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The impact of interaction among traders in artificial financial market

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Abstract In financial market considered as one of complex systems, there exists highly nonlinear interaction between heterogeneous traders. Due to this nonlinear interaction, emergent behavior so called *stylized facts* occurs in financial market. To understand impact of interaction between heterogeneous traders in financial market, we propose an agent based model consists of heterogeneous agents such as fundamentalist, optimistic and pessimistic and interaction between them. Fundamentalist make strategy using the fundamental value of market and play a role of stabilizing market. Fundamentalist forecasts future price will converge to fundamental value. While optimistic and pessimistic called by chartist have an investment strategy using the trend of past price and has a role of destabilizing market. Optimistic (pessimistic) forecasts future price will be larger (smaller) than current price. These three type agents change their own types into other types using transition rules dominated by herding and relative payoff strategy. We consider the topology of interaction between traders using complex network which is constructed by regular, random, small world and scale free network. We find that there were observed stylized facts such as power-law tails and long memory property of volatility in scale free network. Also, it was observed that frequent switching behavior between agent types in scale-free network with small clustering coefficient and short average path length. These results support that the heterogeneous and power-law scaling of interaction between traders would be the source of stylized facts in financial market.

Keywords agent based model · complex network · nonlinear interaction

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1 Introduction

In financial market, there exist emergent behaviors called by *stylized facts* such as fat-tails of return and long memory of volatility which neoclassical economic theory cannot explain (Cont 2001, Gabaix et al.(2003))[1][2]. Neoclassical economic theory assumes perfect rationality and homogeneous of economic agent. Furthermore, neoclassical economic theory assumes that there is no interaction between agents and no information asymmetry between agents. However, this assumption is not realistic in financial market. There are heterogeneous agents and nonlinear interaction them. Also, there exists information asymmetry between agents in financial market. Researches of many scientists and economists have been conducted about models reflect heterogeneous agents and nonlinear interaction between agents. Researches about heterogeneous agent models are able to be classified into two approaches such as theoretically approach and computational approach.
Firstly, theoretically approach attempts to solve analytically given equations, which equations are mostly analytical tractable HAM(Heterogeneous Agent Model).This approach has a advantage of providing mathematical framework which can extend to other economic model or agent based models. Kirman (1991,1993) suggest heterogeneous agent model considers herding behavior in market[3]-[4]. Although he did not focus on stylized facts in financial markets, his model has been basic framework of model based on many agent models. Brock and Hommes (1997),(1998) suggest HAM with heterogeneous beliefs or bounded rationality and interaction between agents[5]-[6]. They found bifurcation of two states such as rational equilibrium states and highly irregular equilibrium states which converges to strange attractor. Furthermore, Hommes et al.(2008) extends Brock and Hommes (1998) model to estimate bubbles in asset pricing experiments[7]. Chiarella et al.(2002),(2006),(2011), Chiarella and He (2003) suggest another HAM which based on two agent types such as fundamentalist and chartist and develop their model with switching process between these two types[8]-[11]. Fundamentalist forecasts future price using fundamental value and makes trading strategies. The quantities which widely used as fundamental value are dividends, earnings, macroeconomic growth, etc. Fundamentalist has a role of market price becomes mean-reverted to fundamental value and stabilizing market. Chartist forecasts future price using past price or return information. Chartist has a role of deviating market price from fundamental value and destabilizing market.Chiarella at al.(2006) found that there could exist two states such as stable fundamental steady states and non-fundamental steady state in the specific range of parameters[10]. Furthermore, Chiarella et al.(2014) developed heterogeneous model for the equity market and estimate heterogeneous agents in S&P 500 [12]. Lux (1998) suggests another new HAM with fundamentalist and noise traders combined with chartism strategy and herding[13]. Lux found chaotic strange attractors in a broad range of parameter value. In chaotic region, the distribution of returns exhibit fat-tails. Alfarano et al.(2008) provide analytical solutions of population dynamics of heterogeneous agent model considered herding behavior

based on model introduced by Kirman(1993)[14].
Secondly, computational approach or agent based model attempt to understand market using computer simulation technique. This approach has an advantage of providing understanding market which could not be solved analytically. Lebaron et al.(1999) suggest artificial stock market called by Santa Fe Institute Artificial Stock Market (SFI-ASM) which contains agent's learning process using genetic algorithm[15]. In SFI-ASM, learning speed of agent is the key parameter which converges to HREE(Homogenous Rational Expected Equilibrium) state. SFI-ASM is extended to other agent based model based on genetic programming by Chen and Yeh (2001,2008)[16]-[17]. Lux and Marchesi (1999) suggest multiagent stochastic model consists of fundamentalist and noise trader which is classified into optimistic and pessimistic[18]. According to competition between three agent's population dynamics, fat-tails and long memory of volatility were observed. Furthermore, Lux (2009) applied Alfarano et al.(2008) model to identify of interaction effects in a business climate survey[19]. Chiarella and Iori (2002), and Chiarella et al.(2009) suggest order driven market with heterogeneous trading such as fundamentalism, charistism and noise trading[20]-[21]. They observed fat-tails and long memory by the impact of chartism in market microstructure.
Many researches about the topology of local interaction between agents have been conducted through agent based model on complex network. Agent locates on node in complex network. Link indicates the connection between agents. The topology of complex network has a role of restriction of information flow or cash flow in economic model. Many economists and scientists have been attempt to study interaction between agents using game theory on network. Especially, emergence of coordination or cooperation has been interested by many researchers. This topological effect of complex network in game theory literature has been extensively discussed in surveys by Wilhite(2006)[22]. In Wilhite(2006), PD(Prisoner's Dilemma) on various complex network such as complete,star,ring,grid,tree,small-world and scale free network has been conducted. Small world and scale free network are observed by efficient network structure which can trigger emergence of cooperation.
Researches about agent based models considered local interaction between agents of financial market has been conducted. Kim et al.(2008) suggest agent based model considered interaction structure between agents using spin model in statistical physics[23]. They found emergence of two-phase phenomena in random and scale free network. Kim et al.(2012) suggest simple agent based model which based on spin model considered modular structure in interaction between agents[24]. They found that power-law distribution has similar exponent in real financial market at specific modular size. Panchenko et al.(2014) develop Brock and Hommes (1998) model considered local interaction between agents using various complex networks[25]. They found that asset price dynamics is influenced by network topologies. Tedeschi et al.(2012) suggest artificial order-driven market added herd behavior by using preferential attachment[26].
Previous agent based model which considers local interactions mostly has been focused on mimicking behaviors between agents(Kim et al. (2008), Kim et

