pricing performance comes from using a large number of parameters, so the SV model with the largest number of parameters using the Portfolio of Volatility Smiles method that estimates the parameters individually for each time to maturity is the best. The results are the same whether the measure of error is MAE or RMSE. However, the question remains: will the SV model and the Portfolio of Volatility Smiles methods using many parameters continue to perform well in out-of-sample pricing or hedging without causing over-fitting problems?

4.2 Out-of-sample Pricing Performance

Next, we examine the out-of-sample pricing performance in Table 4. One-day (one-week) ahead out-of-sample pricing errors are calculated by applying the previous day's (week's) estimated parameters to price the current options. In in-sample pricing performance, an option-valuation model with a large number of parameters performs better than a model with a small number of parameters. However, these results can be caused by over-fitting problems. Put differently, an option-pricing

model that explains the current option prices may well be less likely to predict future option prices. For practical purposes, the predictability of future option prices can be more important than the explanatory power of current option prices. Out-of-sample pricing performance verifies how well an option-pricing model predicts the levels of future option prices.

The Portfolio of Volatility Smiles method shows lower errors than the Volatility Surface method, confirming that the results of the in-sample pricing performance are consistently observed in the results of the out-of-sample pricing performance for both MAE and RMSE measures.[2] Then, regardless of the option-pricing model used, the longer the forecasting period, the larger the error. In the Portfolio of Volatility Smiles method, the one-day-ahead and one-week-ahead out-of-sample pricing errors of all the models increase 15 percent (23%) and 42 percent (45%), respectively, based on MAE (RMSE), compared to in-sample pricing errors. In the Volatility Surface method, one-day-ahead and one-week-ahead out-of-sample pricing errors increase by 8 percent (15%) and 26 percent (39%), respectively in terms of MAE (RMSE). The Volatility Surface method has fewer over-fitting problems than the Portfolio of Volatility Smiles method, suggesting that the Volatility Surface method depicts the entire volatility surface at once. The difference between the BS model and the best model decreases in the out-of-sample pricing: In the Portfolio of Volatility Smiles method, the ratio of the BS model to the SV model for MAE is 7.1321 for in-sample pricing errors, while the ratio of the BS model to the SV model and the A1 model decreases to 3.9893 and to 2.7929 for one-day-ahead and one-week-ahead out-of-sample errors, respectively. As the term of the out-of-sample pricing increases, the difference between the BS model and the best model decreases. In the Volatility Surface method, the ratios of the BS model to the SV model for MAE are 5.2586, 3.8803, and 2.7146 for in-sample, one-day-ahead out-of-sample, and one-week-ahead out-of-sample pricing errors, respectively.

In the Portfolio of Volatility Smiles method, the SV model with the largest number of parameters is the best model for one-day-ahead out-of-sample pricing performance. However, when the forecasting

[2] For the SV model, the Volatility Surface method shows better out-of-sample pricing performance than the Portfolio of Volatility Smiles method when the forecasting period is one week.

period increases from one day to one week, the A1 model, which is the simplest of the "absolute smile" approaches, performs best. In other words, the increased forecasting period reveals the over-fitting problem of the SV model, which is the leader in in-sample pricing and one-day-ahead out-of-sample pricing. On the other hand, in the Volatility Surface method, the SV shows the best performance in in-sample, one-day-ahead and one-week-ahead out-of-sample pricing, so this method can alleviate the over-fitting problems that can occur as the forecasting period increases.

In the AHBS models, the "absolute smile" approaches perform better than the "relative smile" approach in both the Portfolio of Volatility Smiles method and the Volatility Surface method, regardless of the forecasting period.[3] As the number of parameters increases, the models' errors decrease, and when the squared term of the strike price (or moneyness) is considered, out-of-sample pricing performance improves. The combination of the strike price (or moneyness) and the time to maturity improves the out-of-sample pricing performance the most, regardless of the forecasting period. However, the squared term of time to maturity generally does not significantly improve out-of-sample pricing performance, and when the combination terms are considered together, the out-of-sample pricing performance deteriorates.

Table 5 reports one-day-ahead and one-week-ahead out-of-sample pricing errors in terms of the time to maturity, measured by mean absolute errors. As Table 1 shows, the short-term options with maturities of T1, T2, and T3 account for 60 percent of our sample, and the trading volume in the options market is concentrated in the short-term options. In addition, previous studies show that the BS model's performance is poor in the short-term options, so the improvement of the alternative options-pricing models is critical for these options. Therefore, the performance in short-term options is more important than that in long-term options.

We find that the Portfolio of Volatility Smiles method performs generally better than the Volatility Surface method for all maturities. In the "absolute smile" approaches, the Portfolio of Volatility Smiles

[3] Because of these results, we do not report the results of the "relative smile" approaches that are inferior to the "absolute smile" approaches.

method performs consistently better than the Volatility Surface method,[4] and in the SV model, the Portfolio of Volatility Smiles method outperforms the Volatility Surface method in one-day-ahead out-of-sample pricing. However, in one-week-ahead out-of-sample pricing, the Volatility Surface method is superior to the Portfolio of Volatility Smiles method for all maturities. The results shown in Table 4 are maintained regardless of expiration.

