Neglected risk and the market for bond insurance*

Ji Yeol Jimmy Oh†

Korea Military Academy

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Abstract

Monoline insurers collapsed in a dramatic manner during the subprime crisis. In this paper, I present a stylized model to account for this market breakdown. The initial neglect of a severe loss outcome by local thinking agents can trigger rating downgrade of insurers. This results in a damaging forced exit of investors with an investment certification constraint. However, the model identifies a more fundamental problem. Even when the agents are rational, bond insurance exerts a negative externality by eliminating the price discount of an uninsured bond. Therefore, excessive focus on proper risk management alone may not improve the market welfare.

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†Assistant Professor of Economics (as a part of military service), Department of Economics and Law, Korea Military Academy, P.O. Box 77-2, Hwarang-ro 574, Nowon-gu, Seoul, Republic of Korea, 139-804. Telephone: +82 (0)2 2197 2788. E-mail address: ohjimmyy@gmail.com, ohjimmyy@kma.ac.kr.

"It didn't need an oracle to predict Ambac's demise (Economist, 2010)."

1. Introduction

One of the dramatic events witnessed during the subprime crisis was the fall of monoline bond insurers such as Ambac and MBIA. Since the 1970s, these firms had retained the AAA rating for decades and specialized in credit enhancements for municipal bond issuers. By guaranteeing a bond's par and tying its credit rating to the firm's own, these insurers essentially lent their AAA rating for business. This line of business proved popular, and they insured around a half of all U.S. municipal bonds as of 2008. Their venture into the structured products turned out to be less successful, and following the slowdown of the U.S. housing market, unexpected losses and difficulties in raising capital led them to a painful series of rating downgrades (Drake and Neale, 2011). This resulted in "a sweeping rating downgrade across financial instruments with a face value of $2.4 trillion (Brunnermeier, 2009, p. 87)," which caused a widespread panic.

This implosion of the market for bond insurance primarily emanated from gross underestimation of credit risks of mortgage-related products. As the major monoline insurers were repeatedly downgraded throughout 2008 and 2009, credit rating agencies emphasized the insurers' insufficient capital coverage, resulting from sharp increases in loss projections.[1] This was inevitable as the overall quality of mortgage loans deteriorated for late-2006 and 2007 vintages (Demyanyk and Van Hemert, 2011) and the statistical relationship between "hard" credit observables and defaults began to break down (Rajan, Seru and Vig, 2014). Given that the quality of unreported, "soft" credit characteristics deteriorated the worst, the underestimation was particularly prominent among low-documentation loans (Ashcraft, Goldsmith-Pinkham and Vickery, 2010).

Given this failure to take into proper account of mortgage-related risks, monoline insurers have been criticized in the popular press. Their decision to branch out into the resi-

[1] For example, whereas Moody's cumulative loss rate projections for Ambac and MBIA's exposures to 2006 vintage mortgage-related products stood at around 14 to 18% in January 2008, the revised expected and stress-case projections jumped to 22% and 30% respectively by September 2008, in the space of less than eight months (Moody's, 2008).

dential mortgage-backed security (RMBS) and collateralized debt obligation (CDO) markets has been branded a mistake, tempted by "the housing market's siren calls (Forbes, 2010)." A popular belief was that these firms should have simply restrained their line of business to more traditional products such as municipal bonds.

However, over the periods of rapid securitization growth, these firms "played an important role in making securities, including those based on sub-prime loans, attractive to a wide range of investors (Schich, 2008, p. 84)." In particular, given the credit ratings' crucial role of investment certification (DeMarzo, 2005; Boot, Milbourn and Schmeits, 2006; Benmelech and Dlugosz, 2009; Bolton, Freixas and Shapiro, 2012), with many pension and money market funds facing explicit rating-based constraints on their portfolio selection,[2] bond insurance provided access to structured products for investors with "conservative" investment remits.[3]

These recent developments call for both positive and normative analysis of the monoline insurers' exact role in the structured products market. Under what circumstances are the structured bonds insured? Does this insurance provision enhance the agents' welfare? When the insurers underestimate the bonds' risks, how do their decisions and the overall social value of bond insurance change? If some agents had properly assessed these risks, would the outcomes have turned out differently? These are all pertinent questions for the future of bond insurance, but they have not been addressed in systematically in the literature. This paper fills this gap by presenting a relatively straightforward theoretical framework that enables such analysis.

In this paper, I incorporate the concept of local thinking (Gennaioli and Shleifer, 2010) into a model of bond insurance. The idea that people evaluate their decisions on the basis of "what first comes to mind (Gennaioli and Shleifer, 2010, p. 1399)" has become particularly relevant in explaining the rapid rise of securitization. For example, in Gennaioli, Shleifer and Vishny (2012), the initial neglect of a severe credit outcome

[2]Cantor, Gwilym and Thomas (2007) report that three-quarters of pension plan managers in their sample face some form of minimum rating requirements for bond purchases.

[3]Nearly all securities insured by a monoline insurer already held a shadow investment grade rating by at least one of three major rating agencies (Schich, 2008). Therefore, the primary question was whether they were rated AAA or not, given the stringent investment certification requirements of many investors.

gives rise to excessive issuance of securitized assets, which subsequently become fragile when the investors are reminded of this unaddressed risk. Furthermore, when it interacts with the investors' desires to pool loans and diversify against idiosyncratic risks, it also accounts for a systematic failure of the shadow banking system (Gennaioli, Shleifer and Vishny, 2013).

Crucially, this framework also allows for a direct identification of the normative implications of neglected risk. In many of the ambiguity-based approaches (e.g., Gilboa and Schmeidler, 1989; Bewley, 2002; Ghirardato, Maccheroni and Marinacci, 2004; Klibanoff, Marinacci and Mukerji, 2005), it is often difficult to define and compare the agents' welfare as their information sets and relevant decision rules change. However, a local thinking framework preserves the functional form of the agents' utility while allowing for their risk perceptions to differ. Thus, we can address more sensitive issues such as whether or not the overall welfare of market agents declined under local thinking insurers relative to the rational benchmark.

With this in mind, I present a three-period, multi-asset model of bond insurance with constant absolute risk aversion (CARA) investors, risk-neutral bond insurers, issuers, and a credit rating agency (CRA). The model tracks their decisions from issuance until maturity. In the model, provision of bond insurance by a AAA-rated insurer not only eliminates a bond's credit risk, but the issuer also gains access to a larger pool of potential investors. In other words, there is an element of "market segmentation among bottom tier and top tier investment grade bonds (Denison, 2003, p. 99)." To generate this effect, the model assumes that a proportion of investors—referred to as "conservative" investors—are constrained to invest only in AAA-rated bonds. Their presence in the market is thus a rationale for bond insurers to maintain a AAA rating.

The credit ratings of insurers, as in practice, are determined by their capital adequacy against tail risk.[4] This leads to the model's first prediction, namely that the neglected risk could trigger an insurer downgrade. The reasoning is as follows. A rational CRA accurately assesses a bond's worst-case losses, so the insurers set aside sufficient capi-

[4]Moody's (2006), for example, explicitly required its AAA-rated bond insurers to maintain sufficient capital to cover for the 99.9 percentile portfolio loss.

tal from the issuance, preventing the prospect of an insurer downgrade at later stages. However, the local thinking agents' initial neglect of a possible severe loss could trigger a rating downgrade if a subsequent reassessment of loss projections by the local thinking CRA leads to a more stringent capital requirement. An insurer then has to weigh the cost of additional capital against the reputational cost associated with a rating downgrade. Thus, an insurer downgrade occurs when the costs of equity capital are high.

Hence, from the insurer's perspective, the decision to accept a rating downgrade is optimal. However, she fails to account for its negative knock-on effect on the investors' welfare. In particular, the conservative investors have to liquidate their holdings of the downgraded bond and exit the market at a "fire-sale price" (Coval and Stafford, 2007). The simulation results reveal that the adverse effect of this forced exit has a dominant negative effect on the overall market welfare. In fact, in most instances, the affected investors should be willing to pay to capitalize the insurer and prevent a rating downgrade.

This result, though in close accordance with observed facts, is not particularly surprising. However, the model uncovers a more surprising fundamental problem with bond insurance. More specifically, even when all agents hold rational expectations, the amount of bond insurance is likely to be excessive relative to the social optimum. This is due to the negative externality associated with an insurance contract. The parties negotiating a contract—the issuer and the insurer—fail to internalize the fact that "aggressive" investors, who can invest in all types of bonds, can potentially benefit from the price discount of an uninsured bond. In contrast, following the insurance, a bond's price immediately rises to its par, and the investors merely receive their reservation utility. Thus, from a social perspective, some actuarially profitable contracts ought not to be accepted.

Interestingly, there is a natural scope for a comparison of this result with Hanson and Sunderam (2013), who argue that the securitization process entails a negative externality through the originator's excessive issuance of informationally-insensitive, "safe" debt securities in good times. This reduces the investors' ex ante incentive to acquire costly information, but as the presence of informed investors is scarce and valuable in bad times, the market outcome is inefficient. Although my paper does not assign any role for information acquisition, its predictions and the consequent policy prescriptions turn out to be broadly comparable to theirs.

More importantly, local thinking agents have a tendency to under-provide bond insurance relative to the rational benchmark when the magnitude of the neglected outcome is sufficiently severe. Since the investors are risk averse, those who acknowledge the presence of such an extreme outcome hold a strong desire to insure against it. In contrast, because the local thinking agents neglect this risk, they perceive the same bond to be safer, reducing the perceived benefits of bond insurance. This is a complementary result to Gennaioli, Shleifer and Vishny (2012), who argue that neglected risk leads to excessive issuance of seemingly safe securities. In this model, because the securities are already perceived to be safer, the issuer sees little benefit in obtaining a credit enhancement.

In this respect, the two main characteristics of bond insurance prior to the crisis, namely the insurers' neglect of a severe credit outcome and their rapid expansion of business in the structured products market, are susceptible to a *post hoc, propter hoc* fallacy. Contrary to popular belief, the strong demand for bond insurance may not have stemmed from neglected risk; in fact, had the agents been fully rational, it is conceivable that bond insurance would have been even more popular.

This also raises an interesting possibility. Given that there is overinsurance relative to the social optimum under the rational benchmark, the local thinking agents' underprovision of bond insurance can actually be welfare-improving. Using a reasonable set of parameter values, I numerically demonstrate its plausibility. Therefore, although neglected risk does carry the risk of an insurer downgrade and the associated deterioration in welfare, there is no guarantee that the market welfare will be improved for sure when the local thinking agents begin to perceive the bonds' credit risks in a more rational manner; the overprovision of insurance by rational agents could conceivably be welfare-harming. This highlights the danger of a disproportionate focus on proper risk management without addressing a more fundamental issue of bond insurance's negative externality.

As a final extension, I also consider a set of scenarios where the agents' risk perceptions are heterogeneous. The analysis emphasizes that the CRA should be capable of proper risk assessments in order to enable ordered and efficient functioning of the market, since the damaging prospect of an insurer downgrade can be prevented only when the CRA is fully rational at issuance. In fact, a combination of rational CRA and local thinking investors generally eliminates the possibility of an insurer downgrade and reverses the rational agents' tendency to overinsure at the same time. Given various issues that compromised the quality of credit ratings prior to the crisis, such as rating shopping (Bolton, Freixas and Shapiro, 2012) and the insufficient disciplining effect of reputation (Mathis, McAndrews and Rochet, 2009; Fulghieri, Strobl and Xia, 2014), this is troubling.

The rest of this paper is organized as follows. Section 2 presents the model and outlines its main assumptions. In Section 3, I present the results under the rational benchmark, with a particular focus on the negative externality of bond insurance. The market outcomes and welfare results are compared to the case of local thinking agents in Section 4. Section 5 extends the results by allowing for a heterogeneity in risk perception among the agents. Section 6 discusses the model's main predictions, policy implications, and possible extensions. Section 7 then concludes the paper.

2. The model

2.1. Asset composition and payoff structure

I consider a discrete-time, three-period model with $t = 0, 1, 2$. At $t = 0$, $J \geq 1$ bonds are issued to the public, denoted B_1 to B_J. These bonds may be thought of as structured products in practice. They all have the identical maturity, which occurs at $t = 2$. For simplification, they are assumed zero-coupon discount bonds with the par at maturity equal to one unit of consumption numeraire.[5] The period t market price of B_i is denoted p^i_t. All bonds' issuance volumes are normalized to 1.

A bond's credit risk is modeled as follows. At maturity, a bond may fail to repay its par value; instead, a unit of bond B_i repays $1 - \Theta_i$, where Θ_i denotes its credit loss. Θ_i can take one of three values: $\Theta_i = \{0,\ \theta_i,\ \mu_i\theta_i\}$, with the respective probabilities

[5]Given the increasingly popular practice of cash flow separation into principal-only (PO) and interest-only (IO) components, this assumption is not problematic.

$\Pi_i = \{\pi_i^g,\ \pi_i^m,\ \pi_i^b\}$, where $\pi_i^b = 1 - \pi_i^g - \pi_i^m$. Furthermore, $\mu_i > 1$ and $\mu_i \theta_i \leq 1$. In other words, a bond can either experience no credit loss, small loss, or large loss, and μ_i measures the relative severity of the tail outcome, i.e., realization of a "large loss". θ_i then reflects the overall "baseline" credit characteristics of a bond, as an increase in θ_i affects both the small and large loss outcomes simultaneously. In addition, the ex ante probabilities satisfy $\pi_i^g > \pi_i^m > \pi_i^b$, which implies that the large loss event is least likely to occur. This payoff structure is similar to Gennaioli, Shleifer and Vishny (2012, 2013).