al.(2012),Pancheko et al.(2014), Tedeschi et al.(2012))[23]-[26]. However, there can exist not only mimicking behaviors but also arbitrary behaviors between the profitability of different strategies. Furthermore, profitability of strategy is one of the important ingredients in economic decision making. For example, in spite of our neighborhoods are chartist, if fundamentalist's profit is larger than chartist's profit, we select our decision as fundamentalist. Therefore, it is more realistic strategies combined with mimicking and profit strategy.
We suggest agent based models simultaneously considered topology of local interaction between agents and comparisons of the profitability of different strategies. We extend Lux and Marchesi (1999) model to local interaction model[18]. Agent type consists of fundamentalist, optimistic and pessimistic.Optimitic (Pessimistic) forecasts future price will be larger(smaller) than current price. Both optimistic and pessimistic forecast future price using the trend of past price. Switching process between agent types using herding and profit of each agent type is considered. We consider topology of interaction between agents using complex networks such as regular, random, small world and scale free network. Agent locates in only one node on network. Link indicates the connection between agents. We assume that link is undirected. Only between agents connected, agents can interact with each other. We found that stylized facts such as fat-tails and long memory of volatility in scale free network. However, in other networks, there were not observed stylized facts such as fat-tails and long memory of volatility.
The remainder of this paper consists of three sections. Section 2 reports complex network model, artificial stock market and methodology applied in analyzing data model generated. Section 3 presents simulations and analysis result. Section 4 presents our conclusion and discussion.

2 Model and Methodology

2.1 Complex Network

There are many interactions between humans in society. Understanding of interaction between humans is one of the good ways of understanding humans. Complex network is one of the good methodologies of understanding of interaction between humans in social system. Complex network has been widely studied in physics and mathematics. Network or Graph indicates collection of interaction line between two points. Network consists of node and link. Node indicates point. Link indicates the connection between two nodes. In social system, node indicates human or social agent and link indicates the connection or interaction between humans. Network in social system depicts the topology of interaction between humans.
In this paper, we consider local interaction between heterogeneous traders in artificial stock market. In our model, link indicates information flow between agents. The connected agents know other agent's type. Therefore, topology of interaction between agents represents characteristics of information flow in

market. For example, complete network all to all connected network focus on global interaction could generate perfect information between agents in market. In other networks such as random, small world, scale free network consider local interaction between agents imperfect information between agents can occur.
We focus on local interaction or imperfect information between agents in this paper. And we consider topology of local interaction such as random, small world and scale free network. We make random and small world network using WS(Watts and Strogatz) model[29]. Scale free network is generated by BA(Barabási and Albert) model[30]. In the next section, we will explain WS and BA model.

2.2 WS(Watts and Strogatz) Model

Small world network has high clustering coefficient and short average path length. This network less complex but more tractable is the *small world model* suggested by Watts and Strogatz (1998)[29]. Small world model starts from the idea of adding 'shortcuts' in the regular lattice which has high clustering coefficient. 'Shortcuts' has a role of connecting between other parts of the network. Due to the 'shortcuts', average path length between nodes is shortened. 'Shortcuts' are generated with rewiring probability(p). Rewiring process is done by attaching one link after removing one link with probability(p). According to the value of p, we can generate various complex networks from regular lattice to random network. If p is zero, network is regular lattice. As p increases, clustering coefficient and average path length of networks are smaller. If p is one, network becomes random network which proposed by Erdős and Rényi(1959,1960)[27][28]. Network with p between 0 and 1 is called by small world network. Fig. 1 depicts clustering coefficient and average path length with rewiring probability(p). We start regular lattice with 4 nearest neighborhoods and 10000 nodes. C(p) is the clustering coefficient with p, which p indicates rewiring probability and L(p) indicates the average path length. Clustering coefficient shows the degree of cluster together in the network and measured by the ratio of the number of triangles consists of three connected nodes and maximum number of possible triangles in the network. Average path length is average value of the shortest path length between two nodes in network. With increasing of p, clustering coefficient and average path length becomes smaller(Fig. 1). We select p as 0.01 in order to generate small world network. Also, we select p as 0(1) to make regular network (random network).

2.3 BA(Barabási and Albert) Model

Network with power-law degree distribution is called by *scale free network*. Degree indicates the number of connected links in nodes. Scale free network were

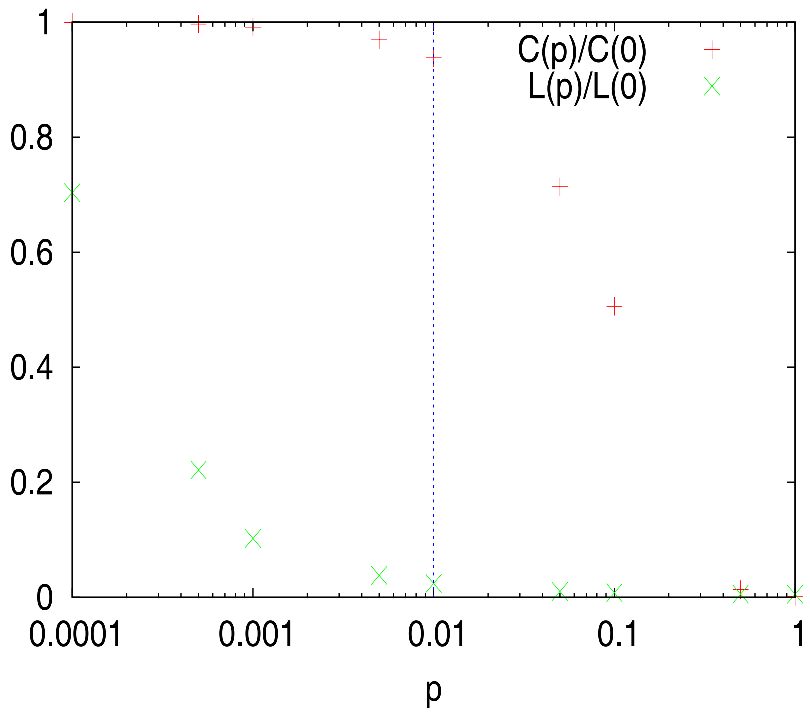


Fig. 1 This figure depicts clustering coeffcient and average path length in small world model. C(p) is clustering coefficient with rewiring probability(p) and L(p) indicates average path length with p. The red cross point indicates C(p)/C(0). The green cross point depicts L(p)/L(0). Vertical blue doted line shows the line with p as 0.01.

