Market-Based Executive Compensation under Asymmetric Information

Guang sug Hahn  
Joon Yeop Kwon

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Abstract

The paper investigates how long term value of publicly trading firm and its stock prices affect managerial compensation under asymmetric information. To do this, we incorporate Grossman and Stiglitz (1980) model into conventional principal-agent problem. We analyze comparative statics of weights on the long term value and stock prices in managerial contract. Each weight is affected by information cost and supply shock. When traders are sufficiently averse to risk, as information cost decreases, the proportion of informed traders increases and this leads to the increases informativeness of stock prices. Then the weight on the long term value decreases and that on the stock prices increases. The changes of supply shock provide the reverse effect to the weights.

Keywords: executive compensation; asymmetric information; information acquisition; price informativeness

JEL Classification: G30, D86
1. Introduction

It is well known that owners of publicly trading firms attempt to monitor the performance of managers by observing stock markets. In a few decades, various studies demonstrate the effects of market based contracts between owners and managers. Holmström (1979) and Holmström and Milgrom (1987) show that increases in liquidity risk make owners offer reduced compensations to managers. Prendergast (2002) and Raith (2003) claim that the increase of uncertainty has influence on managers’s incentives in indirect ways. Market based managerial contracts may incur unexpected problems. Kim and Suh (1993) show that since stock prices contain both public and private information, using the raw price is problematic. According to Goldman and Slezak (2006), managers may find incentive to manipulate private information under market based contracts.

This study examines how the long term value of a publicly trading firm and stock prices affect managerial compensation under asymmetric information in the stock market. To do this, we incorporate Grossman and Stiglitz (1980) model into conventional principal-agent problem such as Holmström and Tirole (1993). It is assumed that before the stocks are traded, the owner of the firm offers contract to a manager and ex ante identical traders choose whether to purchase information about firm’s true value. The firm’s long term value depends on unobservable effort of the manager. In equilibrium, compared to the case where there is no moral hazard problem, the stock price responds less sensitively to firm’s true value and stock supply while proportion of traders who purchase information remains unchanged.

We analyze comparative statics of weights on the firm’s long term value and the stock prices in managerial contract. Each weight is affected by information cost and liquidity shock. When traders are sufficiently averse to risk, as information cost decreases, the manager’s income becomes less sensitive to the long term value and more sensitive to stock prices. In this case, the increase of information cost makes the stock price less informative. Then the owner offers contract, which is less sensitive to stock prices. On the other hand, if supply shock increases, the proportion of informed traders increases and thus price informativeness remains unchanged. However, increases in price sensitivity lead to the increases of weight on firm’s long term value and the decrease of that on stock prices.

Our model is different from the previous literature on market based compensation in two aspects. First, we consider all rational traders participate in trading whether they are informed or uninformed. In Holmström and Tirole (1993), Kang and Liu (2010), and Calcagno and Heider (2014), all rational traders who participate in trading are informed and the others are liquidity traders. In particular, Holmström and Tirole (1993) assume that an informed insider acts as an information monopolist while in our model all rational
traders are ex ante identical and they can purchase information at some costs. Second, our model can explain how the population of informed traders has influence on the managerial contract. It is true that Kang and Liu (2010) consider endogenous information acquisition and attempt to characterize managerial contract by using the number of informed traders. However, their analysis do not find the proportion of informed traders when expected utility of informed and uninformed ones become equivalent. Obviously, Holmström and Tirole (1993) and Calcagno and Heider (2014) are not subject to analyzing the effects of the population of informed traders.

The rest of the paper is organized as follows. In Section 2, we introduce the model of principal agent problem while asymmetric information is present in the stock market. The stock market equilibrium is derived in Section 3. In Section 4, we analyze managerial contract between the owner and the manager. Concluding remarks are given in Section 5. All the proofs are relegated to Appendix.