The "absolute smile" approaches also show better performance than the SV model for the short-term options in both the Portfolio of Volatility Smiles and the Volatility Surface methods. Kim (2009) and Kim and Song (2020), in analyzing the performance of the AHBS models in the short-term options, show that the "absolute smile" approach is superior to the SV model, and we confirm these results again.

We also find that, generally, as the time to maturity increases from T1 to T6, the out-of-sample pricing errors increase regardless of the option-pricing model used. This result is consistent with the increasing nature of option prices as the time to maturity increases. However, some models have more errors in T1, the shortest expiration, than they do in other shorter expirations, T2 and T3. This result is consistent with the results of prior studies that report serious errors in pricing short-term options.

Table 6 reports one-day-ahead and one-week-ahead out-of-sample pricing errors by moneyness, measured by mean absolute errors. As Table 1 shows, the deep OTM put options (S/K >= 1.06) account for half of our sample, and when combined with deep OTM call options (S/K < 0.94), they account for 69 percent of the sample. In the options market, the trading volume is concentrated in the OTM options. In addition, previous studies show that the BS model's performance is poor in the OTM options. Since the improvement of the alternative options-pricing models is essential for the OTM options, the out-of-sample pricing performance in the OTM options is more important than the performance in the options with other moneyness. The Portfolio of Volatility Smiles method performs generally better than the Volatility Surface method for all moneyness: In "absolute smile" approaches, the Portfolio of

[4] In one-week-ahead out-of-sample pricing, the Volatility Surface method outperforms the Portfolio of Volatility Smiles method at the shortest expiration, T1.

Volatility Smiles method performs consistently better than the Volatility Surface method, and in the SV model, the Portfolio of Volatility Smiles method outperforms the Volatility Surface method in one-day-ahead out-of-sample pricing. However, in one-week-ahead out-of-sample pricing, the Volatility Surface method outperforms the Portfolio of Volatility Smiles method. Thus, the results shown in Table 4 are maintained regardless of the moneyness. We also find that the "absolute smile" approaches are superior to other models in the OTM put options, and the SV model demonstrates competitive performance in the OTM call options. In particular, the performance improvement compared to the BS model is important in the OTM put options, so the excellence of the AHBS models is shown again. Finally, as the moneyness moves from OTM to near-the-money options (0.97 < S/K < 1.03), the errors increase regardless of the option-pricing model used because near-the-money options are more expensive than the OTM options.

4.3 Hedging Performance

The hedging performance is as important as the pricing performance in measuring the performance of option-pricing methods. If out-of-sample pricing predicts future levels of option prices, hedging predicts the volatility of future option prices. Among several methods to measure hedging performance, we use the hedging performance measure that Dumas, Fleming and Whaley (1998), Gemmill and Saflekos (2000), and Kim and Kim (2005) propose. Hedging errors are defined as the difference between the change in the market price and the change in the theoretical model price, as follows:

$$\text{Hedging Errors} = \Delta O_i(t,\tau;K) - \Delta O_i^*(t,\tau;K) \tag{27}$$

where $\Delta O_i(t,\tau;K)$ is the change in the reported option price from day t to day $t+1$ or $t+7$, and $\Delta O_i^*(t,\tau;K)$ is the change in the model's theoretical value.

Table 7 reports one-day-ahead and one-week-ahead hedging errors. The parameters and spot volatility implied by all options of the previous day or week are used to obtain the current day's hedged portfolio. We find that the Portfolio of Volatility Smiles method generally outperforms the Volatility Surface method—these results are the same as those of in-sample and out-of-sample pricing[5]—and that, regardless of the options-pricing model and the forecasting period, the "absolute smile" approaches outperform the BS and SV models in both the Portfolio of Volatility Smiles and the Volatility Surface methods. These results are consistent with those of Kim (2009) and Kim and Song (2020). In addition, in the AHBS models, as the number of parameters increases, the hedging errors of the models decrease, and when the squared term of the strike price is considered, hedging performance improves. The combination of the strike price and the time to maturity is the key factor to improving the hedging performance, regardless of the forecasting period.

Table 8 reports hedging errors by time to maturity, measured by mean absolute errors. For all models, the Portfolio of Volatility Smiles method is generally superior to the Volatility Surface method, regardless of maturity.[6] The "absolute smile" approaches also perform consistently better than the BS and SV models for all maturities. Table 9 reports hedging errors by the moneyness. For all models, the Portfolio of Volatility Smiles method is generally superior to the Volatility Surface method, regardless of moneyness.[7]

In sum, the Portfolio of Volatility Smiles method is superior to the Volatility Surface method in the hedging performance, and the "absolute smile" approaches outperform other models regardless of maturity and moneyness.

5. Conclusion

[5] The exception is the SV model's one-week-ahead hedging errors.

[6] The Volatility Surface method is superior to the Portfolio of Volatility Smiles method in one-week-ahead hedging errors of the short-term options (T1, T2, and T3) for the SV model.

[7] The Volatility Surface method is superior to the Portfolio of Volatility Smiles method in the OTM put options ($1.00 < S / K < 1.06$) for the BS and SV models.