frequently observed in real world. In citation networks, the World Wide Web, the Internet, metabolic networks, telephone call graphs and the network of human sexual contracts, power-law degree distribution were observed and these results are well summarized in M.E.J.Newman (2003)[31]. The characteristics of scale free network are heterogeneity of degree distribution or connection between nodes. Additionally, in scale free network there exists hub which has significant large degree.
Barabási and Albert (1999) proposed new random network model with power-law degree distribution, so called by *BA model* using preferential attachment[30]. In BA(Barabási and Albert) model, network starts from initial network with nodes m_0. In each step, network grows with adding degree m. The probability of attaching with other nodes in each node is proportional to the number of degree in each node. Therefore, node with large degree has higher priority of attaching new node. In preferential attachment process, links are only growing without removal in network. In BA model, degree distribution has power-law distribution such as $P(x) \sim x^{-\gamma}$ with $\gamma = 3.0$. In our paper, scale free network is generated with $m_0 = m = 2$.Finally, scale free network is

generated with 10000 nodes. Fig 2 shows degree distribution of regular, random, small world and scale free network. In Fig. 2, heterogeneity of degree or power-law degree distribution and hub are observed in scale free network compared with other networks. The other topological properties of all networks are summarized in Table.1.

2.4 Artificial stock market with local interactions

In order to analyze the impact of interaction between heterogeneous agents or traders, our model is constructed by using agent based modeling. The agent type and transition rules between agent types are based on Lux and Marchesi (1999) model[18]. In our model, there are three agent types such as fundamentalist, optimistic and pessimistic. Fundamentalist has arbitrage strategy between market price and fundamental value. Also, fundamentalist forecasts future price will converge to fundamental value. Therefore, if fundamentalists are dominant in market, price closes to fundamental value. The strategy of optimistic and pessimistic is arbitrage strategy between current market price and past market price trend. Optimistic and pessimistic are kinds of trend follower or chartist. Optimistic (pessimistic) forecasts future price becomes larger (smaller) than current price. Additionally, Optimistic and pessimistic has characteristics of herding behavior. Lux and Marchesi (1999) assumes that all agents are connected globally and focuses on herding behaviors in market[18]. With the different view point of Lux and Marchesi (1999) model, we focus on local interaction between agents. The modified transition rates of agent i are:

$$\pi^i_{+-} = v_1 \frac{n^i_c}{N^i} \exp{(U_1)}, \pi^i_{-+} = v_1 \frac{n^i_c}{N^i} \exp{(-U_1)}, \tag{1}$$

$$U_1 = \alpha_1 x^i + \frac{\alpha_2}{v_1} \frac{dp/dt}{p} \tag{2}$$

$$\pi^i_{+f} = v_2 \frac{n^i_+}{N^i} \exp(U_{2,1}), \pi^i_{f+} = v_2 \frac{n^i_f}{N^i} \exp(-U_{2,1}) \tag{3}$$

$$\pi^i_{-f} = v_2 \frac{n^i_-}{N^i} \exp(U_{2,2}), \pi^i_{f-} = v_2 \frac{n^i_f}{N^i} \exp(-U_{2,2}) \tag{4}$$

$$U_{2,1} = \alpha_3 \{ \underbrace{\frac{r + (1/v_2)(dp/dt)}{p} - R}_{\text{profits of chartist agents from } n_+ \text{group}} - \underbrace{s|\frac{p_f - p}{p}|}_{\text{fundamentalists' profit}} \} \tag{5}$$

$$U_{2,2} = \alpha_3 \{ \underbrace{R - \frac{r + (1/v_2)(dp/dt)}{p}}_{\text{profits of chartist agents from } n_- \text{group}} - \underbrace{s|\frac{p_f - p}{p}|}_{\text{fundamentalists' profit}} \} \tag{6}$$

where $\pi^i_{A,B} \Delta t$ is the transition probability from B type to A type of agent i during a time increment Δt. $x^i = (n^i_+ - n^i_-)/n^i_c$,$n^i_c = n^i_+ + n^i_-$, N^i, n^i_+, n^i_-, n^i_f is

the number of agents,optimistics,pessimistics and fundamentalists connected with agent i. R is the average real return. r is nominal dividend calculated by Rp_f. p is the current market price and p_f is the current fundamental value. s is the discount factor. v_1 and α_1 are the parameters measure of frequency of changing opinion in chartist. In particular, α_1 is the parameter adjusting herding behavior between chartist. α_2 is the parameter measures frequency of revaluation of past price trend information. v_2 and α_3 are measures of the frequency of revaluation of opinion between chartist and fundamentalist. U function contains the profit of fundamentalist, optimistic and pessimistic. The interaction between agents is generated by using complex network. All agents locate on node in complex network. Link indicates the connection between agents. The complex network generated are regular, random, small world and scale free network.(i.e Regular network indicates regular lattice.) Random and small world network are generated using WS model[29].Scale free network is generated by BA model[30]. All complex network generated are assumed to be undirected network. The details of generating and simulation of complex network are written in Section 3.Simulation and Result. (dp/dt) is calculated by average value of $\Delta P/\Delta t$ during [t-0.2,t).
Price changes are determined by the imbalance between demand and supply. The demand and supply are determined by endogenously with the population of optimistic, pessimistic and fundamentalist. The excess demand of chartist(optimistic and pessimistic) and fundamentalist are :

$$ED_c = (n_+ - n_-)t_c, ED_f = n_f \cdot \gamma \cdot \frac{p_f - p}{p} \tag{7}$$