Although I focus on a three-point credit risk model for the ease of exposition, the model is flexible enough to yield tractable results for a larger finite number of possible credit loss realizations, as will be shown in Section 6.3.1. Thus, it may be considered as a reduced version of a more elaborate discrete risk modeling approach employed by the market participants in practice.[6]

Another assumption I maintain is that a bond's credit loss realization is determined independent of other bonds. Of course, this is unlikely in practice, given the strong interdependence among structured products due to their common exposure to regional or macroeconomic factors. However, this interdependence has little impact on the qualitative results of the model due to its structure. Even when the structure is modified in a more realistic manner, isolating the effects of credit risk interdependence is relatively straightforward, which I discuss in Section 6.3.2.

Finally, each bond is issued by a unique issuer, also indexed $i = 1, ..., J$. This assumption is a reasonable approximation of the actual insurance arrangements, given that insurance contracts are usually offered on a deal-by-deal basis in practice. In the absence of any multi-deal bundling or packaging of insurance products, even if a particular issuer issues more than one bond, her optimization problem remains identical to the case where each bond is issued by a different issuer. This, by construction, also rules out the possibility that two bonds are different tranches of a particular RMBS or CDO, which alleviates remaining concerns regarding the assumption of credit risk independence. Given that

[6] For example, Moody's uses a correlated binomial expansion technique to calculate the idealized default probabilities for CDOs. As with any binomial expansion model, the default risk modeling is discrete in nature.

bond insurance generally involved a small subset of mezzanine, junior investment grade tranches, this is not a serious issue.

2.2. Market participants

2.2.1. Bond issuers

As with other models of securitization, bond issuers essentially serve as a "broker." At $t = 0$, issuer $i \in \{1, \ ..., \ J\}$ obtains a pool of loans by paying l_0^i. The loans last for two periods, and repay $1 - \Theta_i$ at $t = 2$. There are no interest payments. The issuers hold no capacity for loss absorption, given the use of special purpose entities (SPEs)—such as real estate mortgage investment conduits (REMICs)—in a securitization deal.[7] Not surprisingly,, this is the source of a bond's credit risk.

Given this set-up, a representative bond issuer's objective is simply to maximize the intermediation spread at $t = 0$. Prior to issuance, the issuer receives an offer to insure her bond from each insurer. If issuer i rejects all offers, then she proceeds without insurance and her intermediation spread is given by $p_{0,U}^i - l_0^i$, where the subscript U denotes that the bond is uninsured. On the other hand, if she accepts insurer j's offer, her intermediation spread is $p_{0,I}^i - \chi_j^i - l_0^i$, where the subscript I denotes an insured bond and χ_j^i is the one-off insurance premium paid to insurer j.[8] An insurance can only be bought at issuance ($t = 0$). Thus, i strictly prefers bond insurance when:

$$p_{0,I}^i - p_{0,U}^i > \chi_j^i. \tag{1}$$

This implies that the issuance price differential ($p_{0,I}^i - p_{0,U}^i$) gives the issuer's maximum reservation price for the insurance premium. As this quantity forms a central part of the subsequent analysis, I denote $\Delta_0^i \equiv p_{0,I}^i - p_{0,U}^i$ as a shorthand.

[7] Not only do SPEs remove the loans from the originators' balance sheet, but they are also not subject to any minimum equity requirement, severely limiting the loss-bearing capacities.

[8] Since the insurance premium is one-off, it is possible to drop the time subscript t. This is a standard practice within the industry (Drake and Neale, 2011).

2.2.2. Bond insurers

In the years prior to the crisis, the bond insurance industry witnessed strong competition among nine firms, including Ambac, MBIA, FGIC, and Assured Guarantee. Therefore, incorporating an element of competition is important. As any competition takes the form of price competition under the model set-up, it suffices to restrict the attention to the case of an insurer duopoly.[9] These two insurers are indexed $j = A$, B.

At $t = 0$, both insurers make an offer to each issuer i. If insurer j's offer is accepted, i pays the agreed one-off premium (χ_j^i) and j, in return, guarantees the bond's par value by covering any credit losses of bond investors at $t = 2$. As these insurers "typically retain most of the risk that they underwrite (Schich, 2008, p. 91)", it is reasonable to ignore the possibility of reinsurance. Thus, an insurer needs to prepare for a possible claim payout at maturity through building up her own capital buffer.

As in practice, capital buffer consists of two components, namely the insurance premium reserve and equity capital. Equity capital may be raised at both $t = 0$ and $t = 1$, and insurer j's total stock of equity capital at t is denoted K_t^j. More importantly, raising equity capital incurs the insurer a capital cost of $c_t^j < 1$ per unit.[10] As denoted, it may differ between the two insurers and also over time.

Furthermore, it has often been noted that these insurers were poorly capitalized in the run-up to the crisis (e.g., Schich, 2008; Brunnermeier, 2009).[11] I model this by assuming that the insurers enter the market at $t = 0$ with no initial capital. In other words, $K_{-1}^j = 0$ for $j = A$, B. This does not imply that insurers held no capital base; instead, it should be interpreted that the insurers' existing capital was tied up in their traditional areas of business when they entered the structured product market.

Reflecting the monoline insurers' long, historical AAA status, both insurers' initial credit ratings are set at AAA. It will also be shown shortly that an insurer downgrade is not an issue at $t = 0$. However, an insurer j can be downgraded by the credit rating

[9] The case of $N > 2$ insurers carries the identical economic intuition.

[10] This cost reflects underwriting and brokerage fees, as well as the well-documented underpricing of seasoned equity offerings (Corwin, 2003; Altinkilic and Hansen, 2003).

[11] At the end of 2006, for example, monoline insurers' capital as a percentage of the net par outstanding averaged around 1%. Even the firm with the highest capital ratio, AGC, held less than 1.4% of the net par outstanding.

agency at $t = 1$. In this instance, a reputational cost of κ_j is incurred. Although the model yields meaningful results even in the absence of an exogenous reputational cost, its inclusion captures the firms' reluctance to accept a rating downgrade in practice due to a loss of trust, reputation, and a loss of future business.

Finally, as in standard models of insurance, both insurers are risk neutral. More formally, at $t = 0$, 1, insurer $j \in \{A, B\}$ maximizes the expected value of her terminal wealth at $t = 2$, denoted V_2^j, defined as:

$$V_2^j \equiv \sum_{i=1}^{J} I_j^i \left(\chi_j^i - \Theta_i\right) - c_0^j K_0^j - c_1^j \left(K_1^j - K_0^j\right), \tag{2}$$

if insurer j retains AAA rating at $t = 1$, and:

$$V_2^j \equiv \sum_{i=1}^{J} I_j^i \left(\chi_j^i - \Theta_i\right) - c_0^j K_0^j - c_1^j \left(K_1^j - K_0^j\right) - \kappa_j, \tag{3}$$

if insurer j is downgraded at $t = 1$. In both (2) and (3), I_j^i is an indicator function that takes the value of 1 if and only if bond i is insured by insurer j. Both equations also implicitly assume no discounting, since an inclusion of discount rate has no impact on the qualitative results of the model.

(2) and (3), however, require an implicit assumption. When an insurer's capital buffer falls short of the claim demand at maturity, limited liability becomes an issue. In other words, an insurer's ex ante decision may incorporate the possibility that she cannot be held responsible beyond her capital buffer in the event of bankruptcy. However, this complicates the optimization problem and renders a solution intractable due to the model's set-up. To overcome this issue, the insurer is assumed to receive a negative utility equal to the size of her capital shortfall when she holds insufficient capital to meet all claims.[12] Given the lengthy negotiations and substantial legal and administrative costs involved in such instances, this prospect of negative utility is not particularly controversial.

[12] A similar assumption is imposed out of necessity or convenience in other theoretical studies such as Brunnermeier and Pedersen (2009).

2.2.3. Credit rating agency

In the model, there is a single credit rating agency (CRA) in charge of rating all bonds and bond insurers. The possibility of "rating shopping" (Bolton, Freixas and Shapiro, 2012; Bongaerts, Cremers and Goetzmann, 2012) is deliberately ruled out as it is not the main interest of the paper. CRA's rating disclosure is not contingent upon payment. Then, due to the lump sum nature of the payment, it is possible to simply assume the ratings are determined free of charge. This also implies that the CRA does not have an explicit objective function.

Given the earlier discussion, I consider a simplified rating structure whereby a bond or an insurer is either rated AAA or below-AAA. All ratings are updated every period. Then, the essence of various rating criteria in practice is distilled in the following set of conditions. Firstly, a bond is rated AAA at t if and only if $E_t(\Theta_i) = 0$. Therefore, unless $\pi_i^g = 1$, a bond B_i cannot be rated AAA on its own merit.

Secondly, a bond insurer's credit rating is solely determined by her capital adequacy. In the U.S. bond market, whether or not an insurer has built up sufficient capital to cover for her portfolio loss remains the most important rating factor, although the CRAs do take into account of other factors such as the insurer's market position and profitability. More specifically, as discussed earlier, the CRAs emphasize the insurer's ability to cover for the tail risk. To reflect this, a bond insurer has to hold sufficient capital to cover for the "worst case" portfolio loss to be rated AAA. Due to the independence of credit risk, a worst case portfolio loss arises when the worst case loss is realized for each insured bond. Crucially, this definition of "worst case" differs depending on whether the CRA neglects tail risk or is fully rational.

Of course, it may be argued that this metric becomes irrelevant as the number of insured bonds in the portfolio increases; a lack of interdependence between bonds' credit risks and the law of large numbers render it a probability zero event as the number of insured bonds tends to infinity. However, the independence assumption is imposed mainly for the purpose of analytical convenience, and it is well known that these bonds are strongly correlated in practice. In this respect, the worst case scenario is not as unrealistic as the simple asymptotics would suggest.

2.2.4. Bond investors

The single most important distinction in this model is made with regards to the investor type. There are two different types of potential investors. Firstly, a proportion of "aggressive" investors are free to invest regardless of a bond's credit rating. The remaining "conservative" investors, however, can only invest in AAA-rated bonds. Thus, if a bond loses its AAA rating, then they are not only prevented from investing in it but also required to sell off any existing holdings.

In order to make bond prices directly comparable, each bond B_i at $t = 0$ attracts a continuum of potential investors of measure one, a proportion $\lambda_i \in [0, 1]$ of whom are aggressive and the remaining $1-\lambda_i$ are conservative. Each bond's investor pool is distinct, reflecting a closed nature of the structured product issuer's potential clients in practice. It also implies that an investor only considers her potential investment decision over one particular bond.[13]

In short, both types of investors can hold a AAA-rated asset; the *ex post* proportions of aggressive and conservative investors are λ_i and $1 - \lambda_i$ respectively. In contrast, for a bond rated below AAA, the actual investor pool consists entirely of aggressive investors of measure λ_i.

All investors follow standard CARA utility, take the market price as given at each period, and consume only at $t = 2$. This means that the investors maximize the expected utility associated with their terminal wealth. More formally, if a representative investor j's cumulative holding of bond B_i at t is denoted $x_{i,t}^k(j)$, where $k \in \{agg,\ con\}$ distinguishes whether the investor is aggressive or conservative, then her objective function is given by:

$$\max_{x_{i,t}^k(j)} -E_t \exp\left\{-\gamma W_2^k(j)\right\}, \tag{4}$$

[13] For private investors with limited access to the network of structured product issuers, this assumption is more appropriate. On the other hand, a mutual or hedge fund often manages a portfolio with a large number of structured products, making the assumption more difficult to defend. Thus, I relax the assumption and discuss its implications in Section 6.3.2.

where $W_t^k(j)$ denotes the wealth of investor j of type k at t, and γ is the risk aversion parameter. The evolution of wealth is mark-to-market, given as follows:

$$W_t^k(j) = W_{t-1}^k(j) + \left(p_t^i - p_{t-1}^i\right) x_{i,t-1}^k(j)\,; \quad k \in \{agg,\ con\}\,. \tag{5}$$

Due to the constant risk aversion, any change in the investors' initial wealth does not affect their optimization problem, and I therefore assume $W_0^{agg}(j) = W_0^{con}(j) = 0$.

Another implicit assumption is that the investors completely disregard an insurer's payout capacity at $t = 0$ if she is rated below AAA at issuance. This shortcut prevents bond insurance by an insurer without AAA rating. For a small number of bonds in the asset universe, and with reasonable parameter values, it is possible to show numerically that the intermediate option of bond insurance without AAA rating is almost always dominated by either retaining the AAA rating or providing bond insurance from the insurer's perspective.

Finally, for each bond $B_i \in \{1, ..., J\}$, the market clearing condition is given by:

$$\int_0^{\lambda_i} x_{i,t}^{agg}(j)\,dj + \int_{\lambda_i}^1 x_{i,t}^{con}(j)\,dj = 1, \tag{6}$$

since the volume of each bond issuance is normalized to 1.

3. Rational benchmark

In this section, I present a case when all agents' risk perceptions are rational in all periods. In other words, their perceived probabilities correspond to the objective probabilities specified in Section 2.1. This provides a benchmark against which the insurance decisions and welfare implications of neglected risk may be compared subsequently.

3.1. Insurance choice at issuance ($t = 0$)

Given that the bond insurance market is a duopoly, I consider without loss of generality whether issuer i has an incentive to accept insurer A's offer or not. A's offer is accepted for sure if the following conditions are met. First, it must be that A's insurance premium offer is less than the issuance price differential ($\chi_A^i < \Delta_0^i$). Second, it must also be less than the competitor's offer ($\chi_A^i < \chi_B^i$). If $\chi_A^i = \chi_B^i < \Delta_0^i$, then the standard assumption applies and both insurers' offers are equally likely to be accepted.