We examine the pricing and hedging performance of several options-pricing models using two parameter-estimation methods to employ cross-sectional options data with multiple maturities. The Portfolio of Volatility Smiles method allows each set of parameters to be estimated separately for each maturity and has the advantage of considering the structure of volatility smile whose characteristics differ at each maturity. The second method is the Volatility Surface method. Regardless of the time to maturity, a single parameter set that describes the entire volatility surface is estimated. This method enjoys the benefit of taking the entire volatility surface structure at once.

The results are as follows. When pricing and hedging options with various times to maturity, the Portfolio of Volatility Smiles method generally outperforms the Volatility Surface method, irrespective of the option-pricing model used, maturity, and moneyness. In other words, considering the volatility smile at each maturity is more effective than employing all of the volatility surfaces at once. In in-sample pricing and one-day-ahead out-of-sample pricing, the SV model performs best, but in one-week-ahead out-of-sample pricing, the SV model is outperformed by the AHBS models, confirming that the over-fitting problem may occur in the SV model. As for the hedging performance, the AHBS models outperform the BS and SV models regardless of time to maturity or moneyness. Finally, the "absolute smile" approaches perform better than the "relative smile" approaches.

We find that, overall, the Portfolio of Volatility Smiles method is superior to the Volatility Surface method. Using cross-sectional option data by maturity is more effective than using panel option data that employs both cross-section and time-series option data for pricing and hedging options.

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Table 1: Options Data

This table shows the average options prices and the number (in parentheses) by time-to-maturity and the moneyness of S&P 500 options. Six expressions of moneyness as S (S&P 500 index)/K (strike price) and six maturities, where T1 is the shortest expiration on each trading day and T6 is the sixth-shortest maturity on each trading day are used.

	<0.94	0.94-0.97	0.97-1.00	1.00-1.03	1.03-1.06	>=1.06	Total
T1	2.2594	4.1378	11.613	13.9098	6.8526	2.4903	6.0115
	(18,377)	(24,377)	(33,638)	(32,180)	(28,485)	(99,944)	(237,001)
T2	3.5213	9.1208	23.1165	26.553	16.9139	4.6211	9.4339
	(43,561)	(28,666)	(29,716)	(27,886)	(24,387)	(166,192)	(320,408)
T3	5.4374	16.7295	33.4245	36.9905	26.3047	7.1233	13.4345
	(45,483)	(20,772)	(22,123)	(20,378)	(17,643)	(135,322)	(261,721)
T4	8.8094	27.666	45.7303	47.3428	35.7563	10.1754	17.1631
	(43,797)	(13,212)	(13,132)	(12,375)	(11,308)	(104,724)	(198,548)
T5	14.1678	44.4315	64.5972	62.588	50.0892	15.3063	23.4724
	(48,315)	(10,196)	(9,652)	(9,329)	(8,489)	(92,081)	(178,062)
T6	20.5171	61.9674	82.5792	77.3904	64.8239	20.5275	29.8087
	(49,376)	(8,169)	(8,062)	(7,526)	(7,091)	(90,048)	(170,272)
Total	10.1467	19.3049	31.8664	33.6822	24.2393	9.1591	15.0997
	(248,909)	(105,392)	(116,323)	(109,674)	(97,403)	(688,311)	(1,366,012)

Table 2: Implied Volatility

This table shows the average implied volatilities of options by time-to-maturity and moneyness of S&P 500 options. Six expressions of moneyness as S (S&P 500 index)/K (strike price) and six maturities, where T1 is the shortest expiration on each trading day and T6 is the sixth-shortest maturity on each trading day are used.

	<0.94	0.94-0.97	0.97-1.00	1.00-1.03	1.03-1.06	>=1.06	Total
T1	0.2643	0.1586	0.1520	0.1833	0.2094	0.3323	0.2486
T2	0.1939	0.1430	0.1550	0.1889	0.2067	0.3186	0.2510
T3	0.1733	0.1493	0.1593	0.1943	0.2093	0.3086	0.2442
T4	0.1687	0.1537	0.1617	0.1996	0.2113	0.3047	0.2433
T5	0.1681	0.1610	0.1694	0.2136	0.2222	0.3069	0.2445
T6	0.1650	0.1629	0.1679	0.2168	0.2248	0.3054	0.2441
Total	0.1802	0.1525	0.1578	0.1935	0.2112	0.3132	0.2465

Table 3: Parameters

This table represents the mean and the standard errors (in parentheses) of the parameter estimates for each model. The average and the standard deviation of R^2s for the AHBS models are reported. Models beginning with "A" represent the "absolute smile" approaches. Models beginning with "R" represent the "relative smile" approaches. In "absolute smile" approaches, the implied volatility is treated as a function of strike price (K) and the time-to-maturity (T) with higher-order terms and/or interaction terms. In "relative smile" approaches, its implied volatility is treated as a function of moneyness (S/K) and the time-to-maturity (T) with higher-order terms and/or interaction terms. For the AHBS models, each parameter is estimated using ordinary least squares every day. $BS_{\text{Portfolio of Volatility Smiles}}$ and $SV_{\text{Portfolio of Volatility Smiles}}$ are the Black-Scholes (1973) option pricing model and Heston's (1993) option pricing model with the stochastic volatility using the Portfolio of Volatility Smiles method. $BS_{\text{Volatility Surface}}$ and $SV_{\text{Volatility Surface}}$ are the Black-Scholes (1973) option pricing model and Heston's (1993) option pricing model with the stochastic volatility using the Volatility Surface method. For the BS and SV models, each parameter is estimated by minimizing the sum of squared errors between the model and market option prices every day.