2. The Model

We consider three periods, indexed $t = 0, 1, 2$. At the initial period, i.e., $t = 0$, a publicly traded firm is established and the firm’s owner hires a manager. The owner offers him an management contract. The true value $\theta$ of the firm consists of managerial effort level $e$ and a factor $\eta$ outside the manager’s control, which is normally distributed with mean zero and variance $\sigma^2_\eta$. Thus the true value $\theta$ has normal distribution with mean $e$ and variance $\sigma^2_\eta$. The stock is issued and traded at time 1. It is assumed that the liquidity $z$ of the stock is normally distributed with mean zero and variance $\sigma^2_z$. At the final period, i.e., $t = 2$, the terminal value $v$ of the firm is realized and the manager is paid. We assume that $v$ is the sum of true value $\theta$ and noise $\varepsilon$: $v = \theta + \varepsilon$ where $\varepsilon$ has normal distribution with mean zero and variance $\sigma^2_\varepsilon$.

2.1. Managerial Contract

Based on Baiman and Verrecchia (1995) and Holmström and Tirole (1993), we assume that there are two performance measures of the manager: the stock price $p$ and the firm’s terminal payoff $v$. The manager’s income is given by

$$I = a_0 + a_1 v + a_2 p,$$

where $a_0$ represents a fixed wage, and $a_1$ and $a_2$ means the sensitivities of the manager’s compensation to $v$ and $p$, respectively. Note that $a_1 v$ means compensation for the firm’s long term value and $a_2 p$ means that for stock prices. The manager is paid $a_0 + a_2 p$ in cash.
and \( a_t v \) is paid in the stock. We assume that the manager chooses his effort level \( e \) at time 0 and has CARA utility function with absolute risk aversion coefficient \( \tau : u_m(w) = -\exp^{-\tau w} \). It is also assumed that the manager is barred from trading. This assumption reflects real world where managers are subject to laws and restriction about stock trading.

2.2. Stock Market and Traders

There are two stocks: a risky stock and a risk-free bond. At period 1, the price of the risky stock and the bond are given by \( p \) and 1, respectively. A trader \( t \) invests his initial wealth \( w_t \) between \( x_t \) shares of the risky stock and \( b_t \) shares of the bond with the budget constraint \( b_t + px_t = w_t \). At the end of the period, i.e., \( t = 2 \), the risky stock gives random payoff \( \tilde{v} - (a_0 + a_1 \tilde{v}) \). The bond gives deterministic payoff 1. Thus his portfolio \((x_t, b_t)\) yields wealth \( w'_t = w_t + (\tilde{v} - (a_0 + a_1 \tilde{v}) - p)x_t \).

There is a continuum of traders denoted by interval \([0, 1]\). All traders are ex ante uninformed and they should decide whether to purchase information about \( \theta \) by paying \( c \) at \( t = 0 \). If the expected utility of informed traders is higher than that of uninformed traders, traders pay \( c \) for this information. If not, they remain uninformed. Thus in equilibrium, both expected utilities should be equal. Informed traders observe realization \((p, \theta)\) of \((\tilde{p}, \tilde{\theta})\) at cost \( c \), while uninformed traders only observe \( p \). Let \( \lambda \in [0, 1] \) denote the fractions of informed. It is assumed that all traders have rational expectations so that they understand the functional relationship \( \tilde{p} \) between \( p \) and \((\theta, z)\) and have CARA utility with the constant degree of risk aversion \( \gamma > 0 \): \( u(w) = -\exp(-\gamma w) \).

The whole process is illustrated in Figure 1 below.

3. Equilibrium

For the optimal portfolio choice, informed trader \( i \) with initial wealth \( w_i \) solves

\[
\max_{x_i} \mathbb{E}[-\exp(-\gamma[w_i + (\tilde{v} - (a_0 + a_1 \tilde{v}) - p)x_i]) | (\tilde{p}, \tilde{\theta}) = (p, \theta)]
\]
and his demand for the stock is given by
\[ x_i = \frac{\theta - (a_0 + (1 + a_2)p)}{\gamma \sigma^2_e}. \]