However, from the insurer's perspective, the insurance premium must be actuarially fair. It is worth noting that, in the absence of a further shock at $t = 1$, an insurer automatically satisfies the AAA rating criteria at $t = 1$ if she has raised sufficient capital at $t = 0$. In other words, a rating downgrade is not an issue at $t = 1$. If so, using (2), and given the CRA's rating criteria, the insurer's reservation price for bond insurance is given by the sum of a bond's expected credit loss and the additional cost of equity capital incurred by the insurance of B_i. Formally, an insurance offer is made whenever:

$$\chi_j^i \geq E_0(\Theta_i) + c_0^j (\mu_i \theta_i - E_0(\Theta_i)), \quad j \in \{A, B\}. \tag{7}$$

I denote the right hand side, namely insurer j's reservation price for the insurance premium of B_i, as $\Gamma_{j,0}^i$. Since both insurers' risk assessments of B_i are identical, any difference in $\Gamma_{j,0}^i$ arises only from a difference in their respective costs of equity capital at $t = 0$. More specifically, $\Gamma_{A,0}^i < \Gamma_{B,0}^i$ for all $B_i \in \{B_1, \ ..., \ B_J\}$ if and only if $c_0^A < c_0^B$ and vice versa. These conditions together imply that B_i will be insured for sure if:

$$\Delta_0^i > \min\left(\Gamma_{A,0}^i, \ \Gamma_{B,0}^i\right). \tag{8}$$

Then, the investors' optimization conditions, when combined with (8), yield Proposition 1, which reveals that bond insurance occurs only when at least one of the insurers' costs of equity capital is sufficiently low:

Proposition 1 (bond insurance at $t = 0$). Let $\varsigma_i \equiv \mu_i \pi_i^g + (\mu_i - 1)\pi_i^m$. Then, $B_i \in \{B_1, \ ..., \ B_J\}$ is insured with certainty at $t = 0$ if and only if the following condition is met:

$$\min\left(c_0^A,\ c_0^B\right) < \frac{\varsigma_i \pi_i^b \exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right) + (\varsigma_i - \mu_i + 1)\,\pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) + (\varsigma_i - \mu_i)\,\pi_i^g}{\varsigma_i \pi_i^b \exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right) + \varsigma_i \pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) + \varsigma_i \pi_i^g} < 1, \tag{9}$$

where $\pi_i^b = 1 - \pi_i^g - \pi_i^m$ as before.

Proof. See Appendix. ■

In other words, bond insurance only becomes viable when an insurer can raise equity capital cheaply enough so that she expects non-negative profits and fulfills the CRA's rating criteria at the same time. The right hand side of (9) then yields the maximum cost of equity capital at which bond insurance would occur for bond B_i, referred to as the "insurance threshold" throughout the paper.

For a better understanding of this proposition, consider a bond B_i with the following set of parameters. Suppose $\pi_i^g = 99\%$, $\pi_i^m = 0.9\%$, and $\pi_i^b = 0.1\%$. The baseline loss, θ_i, is set at 25%, implying a recovery rate of 75%. In addition, $\mu_i = 2$ such that, in the worst case scenario, the bond's recovery rate drops to 50%. In practice, such a bond holds a shadow rating of either AA or A, creating a demand for bond insurance. If the risk aversion parameter (γ) is set at 2 and the proportion of aggressive investors in the potential investor pool is 25%, then (9) implies that B_i will be insured as long as one of the insurers' costs of equity capital is lower than 7.4%.

From (9), it is apparent that, holding the equity market conditions of bond insurers constant, a bond is more likely to be insured when its insurance threshold is high. Then, with a liberal use of terminology, a bond may be referred to as "more likely to be insured" when its insurance threshold increases. As this threshold depends on a number of model parameters, Proposition 2 clarifies their respective relationships:

Proposition 2 (model parameters and the likelihood of bond insurance).

(i) An increase in γ or θ_i makes bond insurance more likely.

(ii) An increase in λ_i makes bond insurance less likely.

(iii) An increase in μ_i has an ambiguous effect on the likelihood of bond insurance.

(iv) For sufficiently large μ_i, a further increase in μ_i makes bond insurance more likely.

Proof. See Appendix. ■

The first two parts of the proposition are straightforward. Firstly, an increase in γ implies the investors are more risk averse, raising their demand for bond insurance. Secondly, an increase in θ_i increases both the small (θ_i) and large ($\mu_i\theta_i$) losses proportionately, and the bond's expected loss increases accordingly. As the risk averse investors are more adversely affected by this than the risk neutral insurers, bond insurance becomes more valuable to them. Finally, an increase in λ_i raises the number of aggressive investors who clear the market in the absence of bond insurance. This again makes bond insurance a less attractive option for the issuer.

However, the effect of μ_i is not monotonic due to the nature of the rating criteria. More specifically, the insurers are required to raise equity capital to cover for the worst case loss ($\mu_i\theta_i$) regardless of whether it is likely to occur or not. In other words, the extra burden of equity capital cost may be disproportionately high compared to the increase in the overall expected loss. Then, even for the risk averse investors, this elimination of tail risk may not be "value for money," particularly when μ_i is close to 1. However, eventually, as μ_i further increases, the investors become strongly concerned about the catastrophic magnitude of the tail outcome. Their desire to insure against such outcome would then be strengthened, making bond insurance more likely.

FIGURE 1 HERE

FIGURE 2 HERE

This is graphically illustrated in Figures 1 and 2. Using the same set of parameter values as in the earlier numerical example, Figure 1 plots how the insurance threshold changes as the four model parameters (μ_i, γ, θ_i, and λ_i) are varied in turn. Each parameter behaves as discussed in Proposition 2. Figure 2 then presents the non-monotonic effect of μ_i on the insurance threshold in closer detail. For values of μ_i close to 1, an increase in μ_i makes insurance *less* likely due to the disproportionate cost of additional capital. However, this pattern is eventually reversed at around $\mu_i \approx 1.4$, and beyond this point, a monotonically positive relationship between μ_i and the likelihood of bond insurance develops.

Lastly, having derived the conditions under which a bond is insured, I briefly explain the outcome of competition in this market. Given the earlier argument, it is apparent that the insurer with a lower cost of equity capital emerges as the winner. More formally, when $c_0^A < c_0^B$, insurer A takes over the entire market and offer $\min\left(\Delta_0^i,\ \Gamma_{B,0}^i\right)$ to every bond B_i that satisfies (9). The opposite scenario arises when $c_0^A > c_0^B$. Finally, when $c_0^A = c_0^B = c_0$, both insurers offer their reservation prices, i.e., $\Gamma_{A,0}^i = \Gamma_{B,0}^i = \Gamma_0^i$, to issuer i and divide the insurance market in half. Since the major monoline insurers were broadly comparable in size, credit ratings, and other firm characteristics prior to the crisis, the last case is of particular relevance. In this instance, all surplus from bond insurance is transferred to the issuers as a result of competition.

3.2. Subsequent decisions at $t = 1$ and $t = 2$

As the agents continue to be fully rational at $t = 1$, no further trading occurs. The insurers' credit ratings remain at AAA, so an insurer downgrade never occurs under full rationality. As a result, both types of investors continue to hold insured bonds, while all uninsured bonds remain in the hands of aggressive investors.

Finally, at maturity ($t = 2$), credit losses are realized. The par of an insured bond is guaranteed under all circumstances, and the aggressive investors bear the brunt of any credit loss realization for uninsured bonds. In short, the anticipated decisions of agents at $t = 0$ remain intact until maturity.

3.3. Welfare implications

How does the provision of bond insurance affect the overall welfare of agents? In order to address this issue, an appropriate measure of "market welfare" must first be constructed, which I denote Ω_t. In order to minimize any normative judgement over the definition of market welfare, it is defined as a simple sum of each agent's expected utility evaluated prior to maturity at a given point in time, i.e., either $t = 0$ or $t = 1$. For the continuum of investors, this implies that the utility of each type of investors is weighted by their respective proportion (λ_i).

Since the agents' choices and information sets are unchanged between $t = 0$ and $t = 1$ under the rational benchmark, it must be that $\Omega_0 = \Omega_1$. Then, a simple way to calculate Ω_t is to add up the agents' welfare derived from each $B_i \in \{B_1, \ ..., \ B_J\}$, which I denote Ω_t^i. In other words, Ω_0 satisfies:

$$\Omega_0 = \sum_{i=1}^{J} \Omega_0^i. \tag{10}$$

This bond-by-bond analysis of market welfare yields Proposition 3, which states that bond insurance may not always be beneficial from a social perspective:

Proposition 3 (negative externality of bond insurance). Let $U_{i,U}^{agg} \in [-1, 0)$ denote the aggressive investors' ex ante utility from investing in $B_i \in \{B_1, \ ..., \ B_J\}$ if B_i is issued without insurance. Then, for all B_i that satisfies

$$\Delta_0^i - \min\left(\Gamma_{A,0}^i, \ \Gamma_{B,0}^i\right) \in \left(0, \ \lambda_i \left(U_{i,U}^{agg} + 1\right)\right), \tag{11}$$

bond insurance occurs but is not socially desirable.

Proof. See Appendix. ■

In other words, under the rational benchmark, the amount of bond insurance is excessive relative to the social optimum. The reason for this is as follows. Since an insured bond is riskless, competition among investors raises its price to the par value. Then, by investing in an insured bond, investors merely receive their reservation utility of -1. On the other hand, when a bond is issued without insurance, aggressive investors have to clear the market on their own. To induce them to do so, the price of an uninsured bond is substantially discounted, which serves as a source of expected utility gain for them.

However, a decision to insure a bond is made solely between the issuer and the insurer, and they fail to internalize that issuing a bond without insurance may enhance the utility of the aggressive investors. In this respect, bond insurance exerts an inherent negative externality, measured by the foregone utility gain of the aggressive investors, i.e., $\lambda_i \left(U_{i,U}^{agg} + 1 \right)$. As a result, the social undesirability of bond insurance is particularly prominent when the insurers' costs of equity capital are close to the insurance threshold. In this instance, the internalized surplus from bond insurance, namely $\Delta_0^i - \min \left(\Gamma_{A,0}^i, \ \Gamma_{B,0}^i \right)$, may be substantially smaller than the aggressive investors' foregone utility gain. Furthermore, although not explicitly included in the model, bond insurance also eliminates the investors' tax loss benefits in the event of a default (Nanda and Singh, 2004), so its magnitude is likely to be even larger in practice.

This calls for a comparison with Hanson and Sunderam (2013), who also find that securitization leads to a negative externality through the underproduction of valuable information. In their model, the originator decides how much of its pool will be issued as debt or equity claims, while the investors initially choose whether to become informed at some cost or not. They find that the originator issues an excessive amount of virtually safe debt in a good state. However, this reduces the investors' ex ante incentive to become informed because there is no benefit from information acquisition when debt is already "safe" and informationally insensitive. This, though, is socially undesirable, because the presence of informed investors can facilitate efficient market functioning in a bad state. In other words, the benefit of their presence is not internalized.

Whereas the source of negative externality in their model is the social value of informed investors, the externality in my model arises from investment certification constraint. Since only a fraction of the investor pool clears the market for an uninsured bond rated below AAA, these investors can demand a price discount, which acts as a source of utility gain for them. Bond insurance wipes away this potential benefit, which is not internalized as the investors have no say in insurance contracting between the issuer and the insurer. Therefore, the potential desirability of "limiting the amount of AAA-rated debt that can be issued in good times (Hanson and Sunderam, 2013, p. 567)" also echoes in this model, albeit through a different mechanism.

I elaborate this point by revisiting the earlier numerical example in Section 3.1. Here, when $\min\left(c_0^A,\ c_0^B\right) = 6\%$, the cost of equity capital lies below the insurance threshold of 7.4% and B_i is insured. However, the internalized surplus from bond insurance is very small at around 0.007, while the negative externality of bond insurance is notably larger at around 0.048. This is a clear example of how bond insurance may not always be socially beneficial even with realistic parameter values.

Thus, when the negative externality term is large in magnitude, it is more likely that an issuer's acceptance of bond insurance is socially undesirable. A lengthy algebraic inspection of $U_{i,U}^{agg}$ derived in the Appendix yields that $\lambda_i\left(U_{i,U}^{agg}+1\right)$ increases in μ_i, θ_i, and γ, but is non-monotonic in λ_i.[14] This non-monotonicity arises from the presence of two conflicting forces. When λ_i is low, a small number of aggressive investors have to clear the market. This leads to a larger price discount and consequently a stronger utility gain for the aggressive investors. However, their utility gain is assigned a lower weight precisely because they are few in number.

FIGURE 3 HERE

Figure 3 graphically illustrates this relationship using the identical set of parameter values as in Figures 1 and 2. In particular, it reveals the non-monotonic effect of λ_i on the magnitude of negative externality. At low values of λ_i, $\lambda_i\left(U_{i,U}^{agg}+1\right)$ increases rapidly, reaching its maximum at around $\lambda_i = 0.149$. Then, a further increase in λ_i leads to a rapid decay of its magnitude, approaching the lower bound of 0 as λ_i tends to 1.

4. Local thinking and neglected risk

In this section, I maintain that all agents homogeneously engage in local thinking. Following Gennaioli, Shleifer and Vishny (2012), I assume that the agents assess the credit risk of a bond at $t = 0$ using only the two most likely states, ignoring the least likely scenario. Given the earlier assumption, this implies that the "large loss" scenario is initially neglected. For B_i, the agents' perceived probabilities of no loss and small loss are

[14] The proof is omitted for the brevity of exposition.

$\pi_i^g/\left(\pi_i^g+\pi_i^m\right)$ and $\pi_i^m/\left(\pi_i^g+\pi_i^m\right)$ respectively, even though the objective probabilities are governed as before.