Panel A: Ad-Hoc Black Scholes Models

	Intercept	K or S/K	K^2 or $(S/K)^2$	T	T^2	Combination	R^2
A1	0.7326	-0.0004					0.9446
	(0.0015)	(1.4270E-06)					
A2	1.4152	-0.0017	5.8926E-07				0.9609
	(0.0114)	(2.1136E-05)	(1.1714E-08)				
A1T1	0.6409	-0.0004		-0.0145			0.8527
	(0.0018)	(1.6356E-06)		(0.0008)			
A1T1C	0.8561	-0.0006		-0.4648		0.0004	0.9196
	(0.0023)	(2.5031E-06)		(0.0030)		(2.6556E-06)	
A1T2	0.6422	-0.0004		-0.0285	0.0181		0.8575
	(0.0019)	(1.6471E-06)		(0.0023)	(0.0023)		
A1T2C	0.8632	-0.0006		-0.5084	0.0505	0.0004	0.9240
	(0.0024)	(2.5273E-06)		(0.0042)	(0.0023)	(2.6822E-06)	
A2T1	0.7109	-0.0005	3.4616E-08	-0.0237			0.8831
	(0.0047)	(8.5001E-06)	(5.5826E-09)	(0.0009)			
A2T1C	0.9627	-0.0007	7.5983E-08	-0.4639		0.0004	0.9390
	(0.0037)	(5.7685E-06)	(3.6975E-09)	(0.0030)		(2.4651E-06)	
A2T2	0.7159	-0.0005	3.6736E-08	-0.0464	0.0284		0.8882
	(0.0048)	(8.6301E-06)	(5.6665E-09)	(0.0025)	(0.0023)		
A2T2C	0.9762	-0.0008	8.1178E-08	-0.5177	0.0631	0.0004	0.9437
	(0.0038)	(5.7082E-06)	(3.6338E-09)	(0.0042)	(0.0023)	(2.5291E-06)	
R1	-0.2309	0.4267					0.9405
	(0.0016)	(0.0015)					
R2	-0.0924	0.1043	0.1833				0.9416
	(0.0093)	(0.0187)	(0.0094)				
R1T1	-0.0778	0.2942		-0.0416			0.8000
	(0.0014)	(0.0008)		(0.0009)			
R1T1C	-0.3213	0.5180		0.4416		-0.4374	0.9009
	(0.0020)	(0.0014)		(0.0027)		(2.4581E-03)	

R1T2	-0.0767	0.2941		-0.0498	0.0079		0.8059
	(0.0015)	(0.0008)		(0.0026)	(0.0025)		
R1T2C	-0.3187	0.5253		0.3701	0.1115	-0.4510	0.9078
	(0.0021)	(0.0015)		(0.0031)	(0.0025)	(0.0027)	
R2T1	-0.4456	0.8825	-0.2294	-0.0224			0.8799
	(0.0044)	(0.0065)	(0.0027)	(0.0009)			
R2T1C	-0.5219	0.8814	-0.1630	0.3462		-0.3419	0.9290
	(0.0037)	(0.0052)	(0.0020)	(0.0021)		(0.0019)	
R2T2	-0.4395	0.8783	-0.2275	-0.0545	0.0407		0.8851
	(0.0045)	(0.0066)	(0.0028)	(0.0024)	(0.0023)		
R2T2_C	-0.5138	0.8770	-0.1584	0.2863	0.0967	-0.3551	0.9349
	(0.0038)	(0.0053)	(0.0020)	(0.0028)	(0.0025)	(0.0021)	

Panel B: Structural Models

	σ
$BS_{\text{Portfolio of Volatility Smiles}}$	0.1891
	(0.0004)
$BS_{\text{Volatility Surface}}$	0.1906
	(0.0008)

	κ	θ	σ_v	ρ	v_t
$SV_{\text{Portfolio of Volatility Smiles}}$	0.7975	0.1292	0.7606	-0.6957	0.0409
	(0.0153)	(0.0021)	(0.0047)	(0.0009)	(0.0006)
$SV_{\text{Volatility Surface}}$	0.8840	0.1049	0.7864	-0.7538	0.0460
	(0.0165)	(0.0015)	(0.0050)	(0.0011)	(0.0007)

Table 4: Pricing Performance

This table reports in-sample pricing, one-day-ahead and one-week-ahead out-of-sample pricing errors for the S&P 500 options. Each model is estimated every day during the sample period. In-sample pricing errors are calculated by applying the current estimated parameters to price the current options. One-day-ahead (one-week-ahead) out-of-sample pricing errors are calculated by applying the previous day's (week's) estimated parameters to price the current options. In the Portfolio of Volatility Smiles method, each set of parameters is estimated separately for each maturity, and in the Volatility Surface method, a single parameter set is estimated for every maturity. Models beginning with "A" represent the "absolute smile" approaches. Models beginning with "R" represent the "relative smile" approaches. In "absolute smile" approaches, the implied volatility is treated as a function of strike price (K) and the time-to-maturity (T) with higher-order terms and/or interaction terms. In "relative smile" approaches, its implied volatility is treated as a function of moneyness (S/K) and the time-to-maturity (T) with higher-order terms and/or interaction terms. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with the stochastic volatility. Panel A represents mean absolute errors and Panel B does root mean squared errors.