Similarly, uninformed trader \( u \) with initial wealth \( w_u \) solves
\[ \max_{x_u} x_u \mathbb{E}[v - (a_0 + a_2p) - p|p] - \frac{\gamma}{2} x_u^2 \text{Var}[v - I - p|p] \]
and his demand for the stock is given by
\[ x_u = \frac{\mathbb{E}[v|p] - (a_0 + (1 + a_2)p)}{\gamma \text{Var}[v|p]}. \]

We adopt the notion of rational expectations equilibrium in Grossman and Stiglitz (1980). A rational expectations equilibrium stock price function \( \hat{p} \) satisfies the market clearing condition: for every \( p = \hat{p}(\theta, z) \),
\[ \lambda x_i(p, \theta) + (1 - \lambda) x_u(p, \hat{p}) = z. \]

Following Grossman and Stiglitz (1980), we define a compound signal function \( \hat{s} : (\theta, z) \mapsto s \), which encapsulates \( \theta \) and \( z \):
\[ \hat{s}(\theta, z) = \begin{cases} \theta - \frac{\gamma}{2} \sigma^2_e & \text{if } \lambda \in (0, 1], \\ z & \text{if } \lambda = 0. \end{cases} \]

Clearly, \( \hat{s} \) is normally distributed with mean \( \mu \) and variance \( \sigma^2_s = \sigma^2_n + \gamma^2 \sigma^4_n \sigma^2_z / \lambda^2 \). We define equilibrium stock price function \( P : \mathbb{R} \mapsto \mathbb{R} \) by \( \hat{p}(\theta, z) = P(\hat{s}(\theta, z)) \) and conjecture that \( P \) strictly increases in signal \( s \), which is verified by Proposition 3.1 below.

**Proposition 3.1.** For a given \( \lambda \in [0, 1] \), we obtain equilibrium stock price function \( P \), which is given by
\[ P(s) = \frac{1}{1 + a_2} [(1 - \alpha)e + \alpha s] - \frac{a_0}{1 + a_2} \]
where
\[ \alpha = \frac{\lambda (\lambda \sigma^2_n + \gamma^2 \sigma^2_n \sigma^2_z + \gamma^2 \sigma^4_n \sigma^2_z)}{\lambda^2 \sigma^2_n + \lambda \gamma^2 \sigma^2_n \sigma^2_z + \gamma^2 \sigma^4_n \sigma^2_z}. \]

Note that in the absence of this moral hazard problem, the stock price function becomes \( P_0 = (1 - \alpha)e + \alpha s \). We call \( \alpha \) pure price sensitivity to \( s \). Thus the stock price becomes less sensitive to \( s \) due to the manager’s moral hazard.

Let \( \nu_i \) denote the wealth of informed traders and \( \nu_u \) denote that of uninformed traders at \( t = 3 \). To find overall equilibrium, we find \( \mathbb{E}[u(\nu_i)] / \mathbb{E}[u(\nu_u)] \).

\[ ^1 \text{We verify that } \alpha_2 > 0 \text{ in Section 4.} \]
Proposition 3.2. The ratio of expected utility between informed and uninform ed ones is given by

\[
\frac{\mathbb{E}[u(\nu_i)]}{\mathbb{E}[u(\nu_u)]} = e^{\gamma c} \sqrt{\frac{\text{Var}[v|\theta]}{\text{Var}[v|s]}} \equiv \phi(\lambda).
\]

It is clear that if \( \phi(0) > 1 \), no traders become informed (i.e., \( \lambda = 0 \)) and if \( \phi(1) < 1 \), all traders become uninformed (i.e., \( \lambda = 1 \)). Moreover, if \( \phi(1) = 1 \), a part of traders become informed. (i.e., \( \lambda \in (0, 1) \)). Note that \( \phi \) is a strictly increasing function of \( \lambda \).