where n_+, n_-, n_f is the total number of optimistic, pessimistic and fundamentalist in market, t_c is the average trading volume per transaction, γ is a parameter for the strength of convergence to fundamental value. Finally, price change is determined by transition probability consists of excess demand terms. The price changes by a small percentage $\Delta p = \pm 0.001p$. The transition probability of price change is during a time increment Δt:

$$\pi_{p\uparrow} = max[0, \beta(ED + \mu)], \pi_{p\downarrow} = -min[\beta(ED + \mu), 0] \tag{8}$$

where μ is random white noise term of speculators and $\mu \sim N(0, \sigma_\mu)$, β is a parameter for adjusting speed of price and $ED = ED_c + ED_f$ is overall excess demand.
We assume that fundamental value at time t $p_{f,t}$ follows geometric Brownian motion and it updates at integer times: $ln(p_{f,t}) = ln(p_{f,t-1}) + \epsilon$ and $\epsilon \sim N(0, \sigma_\epsilon)$.
To avoid absorbing state, $n_c = 0(n_f = N)$ and $n_f = 0(n_c = N)$, we assume that agents cannot change out of one strategy has less than 0.8 % of total population N.

2.5 Detrended Fluctuation Analysis

To investigate the properties of temporal correlation, we used a Detrended Fluctuation Analysis(DFA)[32]. The average value of the time series was subtracted from each value $u(i)(i = 1, ...N_{max})$,where N_{max} is the number of time series accumulated in the data.

$$y(j) = \sum_{i=1}^{j}[u(i)- < u >], \tag{9}$$

where

$$< u >= \frac{1}{N_{max}} \sum_{i=1}^{N_{max}} u(i), \tag{10}$$

and is divided into boxes of equal size n. In each box, we fit the integrated time series by using a first order polynomial function,$y_{fit}(i)$,which is called the local trend. According to the value of l, DFA is called DFA-l. We detrended the accumulated time series $y(i)$ by subtracting the local trend $y_{fit}(i)$ in each box, and we calculate the detrended fluctuation function.

$$Y(i) = y(i) - y_{fit}(i). \tag{11}$$

For a given box size n, we calculated the root mean square(rms) fluctuation

$$F(n) = \sqrt{\frac{1}{N_{max}} \sum_{i=1}^{N_{max}} [Y(i)]^2} \tag{12}$$

The above calculation is repeated for box sizes n (different scales) to provide a relationship between $F(n)$ and n. A power-law relationship between $F(n)$ and the box size n indicates the presence of scaling:$F(n) \sim n^H$. The parameter H, called the Hurst exponent, represents the temporal correlation property of the signal: if H=0.5, the signal is uncorrelated(white noise); if $H < 0.5$, there exists anti-correlation or short memory within the signal; if $H > 0.5$, there exists positive correlation or long memory within the signal.
In the sequel, we measured the Hurst exponent using the DFA-1 method whose local trend function is a first-order polynomial function.

3 Simulation and Result

We set parameters as these values, N(Number of agent) = 10000, v_1 = 2, v_2 = 0.6, β = 4, t_c = 0.001, γ = 0.01,α_1 = 0.6, α_2 = 1.5, α_3 = 1, $r = Rp_f$, R = 0.0004, s = 0.75, Δt = 0.01, σ_μ = 0.05, σ_ϵ = 0.005. We generate local interaction between agents with complex network such as regular lattice, random, small world and scale free. Regular network is constructed by 4 nearest neighbors regular lattice. Node indicates agent. Link indicates connection between agents. Only between agents connected, agent knows other agent type in each other. Small world network is generated by WS model with 4 nearest neighbor and 0.01 rewiring probability of link in network. Also, random network is generated by WS model with 4 nearest neighbor and 1.0 rewiring probability of link in network. Scale Free network is generated by BA model with 2 preferential attachments of links in growing process in network. In order to measure the only impact of topological properties of network, all networks are normalized with same number of node and link such as N and 2*N. The topological properties of complex networks simulated are summarized in Table 1.The number of node, link and mean degree are fixed in all complex networks (Table 1).The significant difference between regular and small world network is clustering coefficient and average path length(Table 1).Clustering coefficient is measured by the ratio of number of triangles which generated with each connected three nodes in network and possible triangles in network. And it measures the degree of tendency of nodes to clump together in network. Average path length is average value of shortest path length between two nodes in network. If the average path length in network is short, the speed of opinion spreading between nodes is fast. Small world network is more clustered than random network, however has longer average path length than random network(Table 1). The significant difference between scale free and other networks are maximum degree (Table 1). The maximum degree of scale free network is 550 which value is the largest in all complex networks. Average path length of scale free network is the shortest of all networks. In addition, clustering coefficient is almost zero. Therefore, scale free network is very efficient structure spread of opinion.

In the model, the feature which the most effects on between agent interactions is degree distribution. Fig. 2 depicts degree distribution of all complex networks generated. In the order of scale free, random, small world and regular network, dispersion of degree distribution is wide(Fig. 2). Therefore heterogeneity of interaction in scale free network is the largest in given all complex networks. In particular, scale free network has power-law tails such as $P(x) \sim x^{-\gamma}$, γ is measured by 3 in our model (Fig. 2). In other words, the power-law structure in the degree distribution between financial objects should influence on the transmission of information in financial system. Furthermore, scaling behavior of information flow in financial market could enhance probability of emergent scaling behaviors of market.