Panel A: Mean Absolute Errors

	Portfolio of Volatility Smiles				Volatility Surface		
	In	One Day	One Week		In	One Day	One Week
BS	5.5894	5.6456	5.7897	BS	5.7540	5.7855	5.8746
A1	1.4455	1.6473	2.0597	A1T1	2.3715	2.4695	2.7315
				A1T1C	1.7737	1.9013	2.2426
				A1T2	2.3124	2.4505	2.7804
				A1T2C	1.7322	1.9049	2.3330
A2	1.2875	1.5704	2.0730	A2T1	2.2031	2.3015	2.5643
				A2T1C	1.6886	1.8306	2.1918
				A2T2	2.1374	2.2753	2.6034
				A2T2C	1.6566	1.8387	2.2826
R1	2.2524	2.5113	3.0511	R1T1	3.3597	3.5102	3.9063
				R1T1C	2.5470	2.7444	3.2335
				R1T2	3.3112	3.5106	3.9988
				R1T2C	2.5699	2.8016	3.3868
R2	1.4386	1.9163	2.7321	R2T1	2.3112	2.5335	3.0499
				R2T1C	1.8712	2.1285	2.7037
				R2T2	2.2548	2.5074	3.0798
				R2T2C	1.8767	2.1646	2.8293
SV	0.7837	1.4152	2.4423	SV	1.0942	1.4910	2.1641

Panel B: Root Mean Squared Errors

	Portfolio of Volatility Smiles				Volatility Surface		
	In	One Day	One Week		In	One Day	One Week
BS	7.4891	7.6555	8.0590	BS	7.7672	7.8831	8.1964
A1	2.9121	3.2127	3.6787	A1T1	3.9577	4.1897	4.7129
				A1T1C	3.2424	3.5676	4.2808
				A1T2	3.8529	4.4920	5.1738
				A1T2C	3.3188	4.4604	5.6576
A2	2.4360	3.3506	3.8094	A2T1	3.7296	3.9901	4.5321
				A2T1C	3.0746	3.5497	4.5440
				A2T2	3.5806	4.1953	4.9200
				A2T2C	3.1493	4.3633	5.8895
R1	3.9364	4.3311	5.0842	R1T1	5.4015	5.7306	6.5432
				R1T1C	4.2792	4.6894	5.6364
				R1T2	5.3357	5.9889	6.9899
				R1T2C	4.4988	5.3941	7.1043
R2	2.7102	4.6721	5.6512	R2T1	3.9250	4.3443	5.2845
				R2T1C	3.1960	3.6963	4.7385
				R2T2	3.7487	4.5100	5.5630
				R2T2C	3.2732	4.3355	5.9116
SV	1.5826	2.6280	4.2729	SV	1.9904	2.5907	3.7374

Table 5: Pricing Performance by Maturity

This table reports one-day-ahead and one-week-ahead out-of-sample pricing errors in terms of the time to maturity, measured by mean absolute errors. One-day-ahead (one-week-ahead) out-of-sample pricing errors are calculated by applying the previous day's (week's) estimated parameters to price the current options. In the Portfolio of Volatility Smiles method, each set of parameters is estimated separately for each maturity, and in the Volatility Surface method, a single parameter set is estimated for every maturity. Six expressions of moneyness as S (S&P 500 index)/K (strike price) and six maturities, where T1 is the shortest expiration on each trading day and T6 is the sixth-shortest maturity on each trading day are used. Models beginning with "A" represent an "absolute smile" approaches. In "absolute smile" approaches, the implied volatility is treated as a function of strike price (K) and the time-to-maturity (T) with higher-order terms and/or interaction terms. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with the stochastic volatility. Panel A and Panel B represent one-day-ahead and one-week-ahead out-of-sample pricing performance, respectively.

Panel A: One-day-ahead Out-of-Sample Pricing

	Portfolio of Volatility Smiles							Volatility Surface					
	T1	T2	T3	T4	T5	T6		T1	T2	T3	T4	T5	T6
BS	2.1242	3.5442	4.9161	6.4370	8.7315	11.4726	BS	2.4390	3.6965	4.9912	6.4850	8.8042	11.6232
A1	0.9809	1.0130	1.2870	1.7567	2.4901	3.3132	A1T1	1.9049	2.0527	2.0217	1.9606	2.8597	4.9138
							A1T1C	1.1711	1.1905	1.4735	1.8264	2.4252	4.4522
							A1T2	1.9576	2.0418	1.8754	1.8180	2.8452	5.1147
							A1T2C	1.2663	1.1794	1.3108	1.7260	2.4445	4.7165
A2	1.2026	0.8984	1.1971	1.5970	2.2999	3.1267	A2T1	1.8022	1.8675	1.7821	1.7384	2.5996	4.9561
							A2T1C	1.1333	1.0628	1.2802	1.6542	2.3383	4.7669
							A2T2	1.8630	1.8591	1.6385	1.6357	2.7364	4.8746
							A2T2C	1.2436	1.0531	1.1367	1.6145	2.4714	4.8237
SV	1.1161	1.0435	1.1918	1.3936	1.8510	2.4434	SV	1.1811	1.0998	1.2001	1.4525	1.9522	2.6686