Let us define

\[
m = \frac{\gamma^2 \sigma_\lambda^4 \sigma_\varepsilon^2}{\lambda^2 \sigma_\theta^2 \sigma_\varepsilon^2} \quad \text{and} \quad n = \frac{\sigma_\eta^2}{\sigma_\varepsilon^2}.
\]

Note that squared correlation \( \rho_{\theta,p}^2 \) between \( \theta \) and \( p \) is given by \( 1/(1 + m) \) and squared correlation coefficient \( \rho_{\theta,v}^2 \) between \( \theta \) and \( v \) is given by \( n/(1 + n) \). Thus the former can be interpreted as the informativeness of stock prices and the latter can be interpreted as information quality of \( \theta \). Suppose that informed traders and uninformed traders coexist (i.e., \( \lambda \in (0, 1) \)). Then relationship

\[
m = \frac{e^{2ac} - 1}{1 + n - e^{2\gamma c}}
\]

holds in equilibrium. Since \( m \) and \( n \) are not the function of contract variables (i.e., \( a_0, a_1, \) and \( a_2 \)), the equilibrium proportion of informed traders is irrespective of the presence of the moral hazard problem. From (3.1), we consider two ways to affect the proportion \( \lambda \) of informed traders. The first is changing the information cost \( c \). If information cost \( c \) increases, \( m \) should increase and then the proportion \( \lambda \) of informed decreases. The second is changing liquidity shock \( \sigma_z^2 \). Note that the right hand side of (3.1) is not the function of \( \lambda \) and \( \sigma_z^2 \). Thus, the increase of \( \sigma_z^2 \) leads to the increase of \( \lambda \) without changing price informativeness.

4. The Manager’s Contract

This section is devoted to analyze the incentive contract between the owner and the manager. It is assumed that the reservation value of the manger equals to zero. Following

\[ ^2 \text{We omit the proof of Proposition 3.2 since it is similar to that of Grossman and Stiglitz (1980).} \]

\[ ^3 \text{See Grossman and Stiglitz (1980).} \]
Holmström and Tirole (1993), we set up the principal’s problem as follows:

$$\max_{a_0, a_1, a_2, e} \mathbb{E}[v - I]$$

s.t. $$\mathbb{E}[I] - \frac{\tau}{2} \text{Var}[I] - \frac{1}{2} ke^2 \geq 0,$$

$$e = \arg \max_{e'} \mathbb{E}[I] - \frac{\tau}{2} \text{Var}[I] - \frac{1}{2} ke'^2.$$

**Proposition 4.1.** In equilibrium, compensation contract is given by

$$a_1 = \frac{\alpha^2(1 + m)}{\sigma^2 + \sigma^2(1 + m)(1 + k\tau(\sigma^2 + \sigma^2))},$$

$$a_2 = \frac{\sigma^2 + \sigma^2}{\alpha^2(1 + m)(1 + k\tau(\sigma^2 + \sigma^2))}.$$

Since $a_1 > 0$ and $a_2 > 0$, the stock price $P$ becomes less sensitive to $s$ with the manager’s moral hazard problem than without it. In Holmström and Milgrom (1987) and Kang and Liu (2010), coefficient $a_1$ of firm’s future value $v$ is less than zero. In the literature, owners believe that high $v$ is due to high exogenous factors and the effort levels of managers are low. However, our model shows that the compensation of the manager increases in both $a_1$ and $a_2$. Clearly, as the manager exhibits higher degree of risk aversion, his expected payoff decreases. It is noted that $a_1$ increases in $\alpha^2(1 + m)$ and $a_2$ decreases in $\alpha^2(1 + m)$. This implies that as pure price sensitivity increases or stock price informativeness decreases, the manager’s compensation responds to the firm’s long term value $v$ more sensitively but the stock price $p$ less sensitively.

**Proposition 4.2.** Suppose traders’ degree of risk aversion is sufficiently high such that

$$\gamma > \frac{1}{\sqrt{\sigma^2 \sigma^2}}. \quad (4.1)$$

As information cost $c$ decreases, $a_1$ decreases and $a_2$ increases.

As we have seen in Section 3, if information cost $c$ decreases, the proportion $\lambda$ of informed traders increases and then the informativeness of stock price increases (i.e., $m$ decreases). On the other hand, the degree $\gamma$ of risk aversion increases in $\lambda$ since

$$\frac{\partial \alpha}{\partial \lambda} = \frac{[(2 - \lambda)\lambda \sigma^2 + \gamma^2 \sigma^2 \gamma^2 \sigma^2 + \gamma^2 \sigma^2 \gamma^2 \sigma^2]}{(\lambda^2 \sigma^2 + \lambda \gamma^2 \sigma^2 \gamma^2 \sigma^2 + \gamma^2 \sigma^2 \gamma^2 \sigma^2)^2} > 0.$$
Proposition 4.3. As liquidity shock $\sigma_z^2$ increases, $a_1$ increases and $a_2$ decreases.