Fig. 3 depicts the dynamics of market return and population dynamics of agent type. Return is defined by : $r(t) = ln(p(t)) - ln(p(t-1))$ and $p(t)$ is price at time t. In regular network, switching process between agents is stable without large fluctuation because of the weakness of herding effect by strong clustered with local connection in regular network. In large clustered network, information between agents spread very slowly into whole system. Therefore, chance of large change of agent type is very small in regular network. With the loss of large change of agent type, large fluctuation of price does not exist. In small world and random network, most agents become fundamentalist after about 10000 times. The speed of fundamentalist in random network is faster than in small world network. This difference of speed between random net-

work and small world network occurs by the difference of clustering coefficient between two networks. Small world network is more clustered than random network. Therefore, change of agent type in small world network does not occur more frequently than random network. Contrary to regular network, in small and random network, there are some chance to occur herding behavior due to short cut and more large degree than regular network. The fast speed of convergence of opinion in small world network was observed in previous researches[22]. However, herding effete is not larger than arbitrary profit term between price and fundamental value. Therefore, most agents change their own type into fundamentalist by the advantage of profit using fundamental value with weekly herding effect. By dominant population of fundamentalist in market, market price follows fundamental value and there is no large fluctuation of market price and volatility clustering in market. If there exists only chartist type in our model, large fluctuation of return and volatility clustering could be observed in small world network. In scale free network, large changes of agent type population compared with other networks were observed. The significant difference between scale free network and other networks is degree distribution(Fig. 2). In scale free network, there exists power-law scaling of degree distribution and hub which has very large number of link. This hub has a role of spreading her opinion to others simultaneously. Due to the characteristics of hub, herding behavior is enhanced. The fast speed of opinion spreading in scale free network is observed in previous researches[22]. The change of hub agent type causes dramatic change of population of agent type and it does impact on price movement. For example, if hub agent is optimistic(pessimistic), probability of changing optimistic(pessimistic) of the other agents which are connected into hub increases. Hub enhances herding behaviors in short time and switching types during short time. In this case, price will be large than current price. If hub agent is fundamentalist, probability that price converges to fundamental value increases. Consequently, price becomes large fluctuated and volatility is clustered in scale free network. This phenomenon is observed in real financial markets[1][2]. However, in other networks such as regular, random and small world which herding behavior is not dominant in market, there does not exist large fluctuation of return.
Fig. 4 depicts the CDF(Cumulative Distribution Function) of volatility in market with various complex networks. Volatility is measured by absolute value of return and rescaled by standard deviation. In scale free network contrary to other networks, power-law tails of volatility is observed(Fig.3). Fat-tails is fitted with power-law function, $P(x) \sim x^{-\alpha}$ and α is measured by 2.215 ± 0.035(Fig.4). The power-law scaling in volatility is measured in real financial market[1][2]. The power-law exponent is in the region of infinite second moment or variance($\alpha < 3$). However, in the real financial market, power-law exponent is measured in the finite variance region($3 \leq \alpha < 4$)(Gabaix et al.(2003))[2]. This infinite variance of volatility would indicate that market is in the critical phase transition region from stable market phase to unstable market phase such as large fluctuating market phase. Power-law tails in volatility indicates self-similarity of price fluctuations. This self-similarity of

volatility does not occur only due to the existence of hub. For example, in star network, hub exists. By the hub, dramatic change of price could occurs. However, there does not exist power-law tails in volatility by the lack of power-law scaling degree distribution in star network. Consequently, power-law scaling in degree or power-law scaling in strength of connection between agents in financial market can generates power-law tails in volatility.
We analyze the characteristics of temporal correlation in market using DFA(Detrended Fluctuation Analysis). Fig. 5 depicts scale function F(s) with box size s. In regular, random and small world network, power-law scaling with $F(s) \sim s^H$ were found in $10 \leq s \leq 1000$(Fig. 5). In scale free network, power-law scaling was observed in $100 \leq s \leq 10000$(Fig. 5). Power-law scaling is observed at the range of $100 \leq s \leq 1000$ in all complex networks. The scaling exponent H is called Hurst exponent. H indicates temporal correlation of time series.(i.e If H<0.5,time series is anti-correlated or time series has short memory. If H>0.5, time series is long-range correlated or time series has long memory. If H = 0.5, time series has no correlation or time series has no memory.).The details of DFA are written in Section 2. We measure Hurst exponent of return, volatility, fundamental value return and fundamental value volatility. The result of Hurst exponents is summarized in Table 2. In scale free network, strong long memory property was observed in volatility which value is 0.829(Table 2). No memory in return and long memory in volatility is one of stylized facts in real financial market[1].It can be showed that long memory of volatility in scale free network generated by herding behaviors. Hurst exponents of volatility in small world and random network is 0.540 and these value is larger than 0.5. In small world and random network, there exists very small long memory in volatility. The weak long-range correlation in volatility would be originated by the weak herding behaviors.

4 Conclusion and Discussion

In financial market, existence of nonlinear interaction between agents is a well known fact. In order to analyze the impact of interaction between agents in financial market, we propose artificial stock market using agent based modeling. Agents consist of three types such as optistic, pessimistic and fundamentalist. Optimistic and pessimistic traders make strategy with historical price value or trend. Because of this feature, optimistic and pessimistic are often called by chartist. Optimistic(pessimistic) forecasts future price will be larger(smaller) than current price. Contrary to chartist, fundamentalist makes strategy with fundamental value which is not dependent on market state or price. Fundamentalist forecasts future price converges to fundamental value. In our model, we assume that fundamental value follows geometric Brownian motion. Agent type changes into other type according to transition probability. Transition probability consists of herding term which reflects mimicking behavior of near agents and profit term of each strategy. Profit term consist of optimistic, pes-