4.1. Insurance choice with local thinking ($t=0$)

This initial neglect of the large loss outcome leads to a change in both the uninsured price of a bond as well as the insurers' reservation prices. Following the procedure as in Section 3.1 leads to a new insurance condition, as stated in Proposition 4:

Proposition 4 (bond insurance with local thinking). If the agents engage in local thinking and neglect the possibility of a "large loss", $B_i \in \{B_1,\ ...,\ B_J\}$ is insured with certainty at $t=0$ whenever:

$$\min\left(c_0^A,\ c_0^B\right) < \frac{\pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) - \pi_i^m}{\pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) + \pi_i^g} < 1. \tag{12}$$

Proof. See Appendix. ■

Now, let this new insurance threshold be denoted $\tilde{c}_0^i$. Then, the key question is whether this threshold lies above or below the insurance threshold under full rationality (denoted $\bar{c}_0^i$), as it determines whether the local thinking makes bond insurance more or less likely relative to the rational benchmark. An inspection of (9) and (12) reveals that neither threshold is unambiguously higher or lower than the other. Nevertheless, an important result is obtained:

Proposition 5 (neglected risk and underinsurance). For sufficiently large values of μ_i, the initial neglect of a large loss outcome makes bond insurance less likely relative to the rational benchmark.

Proof. See Appendix. ■

This is a crucial result of the paper. Monoline insurers were similarly criticized for expanding their business toward a market where they held little previous experience of risk management and consequently neglecting the possibility of a catastrophic loss. However, Proposition 5 reveals that, had the risks been properly accounted for, there would have been even greater demand for bond insurance under such circumstances. In other words, the popularity of bond insurance prior to the crisis ought not to be attributed to the insurers' neglect of tail risk.

The logic behind this result is as follows. When μ_i is high, there is a possibility that the bond would suffer a severe credit loss. As the magnitude of this loss increases, the risk averse investors' desire for insurance also increases. Conversely, when this risk is neglected, the bond is perceived to be safer and their desire for insurance subsides. In this respect, the monoline insurers' active presence in the market and their neglect of the tail outcome are susceptible to a *post hoc, propter hoc* fallacy; the fact that these two characteristics coexisted should not be interpreted as suggestive of a causal relationship.

As for other model parameters, i.e., γ, θ_i, and λ_i, the results are less clear cut. A change in one of these parameters affects both $\tilde{c}_0^i$ and $\bar{c}_0^i$ in the same direction, and its effect on the *relative* magnitude of these two thresholds is generally ambiguous. However, unpublished simulations indicate that $\tilde{c}_0^i$ is generally less sensitive to a parameter change in comparison with the threshold under full rationality ($\bar{c}_0^i$).[15]

FIGURE 4 HERE

FIGURE 5 HERE

I now revisit the numerical example in Section 3.1. With this set of parameter values, the insurance threshold decreases to 5.4% under local thinking. In other words, local thinking makes bond insurance less likely relative to the rational benchmark. In fact, Figure 4 reveals that this underinsurance result continues to hold for most parameter values. More importantly, the top-left graph of Figure 4 visually presents the insights of Proposition 5. A strong divergence between the two thresholds is observed as μ_i increases, given that $\bar{c}_0^i$ rapidly increases but $\tilde{c}_0^i$ remains unaffected. Figure 5 presents the identical result in finer detail; $\bar{c}_0^i$ lies below $\tilde{c}_0^i$ only for a restricted parameter range of $\mu_i \in [1.2,\ 1.6]$. As μ_i increases beyond 1.6, the result in Proposition 5 begins to hold.

[15]Simulation codes are available from the author upon request.

4.2. Prospect of insurer downgrade at $t = 1$

In contrast to the rational benchmark, a local thinking insurer in Section 4.1 does not raise sufficient capital to cover for the true worst case loss at issuance; since the CRA also engages in local thinking, raising insurance premium and equity capital amounting to θ_i for each insured B_i is sufficient for an insurer to retain her AAA rating at $t = 0$. In order to discuss the impact of neglected risk on the agents' decisions at $t = 1$, I discuss two possible scenarios.

On one hand, suppose that the agents' perceived probabilities do not change at $t = 1$. Then both insurers retain their AAA ratings and no further trading occurs. At maturity, however, due to the possibility of a large loss realization, an insurer's capital buffer may fall short of the claim payout demand. Using the earlier set of notations, with an indicator function I_j^i denoting whether bond B_i is insured by j or not, the amount of j's capital shortfall at maturity is given by $\max\left(0,\ \sum_{i=1}^{J} I_j^i \left(\Theta_i - \theta_i\right)\right)$. If there is a positive amount of capital shortfall, general arrangements in practice demand that all investors experiencing credit loss and requiring insurance payout take a "haircut", due to the lack of seniority among insured bonds. This haircut is defined as a proportion of their demanded payout, more formally $\sum_{i=1}^{J} I_j^i \left(\Theta_i - \theta_i\right) / \sum_{i=1}^{J} I_j^i \Theta_i$.

On the other hand, consider a more interesting case where an exogenous signal arrives at $t = 1$. Such signal could correspond to an indication of slowdown in the housing market, such as a sudden deterioration in the Case-Shiller Home Price index, for example. I assume that, following this signal, the posterior possibilities of no loss for all bonds decrease to the extent that it is completely discounted by local thinking agents. For notational convenience, further assume that the relative posterior probabilities of small loss and large loss remain identical.[16] If so, the perceived probabilities of local thinking agents for bond B_i at $t = 1$ changes to $\pi_i^m / \left(1 - \pi_i^g\right)$ for small loss and $\left(1 - \pi_i^g - \pi_i^m\right) / \left(1 - \pi_i^g\right)$ for large loss. Let these perceived probabilities be denoted $\tilde{\pi}_i^m$ and $\tilde{\pi}_i^b$ respectively.

In this instance, since the local thinking CRA takes into account of the large loss outcome at $t = 1$, its AAA rating criteria consequently changes. The CRA now demands

[16] All qualitative insights remain unchanged when the relative probabilities are altered.

that an insurer's capital buffer must increase to $\mu_i\theta_i$ for each insured B_i at $t = 1$. This leaves the bond insurers with a choice over whether to defend their current AAA credit rating or not. If the required capital is raised, then their AAA rating is retained. However, an insurer may choose not to raise more capital at $t = 1$ and accept a rating downgrade. Then, from the earlier discussion, it is immediate that:

Proposition 6 (insurer downgrade at $t = 1$). Insurer j's rating is downgraded at $t = 1$ if the equity capital cost arising from the CRA's additional capital requirement is larger than the reputational cost of downgrade. Formally, this occurs when:

$$c_1^j \sum_{i=1}^{J} I_j^i (\mu_i - 1) \theta_i > \kappa_j. \tag{13}$$

Proof. Immediately follows from (2), (3), and the CRA's revised rating criteria. ■

Proposition 6 highlights the important role of the insurers' equity market conditions in determining their ability to defend the AAA rating. Indeed, the first round of rating downgrade for Ambac in January 2008 occurred immediately after its plan to raise \$1 billion of fresh equity capital was canceled due to its depressed share prices.

When the insurers defend their credit ratings, the investors' decisions remain unchanged at $t = 1$, and the market outcome is comparable to Section 3.2. The problem arises following an insurer downgrade. Then, all conservative investors holding an asset insured by the downgraded insurer have to liquidate their holdings and exit the market. This exit price is determined in equilibrium through an interaction of aggressive and conservative investors.

In order to describe how this exit price is calculated in more detail, suppose that there are two bonds in the downgraded insurers' portfolio, namely B_1 and B_2. Since the downgraded insurer holds capital amounting to $\theta_1 + \theta_2$, local thinking aggressive investors are faced with the following scenarios. With probability $\tilde{\pi}_1^m \tilde{\pi}_2^m$, both bonds incur small losses and the capital buffer is sufficient. With probability $\tilde{\pi}_1^m \tilde{\pi}_2^b$, B_1 incurs a small loss but B_2's credit loss is large, so the aggressive investors in each bond take a haircut of $(\mu_2 - 1)\theta_2 / (\theta_1 + \mu_2\theta_2)$. With probability $\tilde{\pi}_1^b \tilde{\pi}_2^m$, haircut amounts to

$(\mu_1 - 1)\,\theta_1/\,(\mu_1\theta_1 + \theta_2)$, and in the remaining case of both bonds incurring large losses, haircut is $\left(\sum_{i=1}^{2}(\mu_i - 1)\,\theta_i\right)/\left(\sum_{i=1}^{2}\mu_i\theta_i\right)$. Using this, the investors' optimization condition yields the price of B_1 upon insurer downgrade, denoted $p_{1,D}^1$:

$$p_{1,D}^1 = \frac{\left[\begin{array}{c} \tilde{\pi}_1^m\tilde{\pi}_2^m + \left(1 - \frac{(\mu_2-1)\theta_1\theta_2}{\theta_1+\mu_2\theta_2}\right)\tilde{\pi}_1^m\tilde{\pi}_2^b \exp\left(\frac{\gamma((\mu_2-1)\theta_2/(\theta_1+\mu_2\theta_2))\theta_1}{\lambda_1}\right) \\ + \left(1 - \frac{\mu_1(\mu_1-1)\theta_1^2}{\mu_1\theta_1+\theta_2}\right)\tilde{\pi}_1^b\tilde{\pi}_2^m \exp\left(\frac{\gamma((\mu_1-1)\theta_1/(\mu_1\theta_1+\theta_2))\mu_1\theta_1}{\lambda_1}\right) \\ + \left(1 - \frac{\mu_1\theta_1\sum_{i=1}^{2}(\mu_i-1)\theta_i}{\sum_{i=1}^{2}\mu_i\theta_i}\right)\tilde{\pi}_1^b\tilde{\pi}_2^b \exp\left(\frac{\gamma\left(\left(\sum_{i=1}^{2}(\mu_i-1)\theta_i\right)/\left(\sum_{i=1}^{2}\mu_i\theta_i\right)\right)\mu_1\theta_1}{\lambda_1}\right) \end{array}\right]}{\left[\begin{array}{c} \tilde{\pi}_1^m\tilde{\pi}_2^m + \tilde{\pi}_1^m\tilde{\pi}_2^b \exp\left(\frac{\gamma((\mu_2-1)\theta_2/(\theta_1+\mu_2\theta_2))\theta_1}{\lambda_1}\right) \\ +\tilde{\pi}_1^b\tilde{\pi}_2^m \exp\left(\frac{\gamma((\mu_1-1)\theta_1/(\mu_1\theta_1+\theta_2))\mu_1\theta_1}{\lambda_1}\right) \\ +\tilde{\pi}_1^b\tilde{\pi}_2^b \exp\left(\frac{\gamma\left(\left(\sum_{i=1}^{2}(\mu_i-1)\theta_i\right)/\left(\sum_{i=1}^{2}\mu_i\theta_i\right)\right)\mu_1\theta_1}{\lambda_1}\right) \end{array}\right]} < 1. \tag{14}$$

In other words, a tractable, closed form price solution exists even in the event of an insurer downgrade. The same technique can be extended to any arbitrary number of bonds in the downgraded insurer's portfolio. Though theoretically possible, it can quickly become computationally cumbersome; if there are N bonds in the downgraded insurer's portfolio, investors need to consider 2^N possible scenarios and compute the insurer's capital shortfall in each instance.

In order to demonstrate how this price is derived, suppose the two bonds' various parameters are as follows. Let $\Theta_1 = \Theta_2 = \{0,\ 0.25,\ 0.5\}$ and the corresponding probabilities are $\Pi_1 = \Pi_2 = \{0.99,\ 0.009,\ 0.001\}$. Furthermore, $\lambda_1 = \lambda_2 = 0.25$ and $\gamma = 2$. In other words, the two bonds are identical to each other in all respects, with the parameter values corresponding to the earlier numerical example. Let $c_0^A = 5\% < c_0^B$ so that these bonds are insured by insurer A at $t = 0$, and further suppose c_1^A increases to 10%. Finally, let $\kappa_A = 0.03$, which is smaller than the additional cost of equity capital. Thus, an insurer downgrade occurs.

FIGURE 6 HERE

In this case, it turns out that $p_{1,D}^1 = p_{1,D}^2 = 0.936$. In other words, both bonds experience a price discount of around 6.2%. Figure 6 also plots the period 1 price of B_1

as a function of various model parameters. While a change in γ, θ_1, or λ_1 does not affect the insurer's downgrade decision, it does affect bond's price upon insurer downgrade. As expected, an increase in risk aversion (γ) or the bond's baseline loss (θ_1) reduces the price of downgraded B_1, while an increase in the proportion of aggressive investors (λ_1) has an opposite effect.

A change in μ_1 is somewhat different; the price of B_1 initially remains at 1 but there is a downward jump to 0.982 at $\mu_1 = 1.2$, where the insurer switches from defending her credit rating to accepting a rating downgrade. Beyond this point, a further increase in μ_1 reduces its price. In fact, through a large increase in μ_1, it is possible to generate a significant price discount in the order of magnitudes witnessed during the heights of the recent crisis. For example, in January 2008, the ABX 7-1 series index for AA-rated securities initiated in January 2007 stood at just over 40 (Brunnermeier, 2009), down from the initiating point of 100 a year earlier. As μ_1 approaches its upper limit of 4, the period 1 price of downgraded bond falls to around 0.431, highlighting the importance of neglected losses in generating large price movements.

4.3. Welfare implications of neglected risk

This prediction of insurer downgrade is in close accordance with the observed facts. It is thus all the more important to analyze how this initial presence of neglected risk affects the market welfare at $t = 1$, and I thus repeat the welfare analysis. For the brevity of exposition, I focus on the case where the arrival of an exogenous signal tilts the perception of local thinking agents towards the large credit loss event.