In equilibrium, if liquidity shock $\sigma_z^2$ increases, the proportion $\lambda$ of informed traders also increases. Then the informativeness of the stock price remains unchanged. However, since pure price sensitivity $\alpha$ increases in $\sigma_z^2$, the manager's compensation becomes more sensitive to $v$ and less sensitive to $p$.

From Proposition 4.2 and Proposition 4.3, we observe that the increase of informed traders' population provides conflicting effects on the manager's compensation. Hence the proportion of informed trader cannot be an indicator itself for the manager's compensation. Depending on the sources which lead to changes of $\lambda$, the manager's compensation differently responds to the firm's long term value and the stock price.

5. Concluding Remarks

The paper investigates the effects of market based compensation in managerial contract by combining conventional principal-agent problem and Grossman and Stiglitz (1980) model. We analyze comparative statics of weights on long term value of a publicly trading firm and stock prices in managerial contract. Each weight is affected by information cost and supply shock. When traders are sufficiently averse to risk, as information cost decreases, the proportion of informed traders increases and this lead to the increases informativeness of stock prices. Then the weight on the long term value decreases and that on the stock prices increases. Consequently, the owner offers contract, which is less sensitive to stock prices. If the degree of supply shock increases, the proportion of informed traders increases and thus price informativeness remains unchanged. However, increases in price sensitivity leads to the increase of weight on firms long term value and the decrease of that on stock prices. Topics of future research may include ambiguous information to examine how the manager manipulates stock market information.

Appendix

Proof of Proposition 3.1. We conjecture that $P$ is a strictly increasing function of $s$. Then information generated by the equilibrium price $p$ is equivalent to that by $s$. Since $\tilde{s}$ and $\tilde{v}$ are normally distributed, we have

$$E[\tilde{v}|\tilde{p} = p] = E[\tilde{v}|\tilde{s} = s] = \frac{\gamma^2\sigma_1\sigma_2\varepsilon}{\lambda^2\sigma_z^2 + \gamma^2\sigma_1^4\sigma_z^2},$$

$$\text{Var}[\tilde{v}|\tilde{p} = p] = \text{Var}[\tilde{v}|\tilde{s} = s] = \frac{\sigma_z^2(\lambda^2\sigma_1^2 + \gamma^2\sigma_2^2\sigma_z^2 + \gamma^2\sigma_1^4\sigma_z^2)}{\lambda^2\sigma_z^2 + \gamma^2\sigma_1^4\sigma_z^2}.$$
By a simple calculation, we obtain

\[ P(s) = \frac{1}{1 + a_2} \left[ ((1 - \alpha)e + \alpha s) - a_0 \right] \]

where

\[ \alpha = \frac{\lambda (\lambda^2 + \gamma^2 \sigma_n^2 \sigma_z^2 + \gamma^2 \sigma_e^4 \sigma_z^2)}{\lambda^2 + \lambda^2 \gamma^2 \sigma_n^2 \sigma_z^2 + \gamma^2 \sigma_e^4 \sigma_z^2}. \]

PROOF OF PROPOSITION 4.1. Certainty equivalent measure of the manager is given by

\[ \mathbb{E}[I] - \frac{\tau}{2} \mathbb{V}[I] - \frac{1}{2} ke^2. \]

We have

\[ \mathbb{E}[I] = a_0 + a_1 e + \frac{e - a_0}{1 + a_2} a_2, \]
\[ \mathbb{V}[I] = a_1^2 \left( \sigma_n^2 + \sigma_e^2 \right) + \frac{a_2^2}{(1 + a_2)^2} \alpha^2 \sigma_z^2. \]