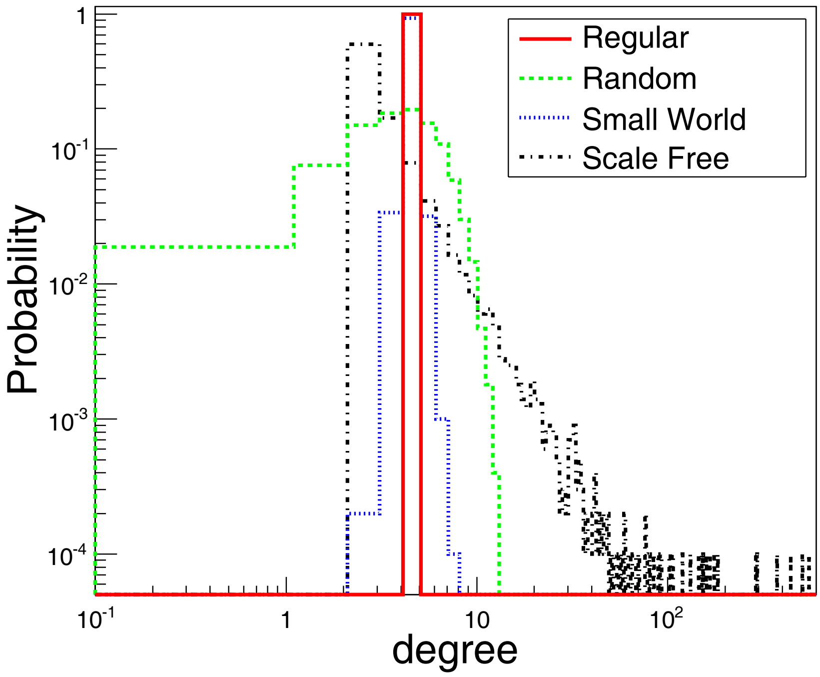
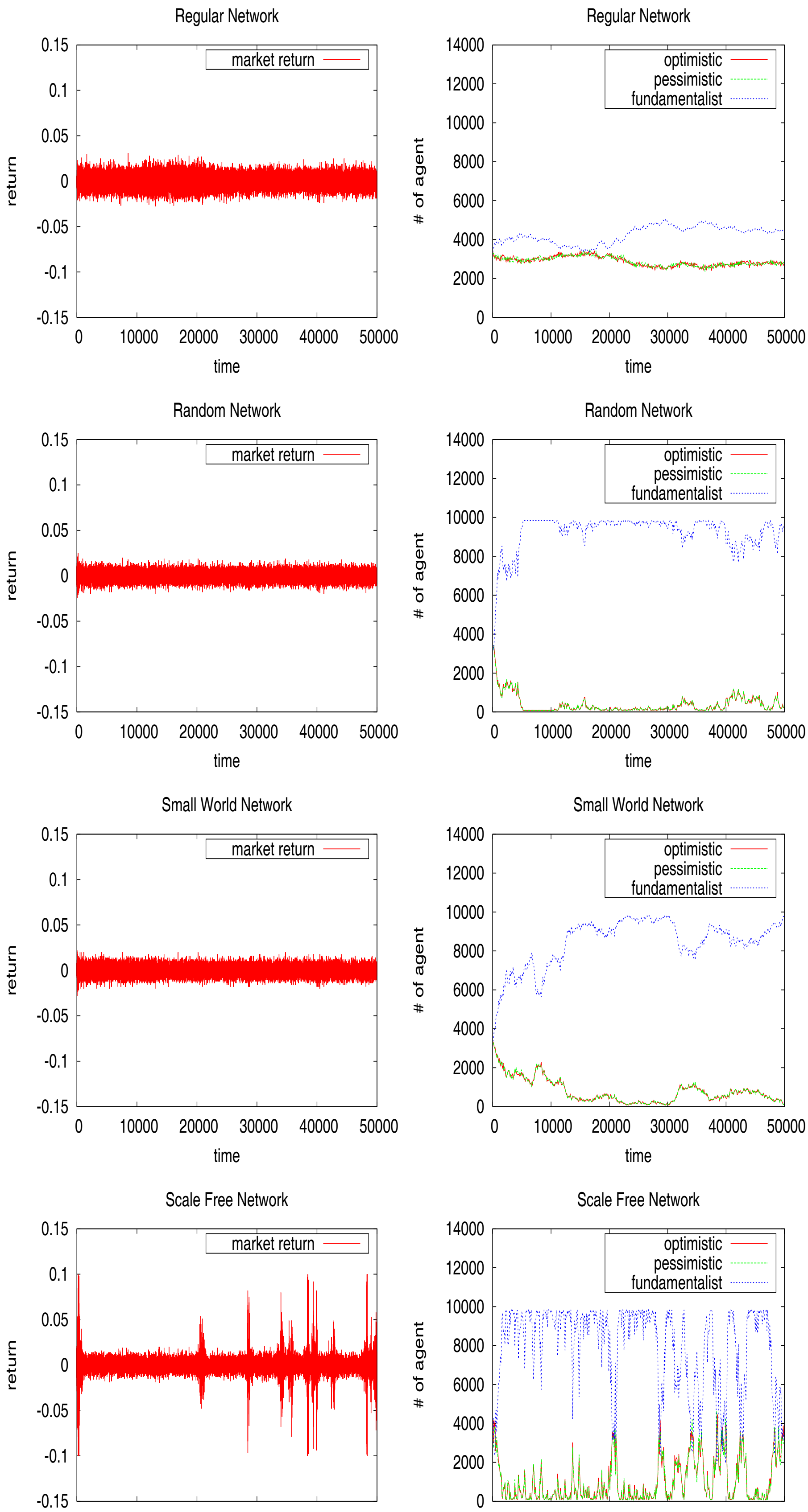


Fig. 2 This figure depicts degree distribution of complex network. Red solid line indicates degree distribution of regular netowrk. Green dashed line indicates degree distribution of random network. Blue dotted line indicates degree distribution of small world network. Black dashed and dotted line indicates degree distribution of scale free network.

simistic and fundamentalist's profit terms. To reflect topological structure of interaction between agents in the model, we apply various complex networks such as regular, random, small world and scale free network in the model. All networks are normalized by fixed number of nodes and links in order to only reflect the topological structure in the model. Artificial stock market is simulated with 10000 agents during 50000 iteration times.
As a result, in scale free network, very well known stylized facts in financial market such power-law tails and long memory property of volatility were observed. In small and random network, week long memory property of volatility was observed with weak herding behaviors by short cut and small clustered network topology. The significant difference between scale free network and other networks are existence of hub and power-law tails of degree distribution. With these results, we conclude that scale free structure in local interaction between agents can be a source of stylized facts such as power-law tails and long memory property of volatility. With these results, our model can provide deep understanding of nonlinear interaction between heterogeneous traders in financial market.