Before the analysis, however, the definition of "welfare" has to be clarified for local thinking agents. Throughout this paper, I maintain that the agents' welfare is computed from an omniscient perspective, using the objective probabilities instead of their perceived probabilities. In other words, when computing the ex ante expected utilities of the market agents for welfare purposes, all possible scenarios are included regardless of whether the agents themselves take into account of a particular credit outcome or not.

4.3.1. Underinsurance and welfare improvement

The first result of the welfare analysis in this section has an important bearing on the regulation of the bond insurance market, which I state in the following proposition:

Proposition 7 (possible welfare improvement under local thinking). Suppose μ_i is sufficiently high that $\tilde{c}_0^i < \bar{c}_0^i$ for bond B_i. If so, the agents' welfare derived from B_i could be higher relative to the rational benchmark when the agents engage in local thinking.

Proof. See the numerical example below. ■

Proposition 7 is best explained by revisiting the numerical example discussed in Section 3.3. Proposition 3 has already stated that bond insurance incurs negative externality by eliminating the aggressive investors' benefit from holding an uninsured bond with significant price discount. However, Proposition 5 has shown that the initial neglect of tail outcome could make a bond less likely to be insured when the magnitude of this neglected loss was sufficiently large. Then, if a bond is issued without insurance as a result of local thinking, the market welfare could be increased accordingly.

I demonstrate this using the earlier numerical example. It was already shown that the insurance threshold under the rational benchmark was around 7.4% but decreased to 5.4% when the agents engaged in local thinking. Thus, if $\min\left(c_0^A,\ c_0^B\right) = 6\%$ as before, a bond would be insured when the agents are rational but not when they engage in local thinking. It is then necessary to compare the welfare associated with insured B_i under the rational benchmark ($\Omega_{0,I}^i$) against the corresponding welfare of uninsured B_i under local thinking ($\tilde{\Omega}_{0,U}^i$).[17] It is then possible to show that:[18]

$$\Omega_{0,I}^i - \tilde{\Omega}_{0,U}^i = 1 - \tilde{p}_{0,U}^i - \min\left(\Gamma_{A,0}^i,\ \Gamma_{B,0}^i\right) - \lambda_i\left(\tilde{U}_{i,U}^{agg} + 1\right), \tag{15}$$

[17]Notice that, under the rational benchmark, $\Omega_{0,I}^i = \Omega_{1,I}^i$. Furthermore, even though the local thinking participants' perceptions change between $t = 0$ and $t = 1$, $\tilde{\Omega}_{0,U}^i = \tilde{\Omega}_{1,U}^i$ since an uninsured bond always remains in the hands of aggressive investors and no further trading occurs at $t = 1$.

[18]Proving this is very similar to the proof of Proposition 4 and is thus left to the reader.

where $\tilde{p}^i_{0,U}$ is the uninsured price of B_i under local thinking, as defined in (A.20) in the Appendix, and $\tilde{U}^{agg}_{i,U}$ is the aggressive investors' expected utility (computed using objective probabilities) from a bond purchase at this price.

Due to the initial neglect of tail risk, it turns out that $\tilde{p}^i_{0,U} = 0.984$ is larger than $p^i_{0,U} = 0.960$. As a result, the aggressive investors' expected benefit from investing in an uninsured bond, i.e., $\lambda_i \left(\tilde{U}^{agg}_{i,U} + 1\right) = 0.005$, is smaller than the rational benchmark, where $\lambda_i \left(U^{agg}_{i,U} + 1\right) = 0.048$. Yet, it still turns out that $\Omega^i_{0,I} - \tilde{\Omega}^i_{0,U} = -0.022$, implying that the market welfare associated with B_i is higher when the agents engage in local thinking. By alleviating the negative externality of bond insurance, the local thinking agents' tendency to underinsure could lead to a welfare improvement even when their perceived probabilities diverge from the objective probabilities and the prices are set "incorrectly" as a result of neglected risk.

4.3.2. Externality of insurer downgrade

Next, I consider the welfare implications of an insurer downgrade in more detail. Whereas the previous analysis primarily considered the case where a bond is not insured as a result of the agents' local thinking, now consider the opposite scenario where a bond is insured at issuance. Then, following the arrival of an exogenous signal at $t = 1$, the insurer has to decide whether to raise additional capital to defend her AAA rating or accept a rating downgrade.

How does this choice affect the market welfare at $t = 1$? Without loss of generality, suppose that insurer A is facing this decision. Furthermore, let $\Omega^A_{1,def}$ denote the total welfare generated by insurer A if she defends her credit rating, and $\Omega^A_{1,down}$ denote the corresponding measure when the insurer accepts a rating downgrade:

$$\Omega^A_{1,def} \equiv \sum_{i=1}^{J} \left(I^i_A \Omega^i_1 \mid \text{insurer } A \text{ retains AAA rating} \right), \tag{16}$$

$$\Omega^A_{1,down} \equiv \sum_{i=1}^{J} \left(I^i_A \Omega^i_1 \mid \text{insurer } A \text{ is downgraded} \right). \tag{17}$$

Then, using a similar line of reasoning as in the proof of Proposition 3, it is possible to show that:

$$\begin{aligned}\Omega^A_{1,down} - \Omega^A_{1,def} &= c^A_1 \sum_{i=1}^{J} I^i_A \left(\mu_i - 1\right)\theta_i - \kappa_A \\ &+ \sum_{i=1}^{J} I^i_A \left\{\lambda_i \left(\tilde{U}^{agg}_{i,U}\left(p^1_{1,D}\right) + 1\right) + \left(1-\lambda_i\right)\left(\tilde{U}^{con}_{i,U}\left(p^1_{1,D}\right) + 1\right)\right\},\end{aligned} \tag{18}$$

where $\tilde{U}^{agg}_{i,U}\left(p^1_{1,D}\right)$ and $\tilde{U}^{con}_{i,U}\left(p^1_{1,D}\right)$ denote the expected utilities of aggressive and conservative investors in B_i following an insurer downgrade.

Crucially, the last term in (18) is the source of externality associated with an insurer downgrade. Whether an insurer defends her credit rating at $t = 1$ or not is solely determined by her cost of equity capital against the reputational cost of rating downgrade. Thus, the first two terms in (18) are fully internalized. The last term, however, represents a knock-on effect on the investors' welfare. An insurer's rating downgrade triggers a forced sale of the conservative investors' holdings at a "fire-sale price" (Coval and Stafford, 2007), with adverse implications on their utility. The aggressive investors also suffer mark-to-market losses on their existing holdings. However, they absorb the exiting conservative investors' positions at a cheap price and benefit from a price discount. In other words, there are opposing forces on market welfare.

I now revisit the numerical example in Section 4.2. With these parameter values, the externality term in (18) amounts to -0.110. In other words, the insurer's acceptance of a rating downgrade harms the investors. Even when the internalized benefit of the insurer is accounted for, the overall welfare impact of an insurer downgrade, i.e., $\Omega^A_{1,down} - \Omega^A_{1,def}$, remains negative at -0.090. In other words, although it is in the insurer's interest not to defend her credit rating, its adverse spillover effect on the investors dominates her private benefit. Further calculations show that the investors, on the whole, are willing to pay the insurer not to be downgraded.

FIGURE 7 HERE

Figure 7 further demonstrates that the social undesirability of insurer downgrade is a general result; regardless of how the parameter values are varied, the welfare differential

$(\Omega^A_{1,down} - \Omega^A_{1,def})$ is always negative, implying that it is always better to encourage the bond insurer to retain her AAA rating from an overall social perspective. Furthermore, Figure 7 is very similar to the price patterns observed in Figure 6. Since the conservative investors are most adversely affected by the fall in period 1 bond price, this suggests that the forced exit of conservative investors exerts a dominant effect on the overall welfare.

The adverse impact of an insurer downgrade is further highlighted in the top-left corner of Figure 7. For low values of μ_i between 1 and 1.2, the insurer chooses to defend her credit rating. However, when μ_i reaches 1.2, the insurer's choice is reversed and she accepts a rating downgrade. At this point, a downward jump in the welfare differential is observed, as the conservative investors are forced into a "fire sale" of their position. As μ_i further increases, the deterioration in the conservative investors' welfare is exacerbated.

5. Heterogeneity in the agents' risk perception

Until now, the agents' risk perceptions were assumed to be homogeneous, as they *all* either engaged in local thinking or full rationality. Given the criticism in the previous literature that the underestimation of risk was a marketwide phenomenon (e.g., Gerardi et al., 2008; Blanchard, 2009), this appears appropriate. However, one cannot completely discount the possibility that their risk perceptions may have been heterogeneous. For example, while the investors engaging in a "search for yield" with limited expertise on valuing securitized assets may be susceptible to underestimating their true risks, the behavior of the CRA or the insurers—with greater familiarity of the securitization industry—may be closer to full rationality.

With this in mind, I briefly discuss the implications of introducing an element of heterogeneity among the agents' risk perception. To make the scenarios realistic, this section focuses on the cases where the investors engage in local thinking but the CRA and/or the insurers are capable of evaluating the bonds' true risks. Since the investors continue to engage in local thinking, this implies that the issuer's reservation price for insurance premium is identical to Section 4.

5.1. CRA and bond insurers with full rationality

If both the CRA and the insurers are fully rational, then the insurers' reservation prices for insuring B_i is the same as in Section 3, namely $\Gamma^i_{A,0}$ and $\Gamma^i_{B,0}$. Then, using the issuer's reservation price, defined in (A.21) in the Appendix and denoted $\tilde{\Delta}^i_0$, it is possible to derive the new insurance threshold:

$$\min\left(c^A_0,\ c^B_0\right) < \frac{\pi^m_i\left(1-\pi^m_i-\left(1-\pi^g_i-\pi^m_i\right)\mu_i\right)\exp\left(\frac{\gamma\theta_i}{\lambda_i}\right)-\pi^g_i\left(\pi^m_i+\left(1-\pi^g_i-\pi^m_i\right)\mu_i\right)}{\varsigma_i\left(\pi^g_i+\pi^m_i\exp\left(\frac{\gamma\theta_i}{\lambda_i}\right)\right)}, \tag{19}$$

where $\varsigma_i \equiv \mu_i\pi^g_i + (\mu_i - 1)\,\pi^m_i$ as before. Since $\Delta^i_0 > \tilde{\Delta}^i_0$, the threshold is lower than the rational benchmark. Moreover, since $\min\left(\Gamma^i_{A,0},\ \Gamma^i_{B,0}\right) > \min\left(\tilde{\Gamma}^i_{A,0},\ \tilde{\Gamma}^i_{B,0}\right)$, this threshold is also lower than the threshold under the homogeneous local thinking scenario in Section 4.1. For example, in the earlier numerical example, while the insurance threshold under the rational benchmark was 7.4% and the local thinking threshold was 5.4%, it further decreases to 2.6% when the investors engage in local thinking but the remaining agents' risk perceptions are rational. In other words, insurance becomes even less likely.

It is not possible to arrive at an unambiguous prediction regarding whether this decrease in the likelihood of bond insurance is welfare-improving or not. Due to Proposition 3, a more stringent insurance threshold can be welfare-enhancing in some circumstances. However, it may also turn out to be a step too far in the other direction, as one cannot rule out the possibility of underinsurance relative to the social optimum.

Nevertheless, there is one notable benefit in comparison with the homogeneous local thinking case in Section 4. Since a rational CRA correctly assesses the credit loss outcomes, the capital requirements for AAA-rated insurers are set appropriately from the issuance. Thus, a possibility of any subsequent rating downgrade of a bond insurer is completely eliminated, which is valuable from the welfare perspective.

5.2. Rational CRA with local thinking bond insurers

In the second scenario I consider, the CRA holds rational beliefs about the bonds' credit risks but the insurers engage in local thinking. Then, even though the insurers' subjective assessments of bond B_i's expected loss at $t = 0$ is simply $\pi_i^m \theta_i / (\pi_i^g + \pi_i^m)$, the CRA demands that the insurers set aside a capital buffer amounting to $\mu_i \theta_i$ for each insured B_i. Thus, insurer j's reservation price for B_i, denoted $\check{\Gamma}_{j,0}^i$, changes to:

$$\check{\Gamma}_{j,0}^i = \frac{\pi_i^m}{\pi_i^g + \pi_i^m}\theta_i + c_0^j \left(\frac{\mu_i (\pi_i^g + \pi_i^m) - \pi_i^m}{\pi_i^g + \pi_i^m} \right) \theta_i, \tag{20}$$

and this, in turn, yields the new insurance threshold:

$$\min\left(c_0^A,\ c_0^B\right) < \frac{\pi_i^m \pi_i^g \left(\exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) - 1 \right)}{\left(\mu_i (\pi_i^g + \pi_i^m) - \pi_i^m\right) \left(\pi_i^g + \pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) \right)}. \tag{21}$$

Assuming $\min\left(c_0^A,\ c_0^B\right) < 1$, $\min\left(\tilde{\Gamma}_{A,0}^i,\ \tilde{\Gamma}_{B,0}^i\right) < \min\left(\check{\Gamma}_{A,0}^i,\ \check{\Gamma}_{B,0}^i\right) < \min\left(\Gamma_{A,0}^i,\ \Gamma_{B,0}^i\right)$, and thus the threshold in (21) is higher than the homogeneous local thinking threshold in Section 4 but lower than the threshold in (19). As for the numerical example, (21) yields the insurance threshold to be 2.7%, slightly higher than 2.6% but substantially below the homogeneous local thinking threshold of 5.4% and the rational benchmark of 7.4%.

Once again, it is difficult to provide a definitive answer regarding the overall market welfare, but as in Section 5.1, the CRA's full rationality and the consequent elimination of any insurer downgrade provide an inherent advantage. Thus, with the exception of a minor change in the insurance threshold, most results would be similar to Section 5.1. In other words, as long as the CRA is capable of rational risk assessments, whether a bond insurer also "gets risk right" or not does not lead to major qualitative differences in the market outcome.