The first order condition of the manager’s problem is given by

\[ a_1 + \frac{a_2}{1 + a_2} - ke = 0 \]

and thus

\[ e = \frac{1}{k} \left( a_1 + \frac{a_2}{1 + a_2} \right). \]

We rewrite objective function of the principal as follows:

\[ \mathbb{E}[v - I] = \frac{1}{k} \left( a_1 + \frac{a_2}{1 + a_2} \right) - \frac{\tau}{2} \left( a_1^2 \left( \sigma_n^2 + \sigma_e^2 \right) + \frac{a_2^2}{(1 + a_2)^2} \alpha^2 \sigma_z^2 \right) - \frac{1}{2k} \left( a_1 + \frac{a_2}{1 + a_2} \right)^2. \]

First order conditions of the firm’s owner are given by

\[ \frac{1 - (1 + a_2)a_1(1 + k\tau(\sigma_n^2 + \sigma_e^2))}{k(1 + a_2)} = 0 \]
\[ \frac{1 - k\tau a_2 \alpha^2 \sigma_z^2 - a_1(1 + a_2)}{k(1 + a_2)^3} = 0. \]

Therefore, we have

\[ a_1 = \frac{\alpha^2 \sigma_z^2}{\sigma_n^2 + \sigma_e^2 + \alpha^2 \sigma_z^2 (1 + k\tau(\sigma_n^2 + \sigma_e^2))}, \]
\[ a_2 = \frac{\sigma_n^2 + \sigma_e^2}{\alpha^2 \sigma_z^2 (1 + k\tau(\sigma_n^2 + \sigma_e^2))}. \]
PROOF OF PROPOSITION 4.2. If $c$ decreases, $\lambda$ increases by (3.1). Note that

$$\frac{\partial(a^2(1+ m))}{\partial \lambda} = \frac{2\gamma^2 \sigma^4 \sigma^2_2 (\lambda \sigma^2_2 + \gamma^2 \sigma^2_2 \sigma^2_2 + \gamma^2 \sigma^2_2 \sigma^2_2 \lambda)}{(\lambda^2 \sigma^2_2 + \lambda \gamma^2 \sigma^2_2 \sigma^2_2 + \gamma^2 \sigma^2_2 \sigma^2_2)^3} \beta$$

where

$$\beta = (1 - \lambda)(\lambda^2 \sigma^2_2 + \gamma^2 \sigma^2_2 \sigma^2_2) - \lambda \gamma^2 \sigma^2_2 \sigma^2_2 \sigma^2_2 - \gamma^4 \sigma^4_2 \sigma^2_2 (\sigma^2_2 + \sigma^2_2).$$

If (4.1) holds, then $\beta < 0$ and thus $a^2(1+ m)$ decreases in $\lambda$. Then $a_1$ decreases in $\lambda$ and $a_2$ increases in $\lambda$.

PROOF OF PROPOSITION 4.3. From (3.1), we know that the increases of $\sigma^2_z$ leads to the increase of $\lambda$ while $\lambda^2/\sigma^2_z$ remains unchanged. Let $\lambda/\sigma_z$ be denoted by $\delta$. Then $a^2(1+ m)$ can be rewritten as

$$\frac{(\sigma^2_2 \delta^2 + \gamma^2 \sigma^2_2)}{(\gamma^2 \sigma^2_2 + \sigma^2_2 \delta^2 + \gamma^2 \sigma^2_2 \delta^2)}.$$ 

Then we obtain

$$\frac{\partial(a^2(1+ m))}{\partial \sigma_z} = \frac{2(\sigma^2_2 \delta^2 + \gamma^2 \sigma^2_2)(\sigma^2_2 \delta^2 + \gamma^2 \sigma^2_2 \delta^2 + \gamma^2 \sigma^2_2 \delta^2)}{(\sigma^2_2 \delta^2 + \gamma^2 \sigma^2_2 \delta^2 + \gamma^2 \sigma^2_2 \delta^2)^3} > 0.$$

Therefore, $a_1$ increases in $\sigma^2_z$ and $a_2$ decreases in $\sigma^2_z$.

References