Panel B: One-week-ahead Out-of-Sample Pricing

	Portfolio of Volatility Smiles							Volatility Surface					
	T1	T2	T3	T4	T5	T6		T1	T2	T3	T4	T5	T6
BS	2.3780	3.7125	5.0455	6.5412	8.8187	11.5477	BS	2.5604	3.8268	5.1004	6.5456	8.8357	11.6524
A1	1.5399	1.3797	1.6786	2.1539	2.80831	3.6777	A1T1	2.0394	2.2332	2.2449	2.2608	3.2458	5.3912
							A1T1C	1.3648	1.4579	1.7196	2.0369	2.8521	5.3471
							A1T2	2.1030	2.2282	2.1407	2.1617	3.1670	6.0630
							A1T2C	1.4590	1.4470	1.6220	2.0088	2.8366	6.1606
A2	2.2400	1.2997	1.6037	1.9992	2.6688	3.4795	A2T1	1.9423	2.0620	2.0278	2.0468	2.9938	5.3538
							A2T1C	1.3291	1.3556	1.5648	1.8967	2.7758	5.6634
							A2T2	2.0162	2.0589	1.9172	1.9659	2.9919	5.8368
							A2T2C	1.4399	1.3473	1.4786	1.9021	2.8220	6.3308
SV	2.2945	1.9313	2.1434	4.1745	2.9399	3.5540	SV	1.6454	1.7635	1.9623	2.1631	2.6669	3.4255

Table 6: Pricing Performance by Moneyness

This table reports one-day-ahead and one-week-ahead out-of-sample pricing errors by moneyness, measured by mean absolute errors. One-day-ahead (one-week-ahead) out-of-sample pricing errors are calculated by applying the previous day's (week's) estimated parameters to price the current options. In the Portfolio of Volatility Smiles method, each set of parameters is estimated separately for each maturity, and in the Volatility Surface method, a single parameter set is estimated for every maturity. Six expressions of moneyness as S (S&P 500 index)/K (strike price) are used. Models beginning with "A" represent an "absolute smile" approaches. In "absolute smile" approaches, the implied volatility is treated as a function of strike price (K) and the time-to-maturity (T) with higher-order terms and/or interaction terms. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with the stochastic volatility. Panel A and Panel B represent one-day-ahead and one-week-ahead out-of-sample pricing performance, respectively.

Panel A: One-day-ahead Out-of-Sample Pricing

	Portfolio of Volatility Smiles							Volatility Surface					
	<0.94	0.94-0.97	0.97-1.00	1.00-1.03	1.03-1.06	>=1.06		<0.94	0.94-0.97	0.97-1.00	1.00-1.03	1.03-1.06	>=1.06
BS	6.8549	5.8585	4.5397	4.6776	6.4159	5.4077	BS	6.7538	6.4634	5.6211	4.7087	6.1030	5.4861
A1	2.0619	3.1942	4.3487	2.0083	1.5687	0.7576	A1T1	2.7644	4.4246	6.0670	3.4456	2.4445	1.3037
							A1T1C	2.3286	3.2534	4.7726	2.2496	1.7352	1.0226
							A1T2	2.7600	4.3947	5.9997	3.4376	2.4410	1.2853
							A1T2C	2.5318	3.2647	4.6788	2.4862	1.8441	0.9172
A2	1.5328	2.7241	3.7238	2.2173	1.7472	0.9153	A2T1	2.4576	3.7614	5.3946	3.4588	2.4466	1.2939
							A2T1C	1.9426	2.8196	4.2319	2.5137	1.9615	1.1056
							A2T2	2.4164	3.7090	5.3192	3.4829	2.4488	1.2734
							A2T2C	2.1766	2.8122	4.1367	2.7805	2.0961	0.9926
SV	1.1836	1.7795	2.9448	2.7983	1.8933	0.8966	SV	1.2279	1.9074	3.092	2.9077	1.9604	0.9598

Panel B: One-week-ahead Out-of-Sample Pricing

	Portfolio of Volatility Smiles							Volatility Surface					
	<0.94	0.94-0.97	0.97-1.00	1.00-1.03	1.03-1.06	>=1.06		<0.94	0.94-0.97	0.97-1.00	1.00-1.03	1.03-1.06	>=1.06
BS	6.9027	6.1100	5.1476	5.2797	6.5935	5.4141	BS	6.7604	6.6538	5.9995	5.1714	6.2202	5.4769
A1	2.3567	3.4290	4.5033	2.6827	2.1676	1.2153	A1T1	2.9962	4.5188	6.1020	3.9020	2.9360	1.5774
							A1T1C	2.7767	3.4190	4.7978	2.7682	2.2425	1.3539
							A1T2	3.0890	4.5424	6.0874	3.9709	3.0102	1.6183
							A1T2C	3.1730	3.5766	4.8341	3.0551	2.4190	1.2891
A2	1.9140	2.9800	3.9124	2.8340	2.3138	1.5256	A2T1	2.7562	3.9367	5.4767	3.8653	2.8806	1.5408
							A2T1C	2.5172	3.0324	4.3032	2.9771	2.4349	1.4293
							A2T2	2.8074	3.9425	5.4502	3.9337	2.9388	1.5844
							A2T2C	3.0159	3.1841	4.3417	3.2697	2.5958	1.3300
SV	2.1561	3.1629	4.4391	4.4576	3.4858	1.6297	SV	1.8689	2.813	4.0441	3.8355	2.9122	1.4819