5.3. Local thinking CRA with rational bond insurers

In the last of the scenarios, the insurers are assumed to be fully rational but the CRA engages in local thinking. In this instance, while the insurers acknowledge the possibility of a "large loss" at $t = 0$, the CRA only demands a capital buffer of θ_i for each insured B_i. Then, the insurers' choices depend on their beliefs about how the CRA's capital requirement would change between $t = 0$ and $t = 1$. In other words, a rational insurer has to form a belief regarding whether the local thinking CRA's information set would subsequently be altered or not.

First, suppose that a rational insurer believes the CRA's capital requirement would not change at $t = 1$. Then, she has no reason to raise any capital beyond the CRA's requirement at $t = 0$, as additional cost of equity capital is incurred without any accompanying benefit. Thus, each insurer j's reservation price for insuring B_i, denoted $\hat{\Gamma}^i_{j,0}$, is now given by:

$$\hat{\Gamma}^i_{j,0} = \pi^m_i \theta_i + \left(1 - \pi^g_i - \pi^m_i\right) \mu_i \theta_i + c^A_0 \left(\left(1 - \pi^m_i\right)\left(1 - \mu_i\right) + \pi^g_i \mu_i\right) \theta_i, \tag{22}$$

where $\mu_i < \left(1 - \pi^m_i\right) / \left(1 - \pi^g_i - \pi^m_i\right)$ is implicitly assumed. This yields the corresponding insurance threshold as follows:

$$\min\left(c^A_0,\ c^B_0\right) < \frac{\pi^m_i \left(1 - \pi^m_i - \left(1 - \pi^g_i - \pi^m_i\right)\mu_i\right) \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) - \pi^g_i \left(\pi^m_i + \left(1 - \pi^g_i - \pi^m_i\right)\mu_i\right)}{\left(1 - \pi^m_i - \left(1 - \pi^g_i - \pi^m_i\right)\mu_i\right)\left(\pi^g_i + \pi^m_i \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right)\right)}. \tag{23}$$

As for the numerical example, this threshold stands at around 5.3%, substantially above the previous two cases and close to the insurance threshold under the homogeneous local thinking case in Section 4.

A more interesting case arises when a rational insurer anticipates the arrival of an exogenous signal at $t = 1$. In other words, even though the CRA is content with the capital buffer of θ_i for each insured B_i at $t = 0$, the insurer knows that the CRA's capital requirement will become more stringent at $t = 1$. Then, the insurer has three options. On one hand, she may wait until the CRA's additional capital requirement at $t = 1$ and raise the required amount of equity capital then. On the other hand, if she believes that the equity market conditions will deteriorate in the meantime, she may raise additional equity capital in excess of the CRA's initial requirement at $t = 0$ in anticipation.

However, there also exists a more disturbing third option. Suppose $\kappa_A = \kappa_B = 0$ so that both insurers would always accept a rating downgrade at $t = 1$ when faced with the requirement for additional capital. In this instance, even though both insurers know that the CRA will call for additional capital at $t = 1$, they fail to do anything about it. More ominously, they enter the bond insurance market by satisfying the local thinking CRA's capital requirement at $t = 0$, even though they fully anticipate that their credit ratings will be downgraded subsequently. Whether this possibility is likely to be prevalent in practice or not, of course, is open to debate.

In general, it is difficult to determine which of these options would prevail when the insurers anticipate the CRA's capital requirement to change between $t = 0$ and $t = 1$. However, it can be shown that the last option will be chosen when their reputational costs are sufficiently close to 0. Thus, the main insight of this section is that the rational risk assessment capabilities of the CRA—not the insurers—is an important prerequisite in preventing the damaging prospect of an insurer downgrade.

6. Discussion

6.1. Main predictions of the model

From the analysis in previous sections, the model has derived four main predictions regarding the bond insurance market. Before I proceed with the discussion, I reiterate these predictions explicitly for the clarity of exposition:

(P1) There is excessive provision of bond insurance relative to the social optimum when the agents' risk assessments are rational.

(P2) When the agents engage in local thinking, there is likely to be an underprovision of bond insurance relative to the rational benchmark, which can be welfare improving.

(P3) However, local thinking also opens up the possibility of a bond insurer's rating downgrade during times of unfavorable equity market conditions. This leads to a fire sale of the rating-constrained investors' position and harms their welfare.

(P4) The credit rating agency's rational risk assessment capabilities are important prerequisite for efficient functioning of the bond insurance market.

6.2. Policy implications

The regulatory response to the crisis in the bond insurance market has largely been confined to damage limitation. State of New York Insurance Department (NYID) brokered a deal between MBIA and FGIC in August 2008, through which MBIA acquired most of the FGIC's relatively safe municipal bond portfolio. Following the deal, FGIC was left with a badly exposed portfolio and the CRAs eventually withdrew their credit ratings, while MBIA was essentially given a lifeline. As for Ambac, the regulators in Wisconsin ordered a segregation of Ambac's liabilities, creating separate accounts for municipal bonds and "toxic" products. The common aim of both actions, which essentially created seniority among the claimholders in favor of municipal bond investors, was to prevent a contagion of mortgage-related problems.

However, due to the near extinct status of the bond insurance industry since the crisis, there has not been much discussion on how to improve the regulation of the industry itself. An exception is Circular Letter No. 19 issued by NYID in September 2008,[19] in which the regulators set strict limits on offering insurance policies for CDOs and non-investment-grade bonds, called for a larger capital buffer against mezzanine junior investment-grade bonds, and encouraged the insurers to re-evaluate their current risk management practices. This guideline remains in effect at the time of writing.

The analysis in this paper offer fresh insights for the future regulation of the bond insurance industry. First, the model argues that, when the issuer and the insurer have little surplus to share from bond insurance, it is often better to leave a bond uninsured in the hands of aggressive investors. Thus, while the regulatory limit on insuring high-yield bonds is in place primarily out of risk management concerns, it may also be beneficial from a welfare perspective. In fact, since uninsured mezzanine investment-grade tranches

[19]With the exception of Ambac, most monoline insurers are domiciled in the State of New York, leaving the primary regulatory responsibilities in the hands of NYID.

often also experience a sizeable price discount,[20] the regulators ought to consider placing a similar, perhaps less stringent limit for these junior investment-grade tranches.

Second, the model highlights the importance of the CRA's "rational" risk assessment capabilities in preventing the potential rating downgrades of bond insurers and enhancing the market welfare. Even so, due to the scope of the regulatory remits, the regulators may focus solely on improving the insurers' risk management practices directly on their watch. As the previous section has shown, this would prove insufficient. Thus, a greater degree of cooperation between the regulatory bodies appears vital, particularly with regards to the interaction between the CRA and the insurers. Following the recent creation of the Federal Insurance Office (FIO) within the U.S. Treasury in response to the calls for a national insurance regulator from various quarters (e.g., Acharya et al., 2009), this would be a natural next step in the overall regulatory design.

Finally, the model has shown that, when an insurer decides to accept a rating downgrade, the ensuing deterioration in the investors' welfare is not internalized. Given that the rating downgrades of Ambac occurred when its desperate plan to raise new capital was canceled due to unfavorable market conditions, it may be worth considering the costs and benefits of providing the claimholders with an option of recapitalizing the insurer as a last resort in times of crisis. On one hand, the claimholders with investment certification constraints should be willing to provide the requisite capital to avoid a damaging forced exit from the market. However, on the other hand, this could be a strong source of moral hazard for the insurers from an ex ante perspective.

6.3. Extensions

6.3.1. Large number of possible credit outcomes at maturity

A salient aspect of this model is its tractability in a more general setting. Suppose that each B_i has more than three possible credit outcomes at maturity. Formally, $\Theta_i = \{\mu_1\theta_i,\ \mu_2\theta_i,\ ...,\ \mu_M\theta_i\}$, where $M > 3$, $\mu_1 = 0$, $\mu_2 = 1$, $\mu_M\theta_i \leq 1$, and $\mu_j > \mu_k$ for all $j > k$. Crucially, M may be arbitrarily large as long as the nature of credit risk remains

[20] See Longstaff (2010) for an example of the initial price offers of different tranches of an ABS CDO.

discrete. The corresponding probability set is given by $\Pi_i = \{\pi_1,\ \pi_2,\ ...,\ \pi_M\}$. Then, under the rational benchmark, the bond's issuance price differential is given by:

$$\Delta_0^i = \frac{\theta_i \sum_{j=1}^{M} \pi_j \mu_j \exp\left(\frac{\gamma \mu_j \theta_i}{\lambda_i}\right)}{\sum_{j=1}^{M} \pi_j \exp\left(\frac{\gamma \mu_j \theta_i}{\lambda_i}\right)}, \tag{24}$$

while each insurer j's reservation price for insuring B_i is given by:

$$\Gamma_{j,0}^i = \theta_i \left[\sum_{j=1}^{M} \pi_j \mu_j + c_0^j \left(\mu_M - \sum_{j=1}^{M} \pi_j \mu_j \right) \right]. \tag{25}$$

A simple inspection of (24) and (25) reveals that, even when there are more than three possible credit outcomes at maturity, the insurance decision will continue to be characterized by a single threshold for the cost of equity capital at $t = 0$, denoted $\mathring{c}_0^i$:

$$\mathring{c}_0^i = \frac{\sum_{j=1}^{M} \pi_j \mu_j \exp\left(\frac{\gamma \mu_j \theta_i}{\lambda_i}\right) - \left(\sum_{j=1}^{M} \pi_j \mu_j\right) \left(\sum_{j=1}^{M} \pi_j \exp\left(\frac{\gamma \mu_j \theta_i}{\lambda_i}\right)\right)}{\left(\mu_M - \sum_{j=1}^{M} \pi_j \mu_j\right) \left(\sum_{j=1}^{M} \pi_j \exp\left(\frac{\gamma \mu_j \theta_i}{\lambda_i}\right)\right)}. \tag{26}$$

The same technique can be applied to the case of local thinking when the agents consider $L < M$ most likely scenarios among the possible credit outcomes. In this instance, an insurer downgrade continues to be a possibility when the CRA neglects the worst possible outcome, i.e., $\mu_M \theta_i$, at issuance. Unpublished simulation results also indicate that, when the neglected losses tend to be sufficiently large in magnitude, the key result of Section 4.1, namely the local thinking agents' tendency to underinsure relative to the rational benchmark, continues to hold. Thus, the qualitative results of this paper are preserved under a more general credit risk setting.

6.3.2. Interdependence of the bonds' credit risks

Due to the nature of the model set-up, any interdependence of the bonds' credit risks has little bearing on the agents' decisionmaking process. First, since the issuer only earns the intermediation spread at issuance, she has little concern over how her bond's credit risk relates to other bonds. Second, the CRA has no incentive to revise its rating criteria if it believes that the bonds' credit risks are positively correlated; if anything, it strengthens the rationale behind its emphasis on the capital coverage for the "worst case" portfolio loss. Third, an increase in the correlation of the bonds' credit risks does not affect the insurers' decisions because of their risk neutrality and the nature of the CRA's rating criteria.

Thus, any effect of the bonds' interdependence is restricted to the investors' optimization problem. However, since the investors in this model only consider investing in one particular bond, its interdependence with other bonds does not affect their decisions. Thus, all of the model's results remain intact regardless of whether the bonds' credit risks are interrelated or not.

Nevertheless, this raises a subtle underlying question: is it appropriate to assume that the investors only have access to one particular bond? For a sizeable proportion of investors such as mutual funds specializing in structured debt instruments, this may not hold. It is not straightforward to characterize the results of a more general model where the investors can invest in multiple bonds, but unpublished simulation results for certain limit cases indicate that bond prices fall as they become more positively correlated to each other. This is not surprising since a greater degree of positive interdependence hinders the investors' ability to diversify. In this instance, whether the neglect of the tail outcome implies a greater or lesser degree of perceived interdependence becomes another important factor in the local thinking agents' decisionmaking process.

6.3.3. Infinite time horizon with repeated bond issuance

Throughout the paper, I considered a one-off issuance of bonds. However, it is possible to extend the model toward a more realistic setting where new bonds are issued every period and the time horizon is infinite. The agents' choices can be modeled in a number of ways, but consider the following scenario. Suppose the insurers maximize the sum of their discounted expected utility while a new pool of potential investors arrive for each bond every period. Further suppose that an exogenous signal arrives at each t for local thinking agents, which tilts their perceived posterior probabilities. Lastly, an insurer's

AAA rating is deemed credible regardless of its past downgrade history as long as she is perceived to hold a sufficient capital buffer for all her outstanding insurance claims.

Then, a simple inspection of this problem reveals that the reputational cost of a downgrade at a certain $t < \infty$, an exogenous parameter in the baseline model, corresponds to the loss of the insurer's expected profit from new insurance business at t. This simple thought experiment demonstrates that the inclusion of an exogenous reputation cost in the model is not an arbitrary choice; it reflects the essence of repeated interactions between the agents in practice.

7. Conclusion

This paper has provided a comprehensive theoretical analysis of the market for bond insurance using the concept of local thinking. Through this model, it was possible to rationalize how the initial neglect of an extreme tail outcome gave rise to a subsequent rating downgrade of a bond insurer, as experienced by the major monoline insurers during the subprime crisis. However, the model revealed a more fundamental issue with bond insurance. By turning mezzanine investment-grade bonds into AAA-rated assets, the provision of bond insurance eliminated the prospect of a price discount, which could have otherwise yielded potential benefits for the investors without an investment certification constraint. Instead, all investors ended up holding a "safe" asset with little additional utility gain.