Regular Network
market return
return
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# of agent
time
Scale Free Network
market return
return
Scale Free Network
optimistic
pessimistic
fundamentalist
# of agent

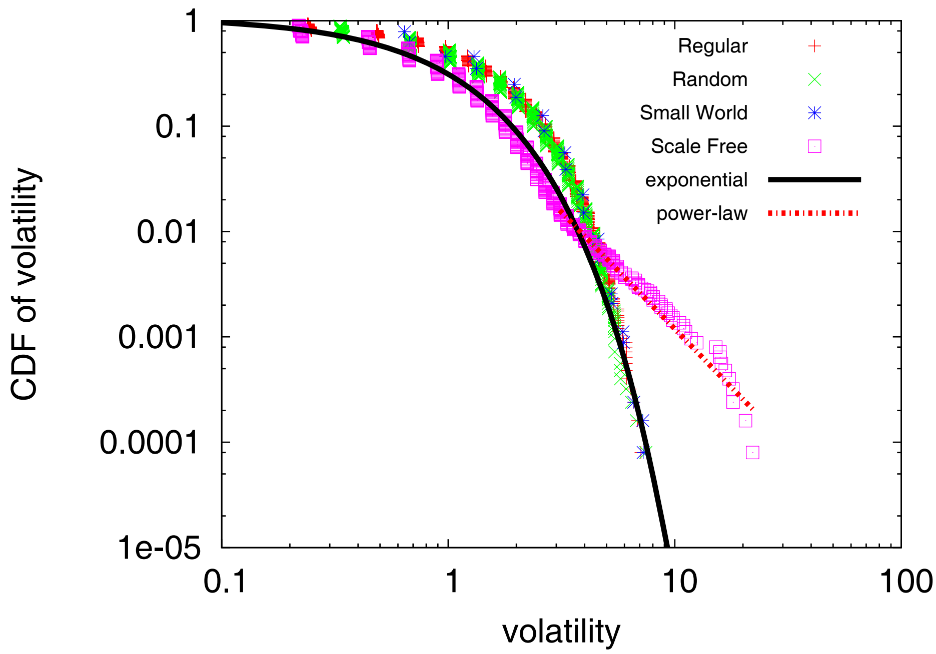

Fig. 4 This figure depicts CDF of volatility. Volatility is resclaed by standard deviation. Red point indicates CDF of volatility in Regular Network. Green point indicates it in Random Network. Blue point indicates it in Small World Network. Magenta point indicates it in Scale Free Network. Black line is the fitted line of CDF in Scale Free Network with exponential function. Red dashed line indiciates the fitted line of CDF in Scale Free Network with power-law function, $P(x) \sim x^{-\alpha}$. α is measrued by 2.215 ± 0.035

However, in order to reach a better elegant model, we should overcome some limitations in our model. First, our model does not reflect dynamical properties of agent interaction. In real financial system, topology of interaction between traders changes at times. However, in our model, topological structure between agents is fixed in all times. Second, strict information asymmetry does not reflect our model. In real system, there exists closed community or group structure. In closed community structure, chance of information flows to other community is not high. In this case, information is almost perfectly blocked. The blocking of information occasionally occurs in real financial market. If these limitations are reflected in our model, we expect that our model will be more elegant model which reflects features of interaction between traders in real financial market.

References

1. Cont,R., Empirical properties of asset returns:stylized facts and statistical issues, Quantitative Finance 1, 223-236, (2001).

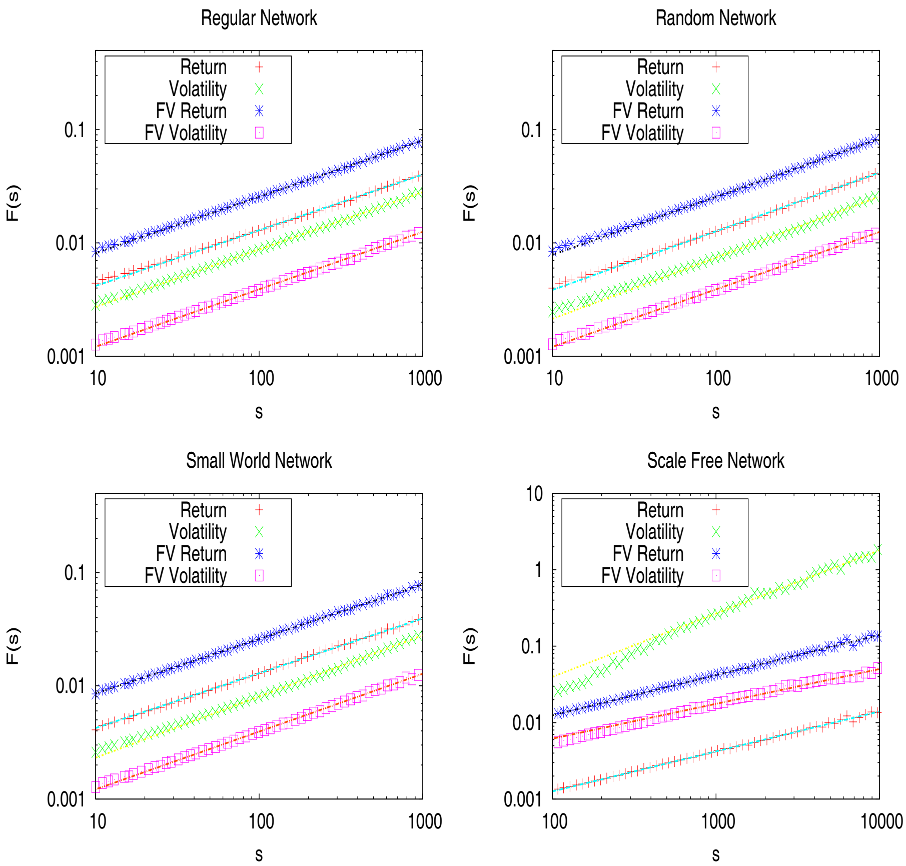

Fig. 5 This figure depicts scale function F(s) with box sizes s. Red point indicates F(s) of return. Green point indicates F(s) of volatility. Blue point indicates F(s) of FV return. Magenta point indicates F(s) of FV volatility. (FV : Fundamental Value)

Table 1 This table depicts topological properties of complex networks. (CC : Clustering Coefficient, l: average path length)

	Regular	Random	Small World	Scale Free
# of node	10000	10000	10000	10000
# of link	20000	20000	20000	20000
CC	0.500	0	0.470	0.001
l	1250.375	6.67	31.658	4.324
Mean Degree	4	4	4	4
Maximum Degree	4	12	7	550