Table 7: Hedging Performance

This table reports one-day-ahead and one-week-ahead hedging errors for the S&P 500 options. Hedging errors are defined as the difference between the change in the market price and the change in the theoretical model price. The parameters and spot volatility implied by all options of the previous day or week are used to obtain the current day's hedged portfolio. In the Portfolio of Volatility Smiles method, each set of parameters is estimated separately for each maturity, and in the Volatility Surface method, a single parameter set is estimated for every maturity. Models beginning with "A" represent an "absolute smile" approaches. In "absolute smile" approaches, the implied volatility is treated as a function of strike price (K) and the time-to-maturity (T) with higher-order terms and/or interaction terms. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with the stochastic volatility. Panel A represents mean absolute errors and Panel B represents root mean squared errors.

Panel A: Mean Absolute Errors

Portfolio of Volatility Smiles			Volatility Surface		
	One Day	One Week		One Day	One Week
BS	0.9139	1.7471	BS	0.9188	1.7751
A1	0.6360	1.1500	A1T1	0.6810	1.2573
			A1T1C	0.6522	1.1919
			A1T2	0.6796	1.2573
			A1T2C	0.6524	1.1975
A2	0.6364	1.1396	A2T1	0.6710	1.2445
			A2T1C	0.6511	1.1915
			A2T2	0.6684	1.2420
			A2T2C	0.6507	1.1962
SV	0.9865	1.8325	SV	0.9924	1.8255

Panel B: Root Mean Squared Errors

Portfolio of Volatility Smiles			Volatility Surface		
	One Day	One Week		One Day	One Week
BS	1.4811	2.7247	BS	1.5018	2.7701
A1	1.1346	1.9866	A1T1	1.2063	2.1243
			A1T1C	1.1720	2.0417
			A1T2	1.1998	2.1187
			A1T2C	1.1705	2.0591
A2	1.1219	1.9435	A2T1	1.1803	2.0935
			A2T1C	1.1578	2.0303
			A2T2	1.1705	2.0836
			A2T2C	1.1534	2.0458
SV	1.8234	3.2433	SV	1.8296	3.2276

Table 8: Hedging Performance by Maturity

This table reports one-day-ahead and one-week-ahead hedging errors by time to maturity, measured by mean absolute errors. Hedging errors are defined as the difference between the change in the market price and the change in the theoretical model price. The parameters and spot volatility implied by all options of the previous day or week are used to obtain the current day's hedged portfolio. In the Portfolio of Volatility Smiles method, each set of parameters is estimated separately for each maturity, and in the Volatility Surface method, a single parameter set is estimated for every maturity. Six maturities, where T1 is the shortest expiration on each trading day and T6 is the sixth-shortest maturity on each trading day are used. Models beginning with "A" represent an "absolute smile" approaches. In "absolute smile" approaches, the implied volatility is treated as a function of strike price (K) and the time-to-maturity (T) with higher-order terms and/or interaction terms. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with the stochastic volatility. Panel A and Panel B represent one-day-ahead and one-week-ahead hedging performance, respectively.

Panel A: One-day-ahead Hedging

	Portfolio of Volatility Smiles							Volatility Surface					
	T1	T2	T3	T4	T5	T6		T1	T2	T3	T4	T5	T6
BS	0.8074	0.8630	0.8961	0.9205	1.0044	1.0787	BS	0.8193	0.8566	0.8956	0.9253	1.0178	1.0953
A1	0.5721	0.5715	0.6074	0.6396	0.7259	0.7894	A1T1	0.7008	0.6586	0.6552	0.6496	0.7020	0.7510
							A1T1C	0.6306	0.5926	0.6102	0.6314	0.7173	0.8151
							A1T2	0.7049	0.6599	0.6499	0.6426	0.6989	0.7512
							A1T2C	0.6361	0.5918	0.6034	0.6244	0.7158	0.8308
A2	0.5704	0.5754	0.6115	0.6401	0.7210	0.7856	A2T1	0.6903	0.6481	0.6449	0.6402	0.6918	0.7421
							A2T1C	0.6283	0.5916	0.6094	0.6292	0.7142	0.8184
							A2T2	0.6956	0.6495	0.6386	0.6316	0.6872	0.7360
							A2T2C	0.6358	0.5914	0.6015	0.6204	0.7104	0.8320
SV	0.8068	0.8870	0.9743	1.0073	1.1409	1.2495	SV	0.8155	0.8902	0.9779	1.0152	1.1502	1.2559