The paper has also shown that the popularity of bond insurance did not stem from the agents' neglect of extreme credit outcomes. Thus, merely encouraging the agents to take a more rational approach to risk assessment was insufficient in achieving a socially optimal outcome. In fact, there was no guarantee that the market welfare would be enhanced as long as the aforementioned negative externality remained unaddressed. In addition, the question of *who* held the capabilities for proper, thorough risk management also turned out to be important, and the paper has highlighted the important role played by the credit rating agencies in this respect.

In light of these findings, the regulators face a number of important practical questions. How should the regulators for monoline insurers cooperate with the regulators for credit rating agencies without creating an unnecessary regulatory overlap? Given the difficulties associated with the quantification of bond insurance's negative externality on the investors, how should the regulators address this problem? These practical issues are beyond the scope of the paper's theoretical framework, and addressing them will require careful consideration and deliberation on the part of the regulators.

Appendix: Proofs

Proof of Proposition 1. I prove the proposition in a number of steps. First of all, I derive issuer i's maximum willingness to pay for bond insurance, given by the issuance price differential with vs. without bond insurance (Δ_0^i). Since the insurer holds sufficient capital to cover for the worst case credit loss, the bond is perceived to be completely riskless if it is insured. If so, $p_{0,I}^i = 1$ follows trivially. Then:

Lemma A.1. The issuance price differential of $B_i \in \{B_1, \ ..., \ B_J\}$ is given by:

$$\Delta_0^i = \frac{\theta_i \pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) + \mu_i\theta_i \left(1 - \pi_i^g - \pi_i^m\right) \exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right)}{\pi_i^g + \pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) + \left(1 - \pi_i^g - \pi_i^m\right) \exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right)}. \tag{A.1}$$

Proof. To compute Δ_0^i, it is necessary to derive the bond's price without insurance, i.e., $p_{0,U}^i$. Since a conservative investor j cannot hold an uninsured bond, $x_{i,t}^{con}(j) = 0$. To derive $x_{i,t}^{agg}(j)$, rearranging (4) and (5) along with $W_0^{agg}(j) = 0$ gives a representative aggressive investor j's objective function as:

$$\max_{x_{i,t}^{agg}(j)} \left[\begin{array}{c} -\pi_i^g \exp\left\{-\gamma\left(1 - p_{0,U}^i\right) x_{i,t}^{agg}(j)\right\} - \pi_i^m \exp\left\{-\gamma\left(1 - \theta_i - p_{0,U}^i\right) x_{i,t}^{agg}(j)\right\} \\ - \left(1 - \pi_i^m - \pi_i^g\right) \exp\left\{-\gamma\left(1 - \mu_i\theta_i - p_{0,U}^i\right) x_{i,t}^{agg}(j)\right\} \end{array} \right]. \tag{A.2}$$

This may be rearranged as:

$$\max_{x_{i,t}^{agg}(j)} -\exp\left(-\gamma\left(1-p_{0,U}^{i}\right)x_{i,t}^{agg}(j)\right)\left\{\begin{array}{c}\pi_i^g+\pi_i^m\exp\left(\gamma\theta_i x_{i,t}^{agg}(j)\right)\\ +\left(1-\pi_i^m-\pi_i^g\right)\exp\left(\gamma\mu_i\theta_i x_{i,t}^{agg}(j)\right)\end{array}\right\}. \tag{A.3}$$

The first order condition yields:

$$\begin{aligned}&\left(1-p_{0,U}^{i}\right)\left\{\pi_i^g+\pi_i^m\exp\left(\gamma\theta_i x_{i,t}^{agg}(j)\right)+\left(1-\pi_i^m-\pi_i^g\right)\exp\left(\gamma\mu_i\theta_i x_{i,t}^{agg}(j)\right)\right\}\\ =\ &\theta_i\pi_i^m\exp\left(\gamma\theta_i x_{i,t}^{agg}(j)\right)+\mu_i\theta_i\left(1-\pi_i^m-\pi_i^g\right)\exp\left(\gamma\mu_i\theta_i x_{i,t}^{agg}(j)\right).\end{aligned} \tag{A.4}$$

Since every investor of the same type are homogeneous in all other respects, I look for a symmetric equilibrium. Then, the market clearing condition requires $x_{i,t}^{agg}(j)=\frac{1}{\lambda_i}$. Using this, a simple rearrangement of (A.4) yields:

$$p_{0,U}^{i}=\frac{\pi_i^g+(1-\theta_i)\,\pi_i^m\exp\left(\frac{\gamma\theta_i}{\lambda_i}\right)+(1-\mu_i\theta_i)\left(1-\pi_i^g-\pi_i^m\right)\exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right)}{\pi_i^g+\pi_i^m\exp\left(\frac{\gamma\theta_i}{\lambda_i}\right)+\left(1-\pi_i^g-\pi_i^m\right)\exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right)}. \tag{A.5}$$

Notice that $p_{0,U}^{i}\in(0,1)$, with $p_{0,U}^{i}=1$ if and only if $\pi_i^g=1$ or $\theta_i=0$. Using $p_{0,I}^{i}=1$ and (A.5), (A.1) can be derived immediately. ■

This quantity ought to be compared to the insurers' respective reservation prices, i.e., $\min\left(\Gamma_{A,0}^{i},\ \Gamma_{B,0}^{i}\right)$. Using the definition of $\Gamma_{A,0}^{i}$ and $\Gamma_{B,0}^{i}$, it must be that:

$$\Gamma_{A,0}^{i}=\pi_i^m\theta_i+\left(1-\pi_i^g-\pi_i^m\right)\mu_i\theta_i+c_0^A\left(\mu_i\theta_i-\pi_i^m\theta_i-\left(1-\pi_i^g-\pi_i^m\right)\mu_i\theta_i\right), \tag{A.6}$$

$$\Gamma_{B,0}^{i}=\pi_i^m\theta_i+\left(1-\pi_i^g-\pi_i^m\right)\mu_i\theta_i+c_0^B\left(\mu_i\theta_i-\pi_i^m\theta_i-\left(1-\pi_i^g-\pi_i^m\right)\mu_i\theta_i\right). \tag{A.7}$$

This in turn implies that:

$$\begin{aligned}\min\left(\Gamma_{A,0}^{i},\ \Gamma_{B,0}^{i}\right)&=\pi_i^m\theta_i+\left(1-\pi_i^g-\pi_i^m\right)\mu_i\theta_i\\ &\quad+\min\left(c_0^A,\ c_0^B\right)\left(\mu_i\theta_i-\pi_i^m\theta_i-\left(1-\pi_i^g-\pi_i^m\right)\mu_i\theta_i\right),\end{aligned} \tag{A.8}$$

Now, let $\varsigma_i \equiv \mu_i \pi_i^g + (\mu_i - 1)\pi_i^m$. Notice that $\varsigma_i > 0$ since $\mu_i > 1$. Using (8), (A.1), and (A.8), the following quantity must be positive, i.e.,

$$\begin{aligned} &-\pi_i^g \left(\pi_i^m + (1-\pi_i^g-\pi_i^m)\mu_i + \varsigma_i \min\left(c_0^A,\ c_0^B\right)\right) \\ &+\left[\pi_i^g \mu_i + (1-\pi_i^m)(1-\mu_i) - \varsigma_i \min\left(c_0^A,\ c_0^B\right)\right]\pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) \\ &+\varsigma_i \left\{1-\min\left(c_0^A,\ c_0^B\right)\right\}(1-\pi_i^g-\pi_i^m)\exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right) > 0. \end{aligned} \tag{A.9}$$

Rearranging the inequality regarding (A.9) in terms of $\min\left(c_0^A,\ c_0^B\right)$ yields (9) in the proposition. ■

Proof of Proposition 2. I prove the last two parts of the proposition first, as this requires more careful consideration. Let $\bar{c}_0^i$ denote the insurance threshold in (9), i.e.,

$$\bar{c}_0^i \equiv \frac{\varsigma_i \pi_i^b \exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right) + (\varsigma_i - \mu_i + 1)\pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) + (\varsigma_i - \mu_i)\pi_i^g}{\varsigma_i \pi_i^b \exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right) + \varsigma_i \pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) + \varsigma_i \pi_i^g}. \tag{A.10}$$

Then, using the chain and quotient rules, a tedious algebraic manipulation yields that the sign of $\frac{\partial \bar{c}_0^i}{\partial \mu_i}$ is determined by the sign of:

$$\begin{aligned} &(1-\pi_i^g-\pi_i^m)\left\{\lambda_i \pi_i^g + \gamma\theta_i(\mu_i - 1)\varsigma_i\right\}\exp\left(\frac{\gamma(\mu_i+1)\theta_i}{\lambda_i}\right) \\ &+\pi_i^g(1-\pi_i^g-\pi_i^m)(\lambda_i + \mu_i\varsigma_i)\exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right) + \lambda_i\left(\pi_i^g\right)^2 \\ &+\lambda_i\pi_i^g(\pi_i^m - \pi_i^g)\exp\left(\frac{\gamma\theta_i}{\lambda_i}\right). \end{aligned} \tag{A.11}$$

The first three terms of (A.11) are always positive but the sign of the last term is ambiguous, depending on whether $\pi_i^m > \pi_i^g$ or $\pi_i^m < \pi_i^g$. In particular, for small values of $1-\pi_i^g-\pi_i^m$ and μ_i close to 1, it is conceivable that the last term dominates. Thus, although an increase in μ_i unambiguously raises $\bar{c}_0^i$ when $\pi_i^m > \pi_i^g$, its effect is otherwise ambiguous. This proves part (iii) of the proposition. However, from (A.11), it is immediately possible to deduce that $\frac{\partial\left(\bar{c}_0^i\right)^2}{\partial^2 \mu_i} > 0$, as the first two terms are always increasing in μ_i but the last two terms are unaffected. This proves part (iv) of the proposition.

As for the remaining parts of the proposition, notice that γ, θ_i, and λ_i do not affect $\bar{c}_0^i$ individually but instead in a combination of $\frac{\gamma\theta_i}{\lambda_i}$. Now, let $\varphi_i \equiv \frac{\gamma\theta_i}{\lambda_i}$ then:

$$\bar{c}_0^i \equiv \frac{\varsigma_i \pi_i^b \exp\left(\mu_i \varphi_i\right) + \left(\varsigma_i - \mu_i + 1\right) \pi_i^m \exp\left(\varphi_i\right) + \left(\varsigma_i - \mu_i\right) \pi_i^g}{\varsigma_i \pi_i^b \exp\left(\mu_i \varphi_i\right) + \varsigma_i \pi_i^m \exp\left(\varphi_i\right) + \varsigma_i \pi_i^g}. \tag{A.12}$$

Then, after some algebraic manipulation, the sign of $\frac{\partial \bar{c}_0^i}{\partial \varphi_i}$ is determined by the sign of

$$\pi_i^m \left(1 - \pi_i^g - \pi_i^m\right) \left(\mu_i - 1\right)^2 \exp\left\{\left(\mu_i + 1\right) \varphi_i\right\} + \pi_i^g \left(1 - \pi_i^g - \pi_i^m\right) \left(\mu_i\right)^2 \exp\left(\mu_i \varphi_i\right)$$
$$+\pi_i^g \pi_i^m \exp\left(\varphi_i\right), \tag{A.13}$$

which is always positive. This implies that $\frac{\partial \bar{c}_0^i}{\partial \varphi_i} > 0$, which in turn implies that $\frac{\partial \bar{c}_0^i}{\partial \gamma} > 0$, $\frac{\partial \bar{c}_0^i}{\partial \theta_i} > 0$, and $\frac{\partial \bar{c}_0^i}{\partial \lambda_i} < 0$, as stated in the proposition. ■

Proof of Proposition 3. Consider a bond $B_i \in \{B_1, \ ..., \ B_J\}$ without loss of generality. The proof is obtained from a comparison of the agents' welfare when B_i is insured against when it is issued without bond insurance. I denote the former welfare measure as $\Omega_{0,I}^i$ and the latter as $\Omega_{0,U}^i$.