Table 2 This table depicts of Hurst exponent of return,volatility,FV return and FV volatility in Regular, Random, Small World and Scale Free Network. (FV: Fundamental Value)

	Regular	Random	Small World	Scale Free
Return	0.492±0.002	0.518±0.002	0.500±0.002	0.523±0.008
Volatility	0.507±0.003	0.540±0.006	0.540±0.004	0.829±0.016
FV Return	0.497±0.002	0.513±0.002	0.497±0.002	0.525±0.008
FV Volatility	0.507±0.002	0.507±0.002	0.507±0.003	0.455±0.008

2. Gabaix, X., Gopikrishnan, P., Plerou, V., Stanley, H.,E., A theory of power-law distributions in financial market fluctuations, Nature 423, 267-270 (2003).
3. Kirman, A., Epidemics of opinion and speculative bubbles in financial markets. In:Taylor, M.P.(Ed.), Money and Financial Markets. Blackwell, Cambridge, pp. 354-368. (1991)
4. Kirman, A., Ant,rationality, and recruitment. Quarterly Journal of Economics 103, 137-156, (1993).
5. Brock, W., Hommes, C.H., A rational route to randomness. Econometrica 65, 1059-1095, (1997).
6. Brock, W., Hommes, C.H., Heterogeneous beliefs and routes to chaos in a simple asset pricing model. Journal of Economic Dynamics and Conrol 22, 1235- 1274, (1998).
7. Hommes, C.H., Sonnemans, J., Tuinstra, J., van de Veldeon, H., Expectations and bubbles in asset pricing experiments. Journal of Economic Behavior and Organizations 67, 116-133, (2008)
8. Chiarella, C., Dieci, R., Gardini, L., Speculative behaviour and complex asset price dynamics:a global analysis. Journal of Economic Behavior and Organization 49, 173-197, (2002).
9. Chiarella, C., He, X.-Z., Heterogeneous beliefs,risk and learning in a simple asset pricing model with a market maker. Macroeconomic Dynamics 7 (4), 503-536, (2003).
10. Chiarella, C., Dieci, R., Gardini, L., Asset price and wealth dynamics in a financial market with heterogeneous agents. Journal of Economic Dynamics and Controls 30, 1755-1786, (2006).
11. Chiarella, C., He, X.-Z., Zheng, M., An analysis of the effect of noise in a heterogeneous agent financial market model. Journal of Economic Dynamics and Control 35, 148-162, (2011).
12. Chiarella, C., He, X.-Z., C.J.Zwinkels, R., Heterogenous expectations in asset pricing:Empirical evidence from the S&P 500. Journal of Economic Behavior and Organization 105, 1-16, (2014).
13. Lux, T., The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions. Journal of Economic Behavior and Organization 33, 143-165, (1998).
14. Alfarano, S., Lux, T., Wagner, F., Time variation of higher moments in a financial market with heterogenous agents:An analytical approach. Journal of Economic Dynamics and Control 32, 101-136, (2008).
15. LeBaron, B., Arthur, W.B., Palmer, R., The time series properties of an artificial stock market. Journal of Economic Dynamics and Control 23, 1487-1516, (1999).
16. Chen, S.-H., Yeh, C.-H., Evolving traders and the business school with genetic programming: A new architecture of the agent-based artificial stock market. Journal of Economic Dynamics and Contrl 25, 363-393, (2001).
17. Chen, S.-H., Huang, Y.-C., Risk preference, forecasting accuracy and survival dynamics:Simulations based on a multi-asset agent-based artificial stock market. Journal of Economic Behavior and Organization 67, 702-711, (2008).
18. Lux, T., Marchesi, M., Scaling and criticality in a stochastic multi-agent model of a financial market. Nature 397, 498-500, (1999).
19. Lux, T., Rational forecasts or social opinion dynamics? Identificiation of interaction effects in a business climate survey. Journal of Economic Behavior and Organization 72, 638-655 (2009)

20. Chiarella, C., Iori, G., A simulation analysis of the microstructure of double auction market. Quantitative Finance 2, 346-353 (2002).
21. Chiarella, C., Iori, G., Perello, J., The impact of heterogeneous trading rules on the limit order book and order flows. Journal of Eonomic Dynamics and Control 33, 525-537 (2009).
22. Whilhite, A., Economic activity on fixed networks. In: Judd, K.L., Tesfatsion, L. (Eds.), Handbook of Computational Economics, Volume 2: Agent-Based Computational Economics, Handbooks in Economics 13, North-Holland, Amsterdam. pp.1013-1045 (2006).
23. Kim, Y., Kim, H.-J., Yook, S.-H., Agent-based spint model for financial markets on complex networks:Emergence of two-phase phenomena. Phyical Review E 78, 036115 (2008).
24. Kim, H., Kim, S., Oh, G., Effects of modularity in financial markets on an agent-based model. Journal of the korean physical society 60(4), 599-603, (2012).
25. Panchenko, V., Gerasymchuk, S., V.,Pavlov, O., Asset price dynamics with heterogenous beliefs and local network interactions. Journal of Economic Dynamics and Control 37, 2623-2642 (2013).
26. Tedeschi, G., Iori, G., Gallegati, M., Herding effects in order driven markets:The ris and fall of gurus. Journal of Economic Behavior and Organization 81, 82-96, (2012).
27. Erdős, P., Rényi, A., On the random graphs, Publicationes Mathematicae **6**, 290-297 (1959)
28. Erdős, P., Rényi, A., On the evolution of random graphs, Pubications of the Mathematial Institutes of the Hungarian Academy of Sciences 5, 17-61 (1960)
29. Watts,D.J., Strogatz, S.H., Collective dynamics of 'small-world' networks, Nature 393, 440-442, (1998).
30. Barabắsi, A.-L, Albert, R., Emergence of scaling in random networks, Science 286, 509-512 (1999).
31. Newman, M., E., M., The structure and function of complex networks, SIAM Review 45(2), 167-256 (2003).
32. Hu, K., Ch.Ivanov, P., Chen, Z., Carpena, P., Stanely H.E,Effects of trends on detrended fluctuation analysis. Physical Review E 64,011114 (2001).