Panel B: One-week-ahead Hedging

	Portfolio of Volatility Smiles							Volatility Surface					
	T1	T2	T3	T4	T5	T6		T1	T2	T3	T4	T5	T6
BS	1.4499	1.6437	1.7808	1.7456	1.9657	2.1244	BS	1.4943	1.6356	1.7728	1.7953	2.0381	2.1831
A1	0.9481	1.0094	1.1355	1.1686	1.3911	1.5027	A1T1	1.1839	1.1739	1.2281	1.2156	1.3846	1.5046
							A1T1C	1.0246	1.0553	1.1474	1.1852	1.4029	1.5841
							A1T2	1.1937	1.1788	1.2176	1.2033	1.3888	1.5060
							A1T2C	1.0381	1.0472	1.1256	1.1698	1.4294	1.6546
A2	0.9045	1.0076	1.1412	1.1655	1.3714	1.4974	A2T1	1.1600	1.1540	1.2120	1.2071	1.3776	1.5190
							A2T1C	1.0141	1.0455	1.1401	1.1832	1.4106	1.6252
							A2T2	1.1723	1.1600	1.1999	1.1930	1.3724	1.5081
							A2T2C	1.0315	1.0411	1.1195	1.1661	1.4257	1.6891
SV	1.4562	1.6055	1.8602	1.8578	2.1912	2.4286	SV	1.3450	1.5893	1.8424	1.9140	2.2508	2.4427

Table 9: Hedging Performance by Moneyness

This table reports one-day-ahead and one-week-ahead hedging errors by moneyness, measured by mean absolute errors. Hedging errors are defined as the difference between the change in the market price and the change in the theoretical model price. The parameters and spot volatility implied by all options of the previous day or week are used to obtain the current day's hedged portfolio. In the Portfolio of Volatility Smiles method, each set of parameters is estimated separately for each maturity, and in the Volatility Surface method, a single parameter set is estimated for every maturity. Six expressions of moneyness as S (S&P 500 index)/K (strike price) are used. Models beginning with "A" represent an "absolute smile" approaches. In "absolute smile" approaches, the implied volatility is treated as a function of strike price (K) and the time-to-maturity (T) with higher-order terms and/or interaction terms. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with the stochastic volatility. Panel A and Panel B represent one-day-ahead and one-week-ahead hedging performance, respectively.

Panel A: One-day-ahead Hedging

	Portfolio of Volatility Smiles							Volatility Surface					
	<0.94	0.94-0.97	0.97-1.00	1.00-1.03	1.03-1.06	>=1.06		<0.94	0.94-0.97	0.97-1.00	1.00-1.03	1.03-1.06	>=1.06
BS	1.2310	1.1675	1.1265	1.0243	0.9937	0.7028	BS	1.2077	1.2351	1.1703	1.0171	0.9665	0.7083
A1	0.7224	0.9423	1.0724	0.9683	0.8217	0.4169	A1T1	0.8174	1.1378	1.1743	0.9711	0.8298	0.4248
							A1T1C	0.7556	1.0065	1.1162	0.9734	0.8286	0.4196
							A1T2	0.8052	1.1385	1.1772	0.9718	0.8298	0.4257
							A1T2C	0.7551	1.0021	1.1221	0.9745	0.8300	0.4195
A2	0.7047	0.9476	1.0730	0.9723	0.8270	0.4217	A2T1	0.7755	1.1104	1.1661	0.9718	0.8310	0.4253
							A2T1C	0.7369	1.0019	1.1142	0.9749	0.8333	0.4242
							A2T2	0.7576	1.1110	1.1692	0.9724	0.8313	0.4259
							A2T2C	0.7316	0.9980	1.1205	0.9762	0.8353	0.4245
SV	0.916	1.475	1.8272	1.6815	1.4453	0.6415	SV	0.9513	1.5651	1.8427	1.5986	1.3884	0.6468

Panel B: One-week-ahead Hedging

	Portfolio of Volatility Smiles							Volatility Surface					
	<0.94	0.94-0.97	0.97-1.00	1.00-1.03	1.03-1.06	>=1.06		<0.94	0.94-0.97	0.97-1.00	1.00-1.03	1.03-1.06	>=1.06
BS	2.2875	1.9643	1.8148	1.8963	1.8958	1.4821	BS	2.2950	2.1288	1.9155	1.8889	1.8479	1.4988
A1	1.3019	1.5467	1.6854	1.7000	1.5053	0.8380	A1T1	1.5230	1.9350	1.9304	1.7607	1.5995	0.8631
							A1T1C	1.3989	1.6762	1.7796	1.7467	1.5540	0.8497
							A1T2	1.5002	1.9364	1.9437	1.7631	1.6054	0.8680
							A1T2C	1.4045	1.6746	1.8099	1.7583	1.5748	0.8500
A2	1.2535	1.5463	1.6783	1.6979	1.4932	0.8376	A2T1	1.4396	1.8968	1.9235	1.7598	1.6060	0.8737
							A2T1C	1.3584	1.6732	1.7823	1.7444	1.5643	0.8618
							A2T2	1.4031	1.8984	1.9387	1.7621	1.6115	0.8782
							A2T2C	1.3540	1.6722	1.8139	1.7579	1.5864	0.8633
SV	1.6351	2.5235	3.2242	3.1379	2.7897	1.2823	SV	1.7093	2.672	3.156	3.004	2.6871	1.2941