Suppose first that B_i is insured. Then, the surplus from bond insurance to be divided between the issuer and the insurer is $p_{0,I}^i - l_0^i - \min\left(\Gamma_{A,0}^i, \ \Gamma_{B,0}^i\right)$. Since $p_{0,I}^i = 1$ and the insured bond's par is always guaranteed, it is trivial to show using (4) and (5) that both types of investors in bond B_i receive ex ante utility of -1. Then, it must be that:

$$\Omega_{0,I}^i = p_{0,I}^i - l_0^i - \min\left(\Gamma_{A,0}^i, \ \Gamma_{B,0}^i\right) - 1 = -\left\{l_0^i + \min\left(\Gamma_{A,0}^i, \ \Gamma_{B,0}^i\right)\right\}. \tag{A.14}$$

On the other hand, suppose now that B_i is not insured. If so, its issuance price is given by $p_{0,U}^i$ and only the aggressive investors hold the asset. Then, the issuer earns $p_{0,U}^i - l_0^i$ while the insurer has zero payoff. Since the conservative investors remain outside the market and consume their initial wealth, their ex ante utility is -1. Finally, the aggressive investors' ex ante expected utility ($U_{i,U}^{agg}$) can be computed using (A.5):

$$U_{i,U}^{agg} = -\left(\pi_i^g + \pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) + \left(1 - \pi_i^g - \pi_i^m\right) \exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right)\right) \exp\left(-\frac{\gamma}{\lambda_i}\Delta_0^i\right). \tag{A.15}$$

Using this, $\Omega^i_{0,U}$ may be computed:

$$\Omega^i_{0,N} = p^i_{0,N} - l^i_0 + \lambda_i U^{agg}_{i,U} - (1 - \lambda_i) \,. \tag{A.16}$$

Obviously, bond insurance is strictly preferred from an overall social perspective whenever $\Omega^i_{0,I} > \Omega^i_{0,U}$. Using (A.14) and (A.16), this condition reduces to:

$$\Delta^i_0 - \min\left(\Gamma^i_{A,0},\ \Gamma^i_{B,0}\right) > \lambda_i \left(U^{agg}_{i,U} + 1\right). \tag{A.17}$$

A quick examination of the CARA utility function yields that the value of $U^{agg}_{i,U}$ has an upper bound of 0. Since the aggressive investors' reservation utility from remaining outside the market is -1, it must be that $U^{agg}_{i,U} \in [-1, 0)$. Therefore, it must be that $\lambda_i \left(U^{agg}_{i,U} + 1\right) \in [0,\ \lambda_i)$. However, as it is apparent from (8) that bond insurance occurs whenever

$$\Delta^i_0 - \min\left(\Gamma^i_{A,0},\ \Gamma^i_{B,0}\right) > 0, \tag{A.18}$$

the relative stringency of (A.17) over (A.18) completes the proof. ■

Proof of Proposition 4. The logic behind the derivation remains identical to Proposition 1. Given that all agents initially neglect the tail outcome, an insured bond will be perceived as riskless by all agents. Thus, the insured price of bond B_i under local thinking, denoted $\tilde{p}^i_{0,I}$, will still be 1. Its uninsured price is now determined by a representative aggressive investor j's optimization condition, given by

$$\max_{x^A_{i,t}(j)} -\frac{\pi^g_i}{\pi^g_i + \pi^m_i} \exp\left\{-\gamma\left(1 - \tilde{p}^i_{0,U}\right) x^{agg}_{i,t}(j)\right\} - \frac{\pi^m_i}{\pi^g_i + \pi^m_i} \exp\left\{-\gamma\left(1 - \theta_i - \tilde{p}^i_{0,U}\right) x^{agg}_{i,t}(j)\right\}, \tag{A.19}$$

as well as the market clearing condition. A similar derivation to the proof of Proposition 1 yields the uninsured price of bond B_i as:

$$\tilde{p}^i_{0,U} = \frac{\pi^g_i + (1 - \theta_i)\, \pi^m_i \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right)}{\pi^g_i + \pi^m_i \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right)}, \tag{A.20}$$

which yields the issuer i's reservation price, namely the issuance price differential:

$$\tilde{\Delta}_0^i = \frac{\theta_i \pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right)}{\pi_i^g + \pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right)}. \tag{A.21}$$

Then, using (7), the reservation prices of both insurers under local thinking, denoted $\tilde{\Gamma}_{A,0}^i$ and $\tilde{\Gamma}_{B,0}^i$, are given as follows:

$$\tilde{\Gamma}_{A,0}^i = \frac{\pi_i^m}{\pi_i^g + \pi_i^m}\theta_i + c_0^A \left(\frac{\pi_i^g}{\pi_i^g + \pi_i^m}\theta_i\right), \tag{A.22}$$

$$\tilde{\Gamma}_{B,0}^i = \frac{\pi_i^m}{\pi_i^g + \pi_i^m}\theta_i + c_0^B \left(\frac{\pi_i^g}{\pi_i^g + \pi_i^m}\theta_i\right). \tag{A.23}$$

Since it must be that $\tilde{\Delta}_0^i > \min\left(\tilde{\Gamma}_{A,0}^i, \tilde{\Gamma}_{B,0}^i\right)$, substituting (A.21), (A.22) and (A.23) into this inequality yields the proposition. ■

Proof of Proposition 5. Part (ii) of Proposition 2 has demonstrated that the insurance threshold under the rational benchmark ($\vec{c}_0^i$) would eventually increase in μ_i when μ_i was sufficiently large. As the insurance threshold with neglected tail risk ($\tilde{c}_0^i$) is unaffected by a change in μ_i, eventually it must be that $\vec{c}_0^i > \tilde{c}_0^i$ for some large enough μ_i. ■

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Figure 1: Insurance threshold as a function of model parameters (rational benchmark)

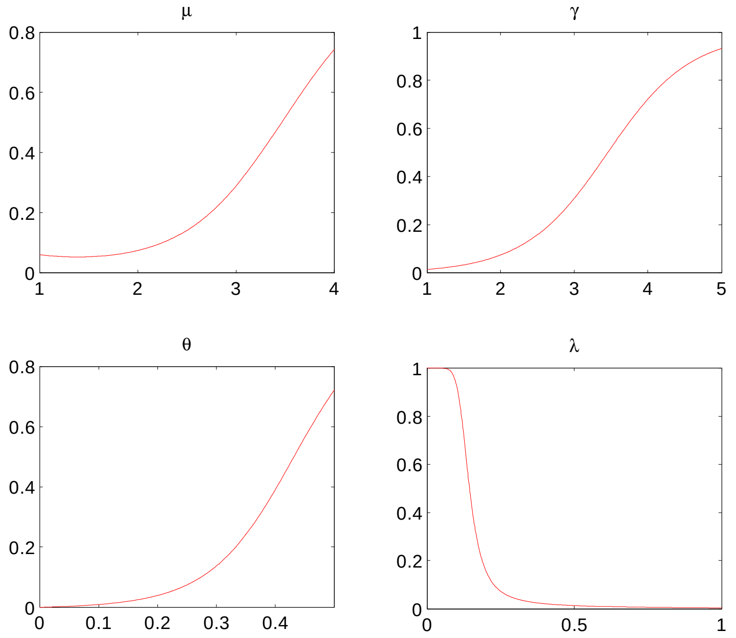

Figure 2: Insurance threshold as a function of μ_i (rational benchmark)

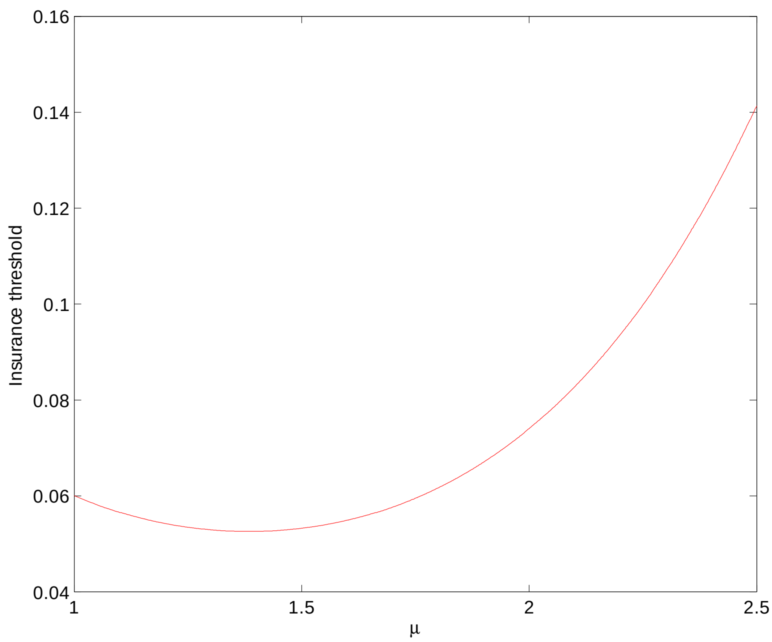

Figure 3: Negative externality of bond insurance as a function of model parameters (rational benchmark)

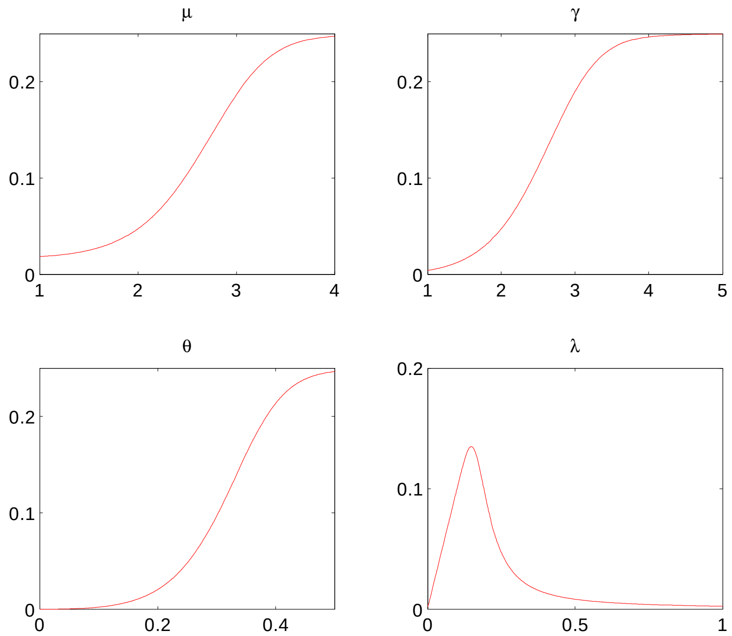

Figure 4: Comparison of insurance thresholds under full rationality and local thinking

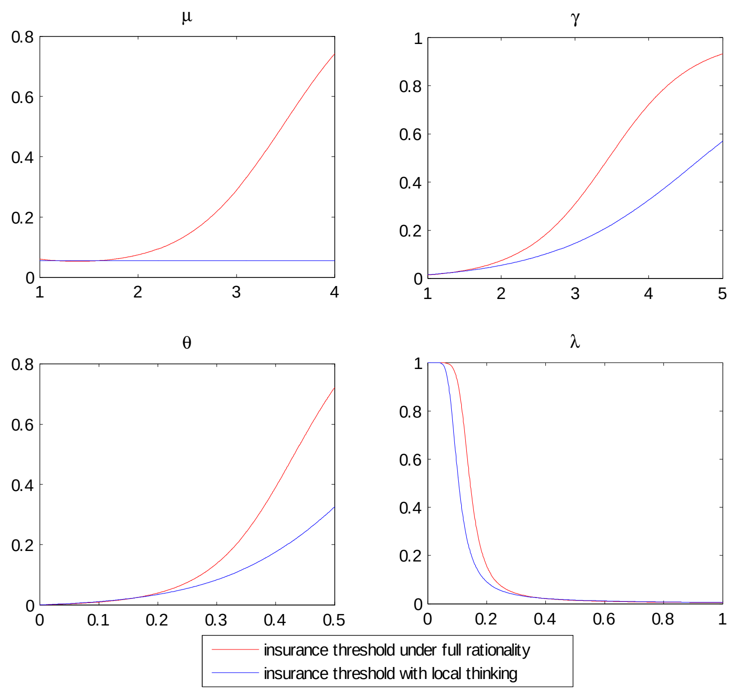

Figure 5: Effect of a change in μ_i on the insurance thresholds

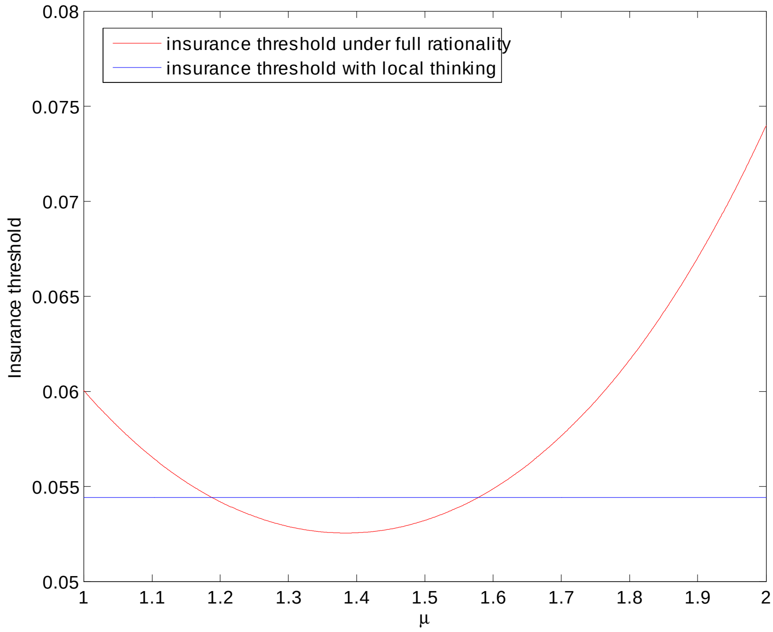

Figure 6: Period 1 price of B_1 and the model parameters (following insurer downgrade)

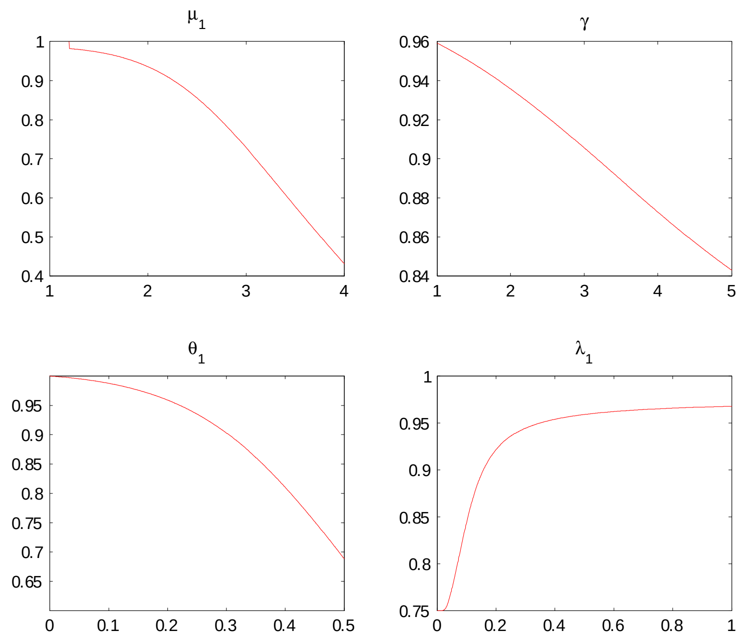

Figure 7: Overall social welfare and the model parameters (following insurer downgrade)